Lecture 27 - The "Long" Metal-Oxide-Semiconductor Field-Effect Transistor *(cont.)*

April 13, 2007

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1. Charge-voltage characteristics of ideal MOSFET *(cont.)*
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Reading assignment:

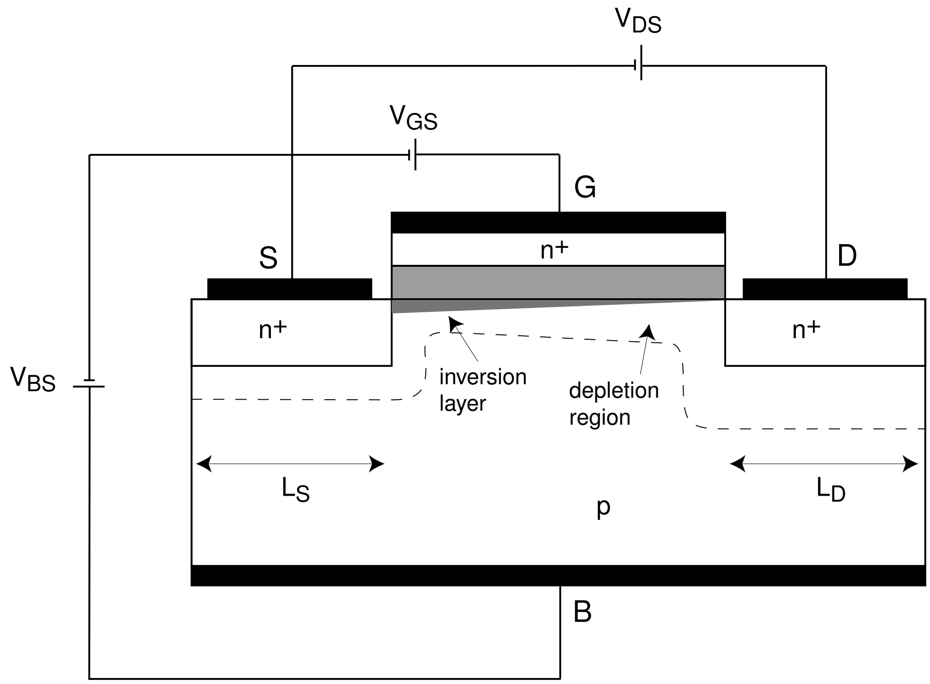
del Alamo, Ch. 9, §§9.5 (9.5.2), 9.6

Key questions

- What are the capacitances associated with the inversion layer charge?
- What is the topology of a small-signal equivalent circuit model for the MOSFET?
- What are the key bias and geometry dependencies of all small-signal elements in the model?
- How does one characterize the frequency response of a transistor?

1. Charge-voltage characteristics of ideal MOSFET *(cont.)*

□ Inversion charge:



For $V_{GS} > V_T$:

$$Q_I = W \int_0^L Q_i(y) dy$$

Change variables to V:

$$Q_I = W \int_0^{V_{DS}} Q_i(V) \frac{dy}{dV} dV$$

$$Q_I = W \int_0^{V_{DS}} Q_i(V) \frac{dy}{dV} dV$$

From channel current equation, we have:

$$\frac{dy}{dV}|_V = -\frac{W\mu_e}{I_D} Q_i(V)$$

Then:

$$Q_I = -\frac{W^2\mu_e}{I_D} \int_0^{V_{DS}} Q_i^2(V) dV$$

Now use charge-control relationship:

