# 6.441 Transmission of Information
# Problem Set 2

Spring 2010
Due date: February 23

**Problem 1** Two semi-working street lamps turn on and off independently as follows: within each one-minute interval, a lamp that is on turns off with probability $p$, and a lamp that is off turns on with probability $p$. At time $t = 0, 1, 2 \ldots$ minutes, an observer records the number $N_t$ of street lamps that are on, as well as the change $D_t = N_t - N_{t-1}$ from the previous recorded number.

(a) Do $N_0, N_1, \ldots$ form a Markov process? What is the entropy rate of this sequence?

(b) Do $D_0, D_1, \ldots$ form a Markov process? What is the entropy rate of this sequence?

**Problem 2** Problem 3.6 in Cover and Thomas (first edition), or 3.10 in Cover and Thomas (second edition).

**Problem 3** Consider a sequence of IID binary r.v.s $A_0, A_1, \ldots$ such that $A_i = 0$ with probability $\xi$ and $A_i = 1$ with probability $1 - \xi$ for some $0 < \xi < 1$. Consider another sequence of IID quaternary r.v.s $\Xi_0, \Xi_1, \ldots$ such that $\Xi_i = 0$ with probability $\frac{1-\theta}{3}$, $\Xi_i = 1$ with probability $\frac{1-\theta}{3}$, $\Xi_i = 2$ with probability $\frac{1-\theta}{3}$, $\Xi_i = 3$ with probability $\theta$ for some $0 < \theta < 1$ . The $\Xi_i$s and the $A_i$s are all mutually independent. Consider a sequence of quaternary r.v.s $Z_0, Z_1, \ldots$ such that $\forall i > 0$

$$Z_i = A_i(\Xi_{i-1} \oplus Z_{i-1}) \oplus \overline{A_i}\Xi_{i-1}$$

and $Z_0$, $\Xi_0$ are IID, where $\oplus$ denotes addition mod 4.

a) What is $H(Z_i|Z_{i-1})$?

b) What is $H(Z_i|Z_{i-j})$?

c) Can you find some form of the AEP that holds for the r.v.s $Z_0, Z_1, \ldots$?

**Problem 4** Problem 4.1 in Cover and Thomas (first or second edition).