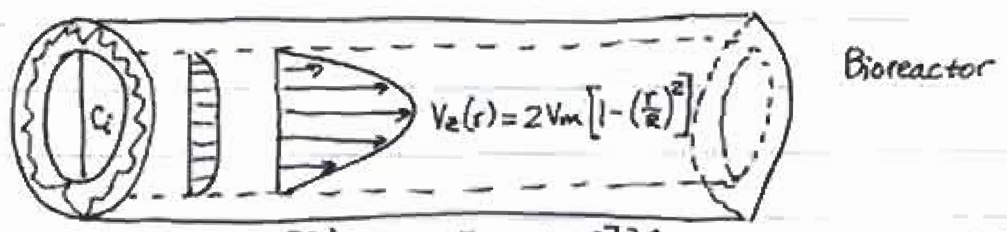
11/24/04



$$0 = D\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c}{\partial r}\right) - 2V_m\left[1-\left(\frac{r}{R}\right)^2\right]\frac{\partial c}{\partial z}$$

@ $z=0$, $C(r)=C_i$

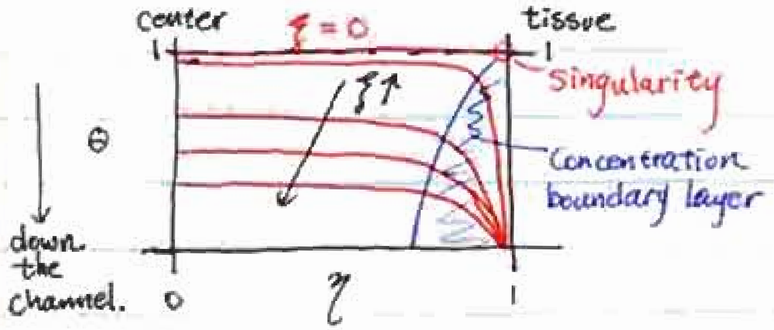
$r=0$, $\frac{\partial c}{\partial r}$

$r=R$, $C=0$.

Solution: $C(r,z) = C_i\,\Theta\left(\frac{r}{R}, \frac{z}{L}\right)$, with $\eta = \frac{r}{R}$, $\xi = \frac{z}{L}$

$$= \sum_{j=1}^{\infty}\left[\int_0^1 \Phi_j(\eta)(1-\eta^2)\,\eta\,d\eta\right]\Phi_j(\eta)\,e^{-\frac{1}{Pe}\lambda_j^2\xi}$$

$\Phi_j(\eta)$, λ_j determined from corresponding E.V. Eqn.



Objective: design to ensure adequate flux of nutrient into tissue throughout channel

Flux @ wall $N_r|_R(\xi) = -D\frac{\partial c}{\partial r}\Big|_R$ — proportional to slope in boundary layer.

moles/area-time — Can calculate from $C(r,z)$

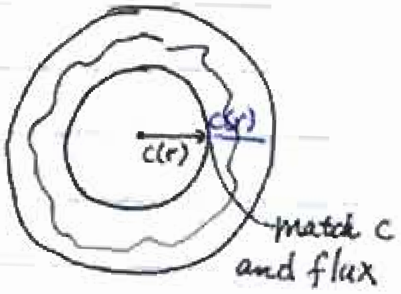
Might desire

$N_r|_R(\xi) \geq n\,\delta\,\mu$

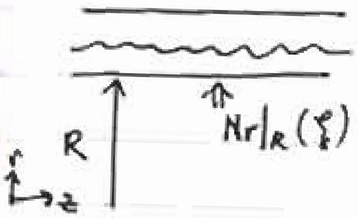
δ: thickness of tissue region

n: cells/vol in tissue region

μ: metabolic uptake rate moles/cell-time cross-section



Offer an alternative perspective
- "mass transfer coefficient"
concept.



avg conc. in bulk fluid [moles/ml]

Define: $N_r|_R(\xi) = \boxed{k_c}(C_{bulk} - C_{wall}) \neq 0$ in our problem

mass transfer coefficient. [distance/time]

Conc. @ wall [moles/ml]

$$\int_0^R 2\pi V_z(r) \frac{\partial c}{\partial z} r\,dr = \int \frac{D}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c}{\partial r}\right) 2\pi r\,dr$$

Take integral of both sides over a cylindrical shell in channel.