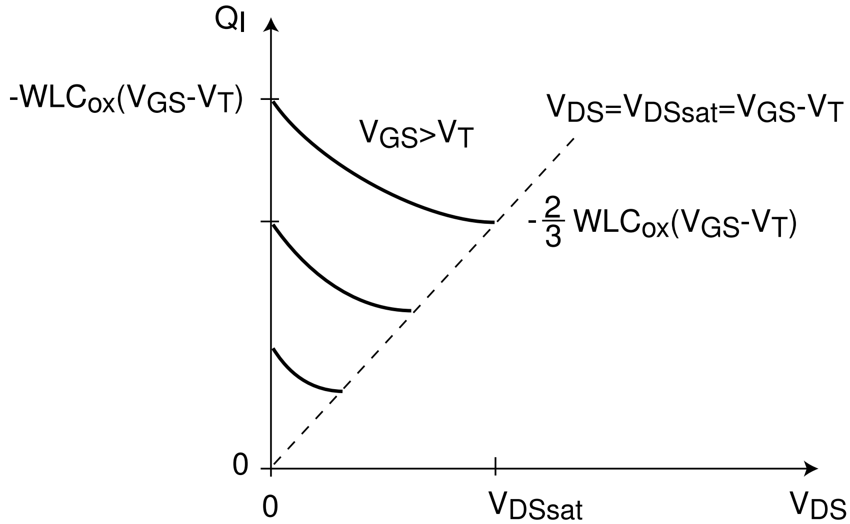
$$Q_i(V) = -C_{ox}(V_{GS} - V - V_T)$$

Finally get:

$$Q_I = -\frac{2}{3} WLC_{ox} \frac{(V_{GS} - V_T)^2 + (V_{GS} - V_T)(V_{GD} - V_T) + (V_{GD} - V_T)^2}{(V_{GS} - V_T) + (V_{GD} - V_T)}$$

$$Q_I = -\frac{2}{3}WLC_{ox}\frac{(V_{GS}-V_T)^2+(V_{GS}-V_T)(V_{GD}-V_T)+(V_{GD}-V_T)^2}{(V_{GS}-V_T)+(V_{GD}-V_T)}$$

Evolution of Q_I with V_{DS}:



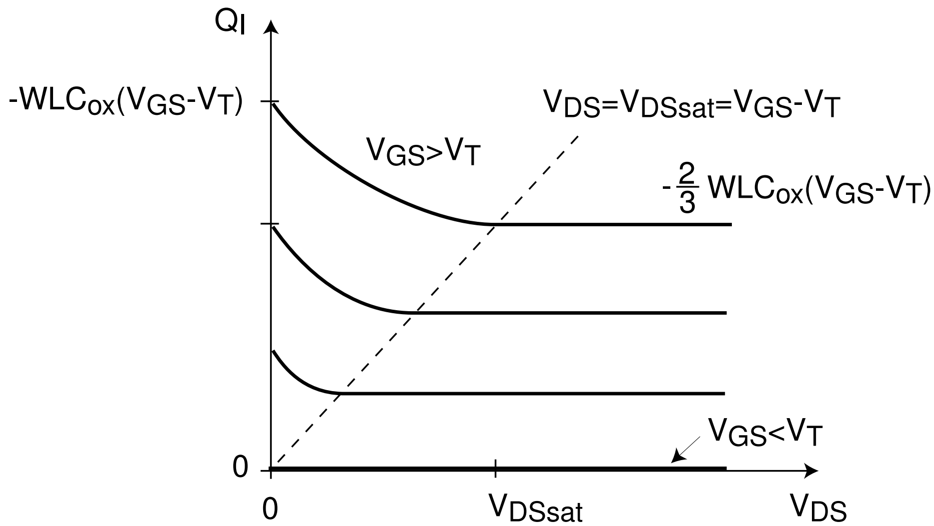
Used fundamental charge control relationship ⇒ expression only valid in linear regime.

For small V_{DS}:

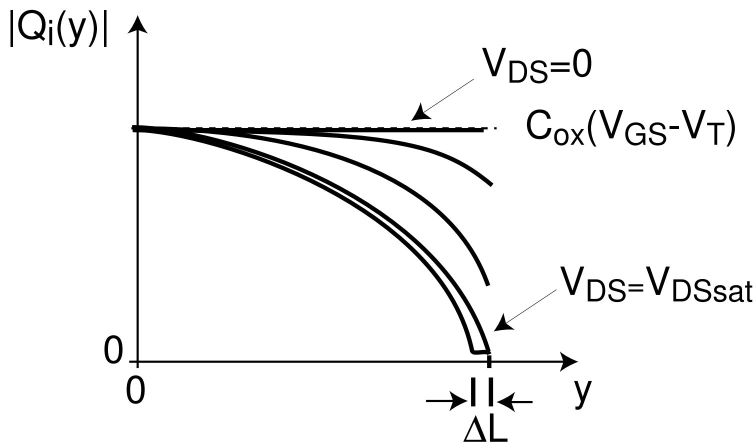
$$Q_I \simeq -WLC_{ox}(V_{GS}-V_T)$$

For saturation: set $V_{DS} = V_{DSsat} = V_{GS} - V_T$ in linear regime expression and get:

$$Q_I = -\frac{2}{3} W L C_{ox} (V_{GS} - V_T)$$



Reduction of $|Q_I|$ towards saturation is another manifestation of *channel debiasing*:



