Fall Term 2002
Introduction to Plasma Physics I

22.611J, 6.651J, 8.613J
Problem Set #7

1. Calculate the potential field due to a stationary *test charge* inserted (at, $\mathbf{x} = 0$, and time, $t = 0$) in a collisionless plasma initially in thermal equilibrium. You may assume that the test charge is sufficiently small that the linear response of the plasma is accurate, and you may also neglect any ion motion.

   First show that the Fourier-Laplace transform of the induced potential is given by,

   $$\phi_{\mathbf{k}\omega} = \frac{S_T(\mathbf{k}, \omega)}{k^2 \epsilon(\mathbf{k}, \omega)} + \frac{S_0(\mathbf{k}, \omega)}{k^2 \epsilon(\mathbf{k}, \omega)}$$

   where, $S_T(\mathbf{k}, \omega)$, is the test charge source,

   $$S_T(\mathbf{k}, \omega) = \frac{4\pi i q_T}{\omega}$$

   and, $S_0(\mathbf{k}, \omega)$, is the initial value source. Use the dielectric response function, $\epsilon(\mathbf{k}, \omega)$, from lecture, or the expression,

   $$\epsilon(\mathbf{k}, \omega) = 1 + \frac{1}{k^2 \lambda_D^2}\left[1 + \frac{\omega}{kv_e} Z\left(\frac{\omega}{kv_e}\right)\right]$$

   Now invert the transforms to find the asymptotic, $t \to \infty$, response, $\widetilde{\phi}(x, t \to \infty)$. Explain why the initial value term doesn't contribute and why the asymptotic limit for, $\widetilde{\phi}$, would be appropriate for most applications.

   Show that the same result obtains if the electron density fluctuation is calculated from the thermal equilibrium response,

   $$f_e = f_{\max}\exp\left(e\widetilde{\phi}/T_e\right) \simeq f_{\max} + f_{\max}\frac{e\widetilde{\phi}}{T_e} + \dots$$

   This is the so-called *Adiabatic Response* that we use previously to derive the Debye cloud. Although collisions and thermal equilibrium were invoked in this prior derivation, you see now that the same shield cloud results in a pure, Vlasov, collisionless plasma.

   Describe in words what this solution represents in terms of physics.

2. *Bump-on-tail Instability:* Consider an electron distribution of the form,

   $$\overline{f_e}(u) = \frac{(n_p + n_b)}{\pi^{3/2} v_e^3}\exp\left(-u^2/v_e^2\right) + \frac{n_b}{\pi^{3/2} v_b^3}\exp\left(-(u - u_b)^2/v_b^2\right)$$

with the orderings, $n_b \ll n_p$, $u_b \gg v_e$, $v_e \sim v_b$. You may assume high frequency modes and neglect ion motion. Make a plot of this distribution function and show where you expect unstable waves to occur. What are the approximate ranges in frequency and wavenumber over which waves are unstable? Calculate, for the fastest growing mode, the frequency, wavenumber and growth rate. You can use the orderings given above and only need to express your answers to leading order in any small parameters.

3. *Electrical Current Driven Instability*: Consider a quasi-neutral electron-proton plasma in which an equilibrium current is flowing. This may be described by a Maxwellian ion distribution at rest,

$$\overline{f_i}(u) = \frac{n}{\pi^{3/2} v_i^3} \exp\left(-u^2/v_i^2\right)$$

where, $v_i^2 = 2T_i/m_i$, and a drifting Maxwellian for the electrons,

$$\overline{f_e}(u) = \frac{n}{\pi^{3/2} v_e^3} \exp\left(-(u - u_d)^2/v_e^2\right)$$

where, $v_e^2 = 2T_e/m_e$, and, $v_i < u_d < v_e$. Make a plot of the distribution functions of ions and electrons on the same scale, look in the region, $v_i < \omega_r/k \ll v_e$, and show where you expect unstable waves might occur. Write an expression for the damping/growth factor, γ/ω_r, including both electron and ion contributions. Show that the condition, $T_e \gg T_i$, is generally required for instability, and determine the growth rate in this limit. What range of real frequencies and wavenumbers do you expect to find?