Obtain LHS $\frac{d}{dz}\int_0^R V_z(r) + C(r)\,dr = \frac{1}{2}R^2 V_m \frac{dC_{Bulk}}{dz}$

$$C_{Bulk} = \frac{\int_0^R c(r)V_z(r)\,r\,dr}{\int_0^R V_z(r)\,r\,dr}$$ "velocity weighted" - bulk concentration

RHS: $\int_0^R D\frac{\partial}{\partial r}\left(r\frac{\partial c}{\partial r}\right)dr = Dr\frac{\partial c}{\partial r}\Big|_0^R = DR\frac{\partial c}{\partial r}\Big|_R = -RN_r|_R(z)$

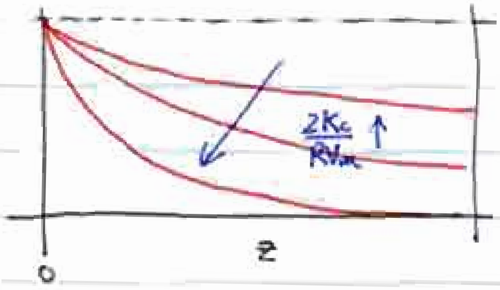
So, $\frac{1}{2}R^2V_m\frac{dC_B}{dz} = -R\left[k_c(C_B - C_w)\right]$. // Assume $k_c \neq k_c(z)$

$\Rightarrow \frac{dC_{Bulk}}{dz} = \frac{-2k_c}{RV_m}(C_B - C_w)$ $z=0,\ C_{Bulk} = C_i$

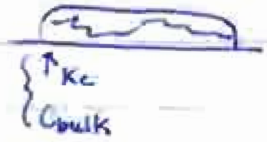
Solution: $\frac{C_{Bulk} - C_w}{C_i - C_w} = \exp\left\{-\frac{2k_c}{RV_m}z\right\}$.   ~~if $C_w = 0$~~

then $C_{Bulk}(z) = C_i e^{\frac{-2k_c}{RV_m}z}$

if $C_w = 0$ $-\frac{2k_c}{RV_m}z$

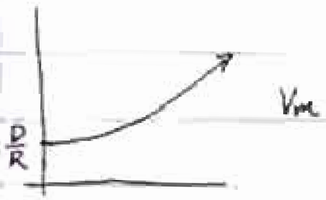


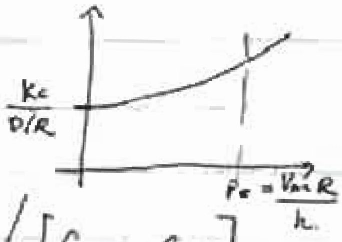
But, what is K_c?



distance/time ; distance²/time ; distance

If $V_m = 0$, have only diffusion, then $K_c = \frac{D}{R}$; $N_r = \frac{D\Delta c}{R}$





How to ascertain K_c under any condition.

Formally, Sherwood number : $Sh \equiv \frac{K_c}{D/R} = -\frac{\partial c}{\partial r}\Big|_{wall} \Big/ \left[C_{bulk} - C_{wall}\right]$

How to determine Sh?

a) Theory

b) Computation - model (yield a number from numerical solution)

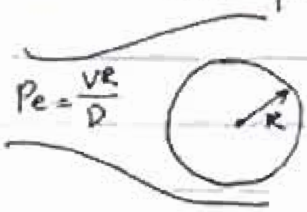
c) Experimental measurements - correlations w/ system parameters.

See sections 9.3 in Deen's text

Examples of a) Theory - problem that we did on Monday (Graetz problem)

$\Rightarrow Sh = \frac{K_c}{D/R} \approx \frac{1}{2}\lambda_1^2 = 3.7$ ← smallest eigenvalue: as $z \to \infty$

Flow around sphere



$Sh = \frac{k_c}{V/2R}$ $Pe \ll 1$