□ Capacitance associated with inversion charge:

Inversion charge supplied by source *and* drain ⇒ need two capacitors

$$C_{gsi} = -\frac{\partial Q_I}{\partial V_{GS}}|_{V_{GD}} = \frac{1}{2}WLC_{ox}(V_{GS} - V_T)\frac{V_{GS} - V_T - \frac{2}{3}V_{DS}}{(V_{GS} - V_T - \frac{1}{2}V_{DS})^2}$$

$$C_{gdi} = -\frac{\partial Q_I}{\partial V_{GD}}|_{V_{GS}} = \frac{1}{2}WLC_{ox}(V_{GS} - V_{DS} - V_T)\frac{V_{GS} - V_T - \frac{1}{3}V_{DS}}{(V_{GS} - V_T - \frac{1}{2}V_{DS})^2}$$

Expressions valid in linear regime.

Note that for small V_{DS}:

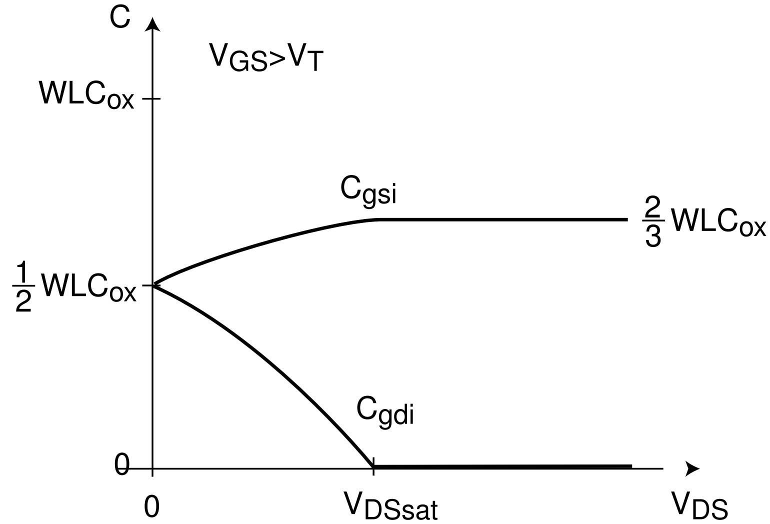
$$C_{gsi} \simeq C_{gdi} \simeq \frac{1}{2}WLC_{ox}$$

For saturation regime, set $V_{DS} = V_{DSsat} = V_{GS} - V_T$ and get:

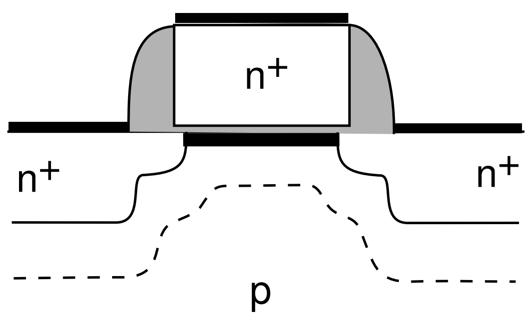
$$C_{gsi} = \frac{2}{3}WLC_{ox}$$

$$C_{gdi} = 0$$

Evolution of C_{gsi} and C_{gdi} with V_{DS}:

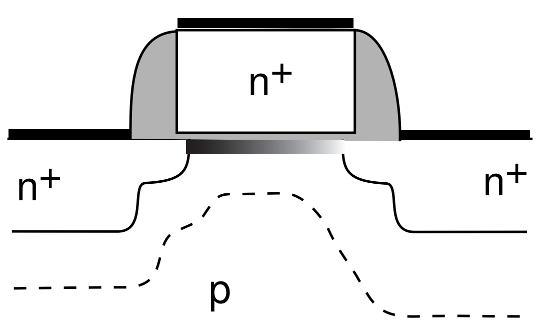


- Linear regime (small V_{DS}): uniform inversion layer charge:



