Simple derivation of Michaelis-Menten enzyme kinetic rate expression

$$E + S \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} (ES) \xrightarrow{k_2} E + P$$

Mass conservation eqns

$$\frac{dS}{dt} = -k_1 E S + k_{-1}(ES)$$

$$\frac{dP}{dt} = k_2 (ES)$$

$$\frac{d(ES)}{dt} = k_1 ES - k_{-1}(ES) - k_2 (ES)$$

$$\frac{dE}{dt} = -k_1 ES + k_{-1}(ES) + k_2 (ES)$$

Sum (ES) + E eqns

$$\frac{d}{dt}[(ES) + E] = 0 \quad \Rightarrow \quad E + (ES) = E_{Total}$$

Apply quasi-steady-state assumption

$$\frac{d[ES]}{dt} \ll \frac{dS}{dt}, \frac{dP}{dt}$$, generally justified by $E_t \ll S$

$$\Rightarrow \frac{d(ES)}{dt} \approx 0$$

$$\therefore \quad 0 = k_1 \left[E_T - (ES) \right] S - (k_{-1} + k_2)(ES)$$

$$\Rightarrow \quad (ES) = \frac{E_T S}{\left[\frac{k_{-1} + k_2}{k_1} \right] + S}$$

$$\therefore \quad \frac{dP}{dt} = -\frac{dS}{dt} = \frac{[k_2 E_T] S}{\left[\frac{k_{-1} + k_2}{k_1} \right] + S}$$

$$= \frac{V_{max} \; S}{K_m + S}$$

V_{max} (maximum reaction velocity) $\equiv k_2 E_T$

K_m (Michaelis-Menten constant) $\equiv \dfrac{k_{-1} + k_2}{k_1}$

This expression is often valid for *in vitro* situations, but less often for *in vivo*; in the latter, the substrate concentration may not necessarily overwhelm the enzyme concentration.

Another caveat is that even when it is valid for individual reactions, if these reactions are coupled it can become invalid.