

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Problem Set No. 5
Fall Term 2006

6.630 Electromagnetics

Issued: Week 5
Due: Week 6

Reading assignment: Section 1.6-1.7, 2.1; J. A. Kong, “*Electromagnetic Wave Theory*,”.
Additional materials: *Waves in media* MATLAB scripts in tools section.

Quiz 1: October 19, Thursday, Open 1 book.

Problem P5.1

A plane wave is propagating in a homogeneous medium with the permittivity ϵ and permeability μ . The magnetic field is

$$\overline{H}_1 = \frac{1}{\omega\mu} \left(-\frac{\sqrt{3}}{2} k \hat{y} + A k \hat{x} \right) \cos\left(\frac{\sqrt{3}}{2} kx + \frac{1}{2} ky - \omega t \right)$$

- (a) Write out the \overline{k} vector and prove $A = \frac{1}{2}$.
- (b) What is the electric field \overline{E}_1 ?
- (c) Let the half space $y > 0$ be filled with another material with the permittivity $\frac{1}{2}\epsilon$ and the permeability $\frac{3}{2}\mu$. The wave \overline{H}_1 is incident on the boundary $y = 0$ from the half space $y < 0$.
 - (i) Is the incident wave a TE or a TM wave?
 - (ii) What is the y component of the transmitted wave vector (k_{ty})?
 - (iii) What is the reflection coefficient?
- (d) Let $\overline{E} = \overline{E}_1 + \overline{E}_2$. Write \overline{E}_2 so that \overline{E} is a right hand circularly polarized wave.

Problem P5.2

Compare the phenomena of total reflection for $\theta > \theta_C$ and total transmission for $\theta = \theta_B$ at an isotropic dielectric interface.

- (a) Total reflection occurs at a range of incident angles larger than the critical angle θ_C ; total transmission of TM waves occurs only at the Brewster angle θ_B .
- (b) Total reflection occurs only when the incident medium is denser than the transmitted medium. The Brewster angle occurs for any two media.
- (c) When an unpolarized wave is totally reflected, the reflected wave is still unpolarized. When the TM wave components of an unpolarized wave are totally transmitted, the reflected wave contains only TE waves.

Suppose a TM wave is incident at an angle θ such that $\theta = \theta_B > \theta_C$. Then the wave is totally transmitted and at the same time it is totally reflected. Is that possible? Explain.

Problem P5.3

Consider a plane wave incident from a dielectric region with permittivity $\epsilon = 3\epsilon_o$ upon a halfspace with $\epsilon = \epsilon_o$ as shown in Figure 1.

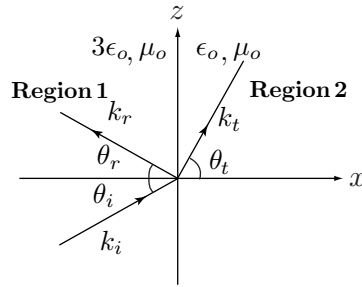


Fig.1

- (a) Find the Brewster angle for Region 1.
 (b) Suppose the transmitted electric field is given by

$$\vec{E}_t = \hat{y} \frac{E_0}{\sqrt{2}} \cos(k_{tx}x + k_{tz}z - \omega t) + E_0 \frac{\hat{z} - \hat{x}\sqrt{3}}{2\sqrt{2}} \sin(k_{tx}x + k_{tz}z - \omega t)$$

- (i) Determine the incident and transmitted angles, θ_i and θ_t .
 (ii) What is the polarization of the transmitted field? Be sure to specify the handedness (left or right) if necessary.
 (iii) What is the polarization of the reflected wave? Be sure to specify the handedness (left or right) if necessary.
 (iv) Give an expression for the incident electric field, \vec{E}_i , and the reflected electric field, \vec{E}_r .