$$C_{gsi} \simeq C_{gdi} \simeq \frac{1}{2} WLC_{ox}$$

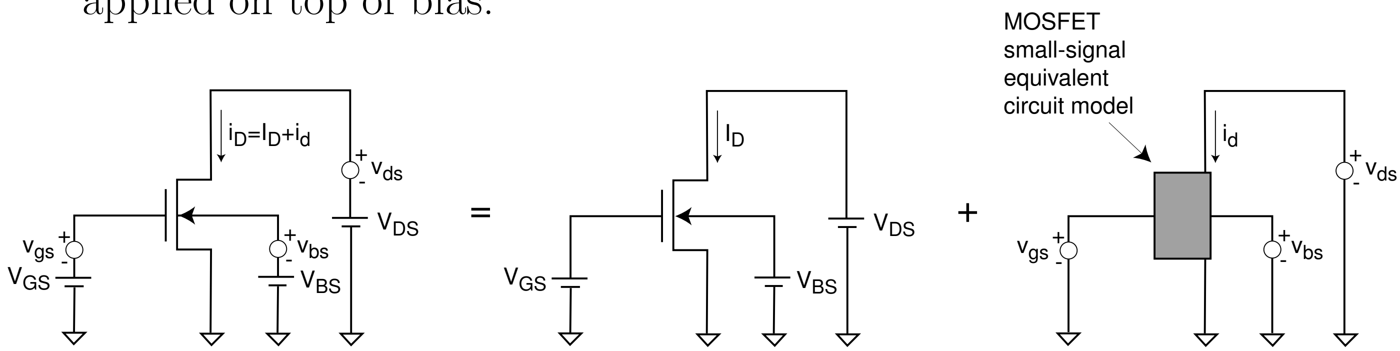
- Saturation regime ($V_{DS} > V_{DSsat}$): channel pinched-off:



$$C_{gs} \simeq \frac{2}{3} WLC_{ox} \qquad C_{gd} \simeq 0$$

2. Small-signal behavior of ideal MOSFET

In many applications, interested in response of device to *small signal* applied on top of bias:



Key points:

- Small signal is *small* ⇒ non-linear device behavior becomes linear.
- Can separate response of MOSFET to bias and small signal.
- Since response is linear, *superposition* applies ⇒ effects of different small-signals independent from each other.

Mathematically:

$$i_D(V_{GS}, V_{DS}, V_{BS}; v_{gs}, v_{ds}, v_{bs}) \simeq$$

$$I_D(V_{GS}, V_{DS}, V_{BS}) + i_d(v_{gs}, v_{ds}, v_{bs})$$

and

$$i_d(v_{gs}, v_{ds}, v_{bs}) = i_d(v_{gs}) + i_d(v_{ds}) + i_d(v_{bs})$$

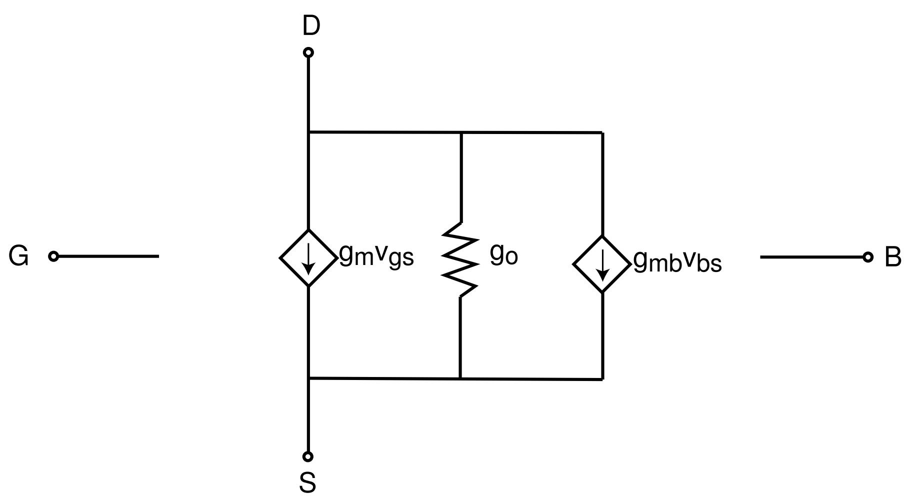
i_d linear on small-signal drives:

$$i_d \simeq g_m v_{gs} + g_d v_{ds} + g_{mb} v_{bs}$$

Define:

$g_m \equiv$ *transconductance* $[S]$
$g_d \equiv$ *output* or *drain conductance* $[S]$
$g_{mb} \equiv$ *back transconductance* $[S]$

Equivalent circuit model representation:



Approach to computing g_m, g_d, and g_{mb}:

$$g_m \simeq \frac{\partial I_D}{\partial V_{GS}}\Big|_{V_{DS},V_{BS}} \quad g_d \simeq \frac{\partial I_D}{\partial V_{DS}}\Big|_{V_{GS},V_{BS}} \quad g_{mb} \simeq \frac{\partial I_D}{\partial V_{BS}}\Big|_{V_{GS},V_{DS}}$$

Will deal with g_{mb} after we have dealt with body effect.

□ **Transconductance**, g_m:

$$g_m \simeq \frac{\partial I_D}{\partial V_{GS}}\Big|_{V_{DS},V_{BS}}$$

- Linear regime:

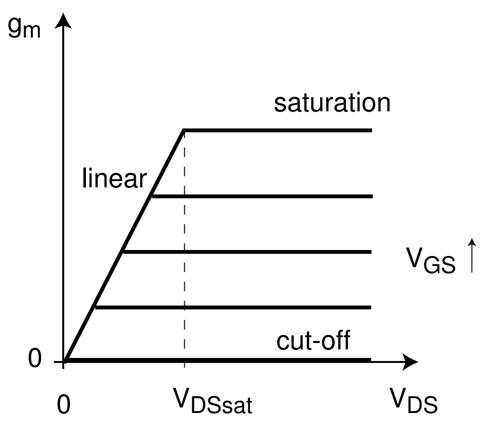
$$I_D = \frac{W}{L}\mu_e C_{ox}(V_{GS} - V_{th} - \frac{1}{2}V_{DS})V_{DS}$$

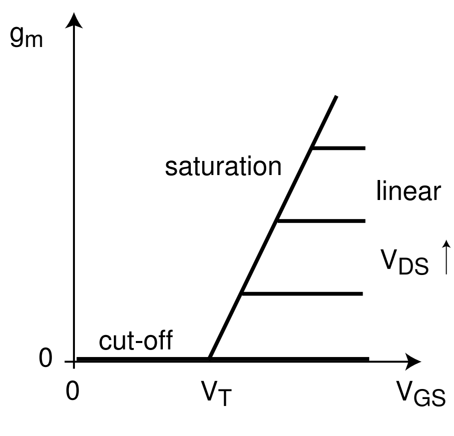
$$g_m = \frac{W}{L}\mu_e C_{ox} V_{DS}$$

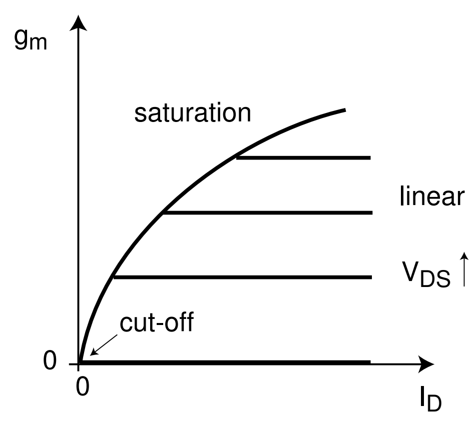
- Saturation regime:

$$I_{Dsat} = \frac{W}{2L}\mu_e C_{ox}(V_{GS} - V_{th})^2$$

$$g_m = \frac{W}{L}\mu_e C_{ox}(V_{GS} - V_{th}) = \sqrt{2\frac{W}{L}\mu_n C_{ox} I_D}$$







□ **Output conductance**, g_d:

$$g_d \simeq \frac{\partial I_D}{\partial V_{DS}}|_{V_{GS},V_{BS}}$$

- Linear regime:

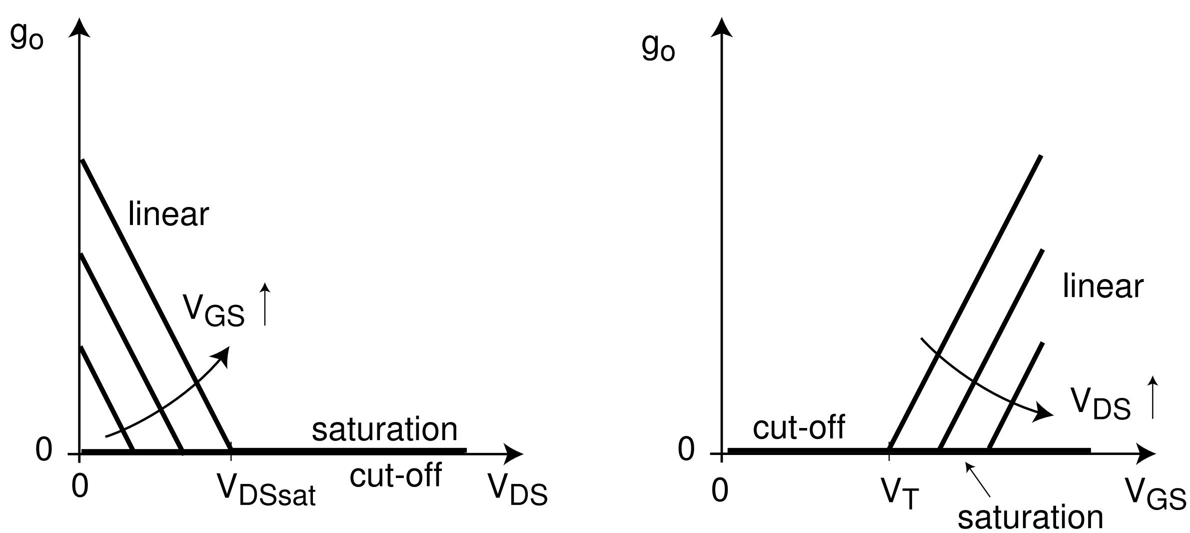
$$I_D = \frac{W}{L}\mu_e C_{ox}(V_{GS} - V_{th} - \frac{1}{2}V_{DS})V_{DS}$$

$$g_d = \frac{W}{L}\mu_e C_{ox}(V_{GS} - V_{th} - V_{DS})$$

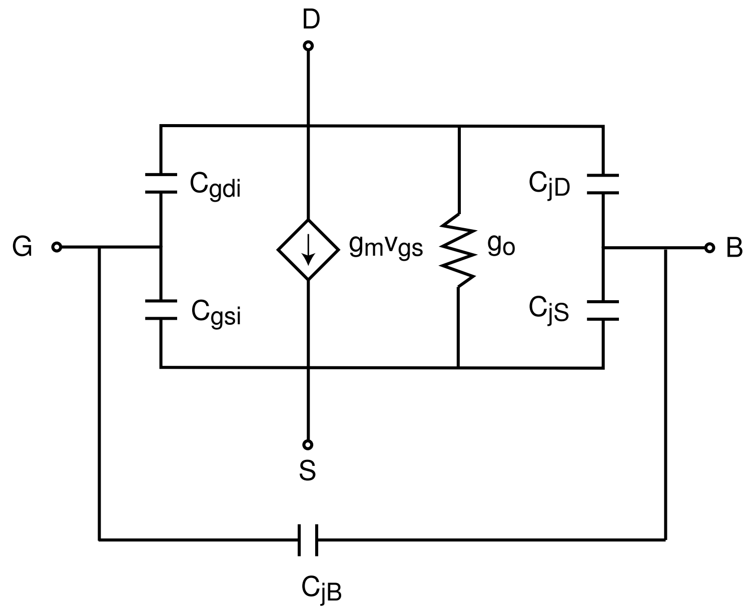
- Saturation regime:

$$I_{Dsat} = \frac{W}{2L}\mu_e C_{ox}(V_{GS} - V_{th})^2$$

$$g_d = 0$$

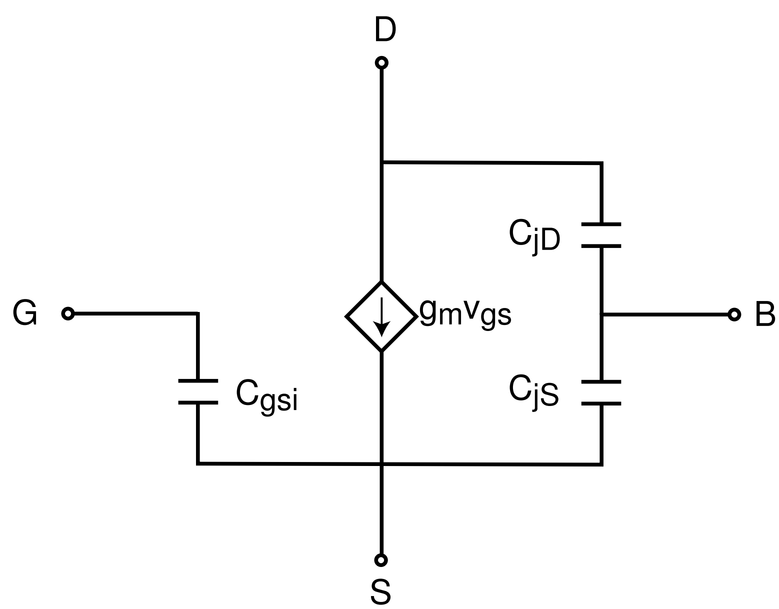


□ Add capacitors:



In saturation:

$C_{gdi} = 0$, $C_{jB} = 0$, $g_o = 0$

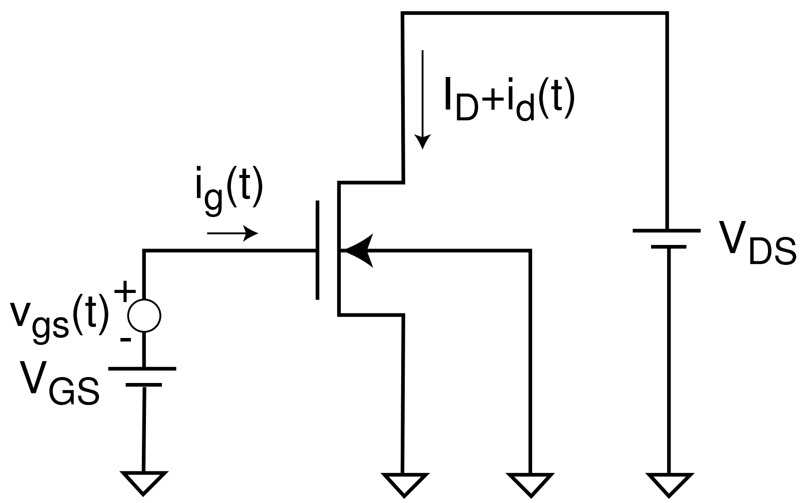


3. Short-circuit current-gain cut-off frequency

f_T: figure of merit to evaluate the frequency response of transistors.

f_T provides insight into electron transport in MOSFET channel

For f_T, MOSFET biased with output shorted from the small-signal point of view:

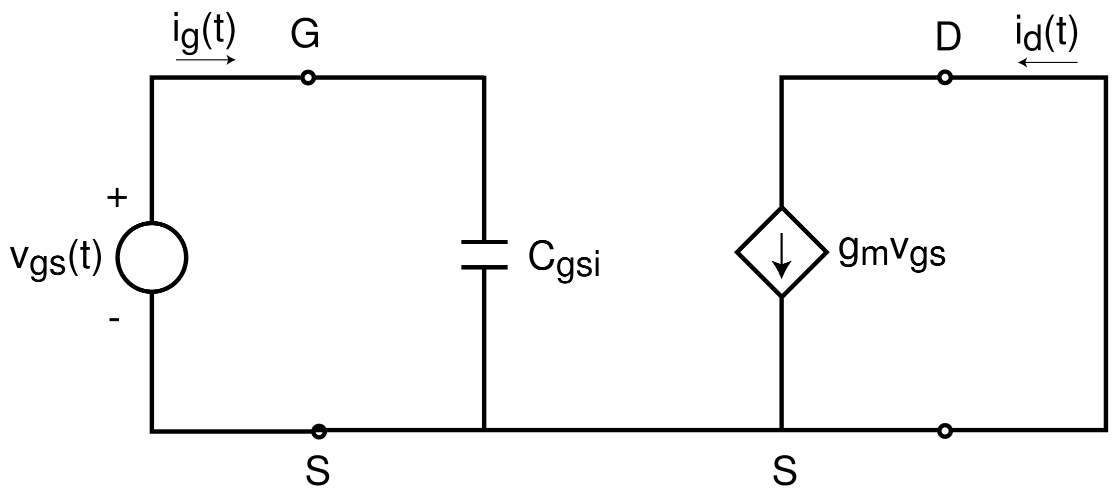


With MOSFET in saturation, f_T defined as frequency at which small-signal *current gain*, h_{21} goes to unity.

$$|h_{21}(f_T)| = |\frac{i_d}{i_g}|_{v_{ds}=0} = 1$$

As $f \downarrow \Rightarrow \; h_{21} \rightarrow \infty$.

□ Computation of f_T: small-signal equivalent circuit model:



i_g and i_d given by:

$$
\begin{aligned}
i_g &= j\omega C_{gsi} v_{gs} \\
i_d &= g_m v_{gs}
\end{aligned}
$$

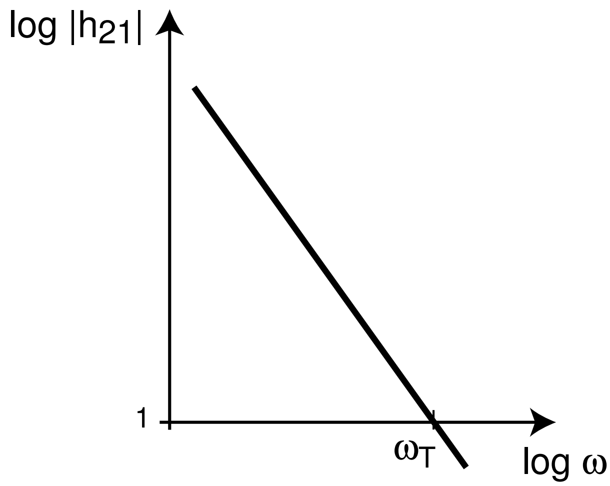
h_{21} given by:

$$h_{21} = \frac{g_m}{j\omega C_{gsi}}$$

Magnitude of h_{21} is:

$$|h_{21}| = \frac{g_m}{\omega C_{gsi}}$$

Evolution of h_{21} with frequency:



$|h_{21}|$ becomes unity at:

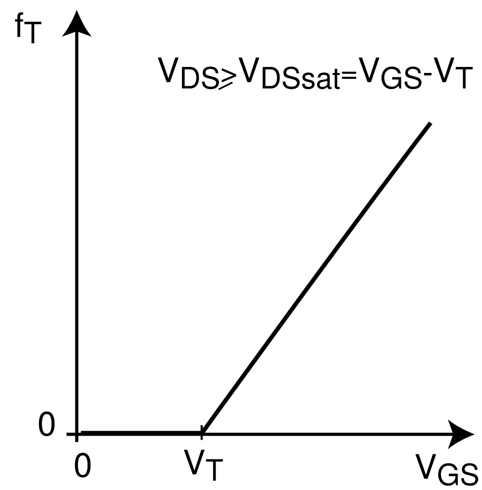
$$\omega_T = \frac{g_m}{C_{gsi}}$$

or

$$f_T = \frac{g_m}{2\pi C_{gsi}}$$

□ Bias dependence of f_T:

$$f_T = \frac{1}{2\pi}\frac{3}{2}\frac{\mu_e(V_{GS} - V_T)}{L^2}$$



Key dependences:

- f_T increases linearly with V_{GS} (g_m linear in V_{GS})
- f_T independent of V_{DS} ($g_m,\ C_{gsi}$ independent of V_{DS})
- f_T increases linearly with μ_e (g_m linear in μ_e)
- f_T scales as L^{-2} (g_m scales as L^{-1}, C_{gsi} scales as L)

□ Physical meaning of f_T

f_T has units of *inverse time.*

• Define *delay time*:

$$\tau_d = \frac{1}{2\pi f_T} = \frac{2}{3}\frac{L^2}{\mu_e(V_{GS} - V_T)}$$

Physical meaning of τ_d:

Delay time is average time for an electron to cross the channel from source to drain.

• Compute delay time directly.

First, time it takes for electrons to travel distance dy drifting at velocity v_e:

$$dt = \frac{dy}{v_e}$$

Then channel transit time is:

$$\tau_t = \int_0^L \frac{dy}{v_e(y)}$$

See Prob. 9.4 for a derivation of transit time:

$$\tau_t = \frac{2}{3}\frac{L^2}{\mu_e(V_{GS} - V_T)} = \tau_d$$

f_T gives idea of intrinsic speed of transistor!

Key conclusions

- In saturation, inversion charge results in:

$$C_{gsi} \simeq \frac{2}{3} W L C_{ox} \ \ and \ \ C_{gdi} = 0$$

- *Transconductance.* In saturation regime:

$$g_m \propto \sqrt{I_D}$$

- *Drain conductance.* In saturation regime:

$$g_d \simeq 0$$

- f_T: figure of merit to characterize frequency response of MOSFET
- f_T reflects intrinsic delay associated with electron transit from source to drain