Fall Term 2002
Introduction to Plasma Physics I

22.611J, 6.651J, 8.613J
MID TERM EXAM

1. **Short Answer (21 points - 3 each)**

   (a) Write the formula for the electron plasma frequency.

   (b) Write the formula for the ion cyclotron frequency.

   (c) Write the formula for the Debye length.

   (d) True or False. Debye shielding makes plasmas *quasi-neutral* on scale lengths much less than the Debye length.

   (e) True or False. Electron plasma oscillations result in electric fields, and *non-neutrality*, on scales much less than the Debye length.

   (f) True or False. The parameter, $\Lambda = n\lambda_{De}^3$, is a large number in a well defined plasma.

   (g) Write down the force balance equation for MHD equilibrium.

2. **(16 points)** A plasma has the dispersion "constant" for transverse waves,

$$D = 1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{k^2 c^2}{\omega^2}$$

   (a) **(6 points)** Make a plot of the dispersion relation, ω *vs.* k, for positive frequencies.

   (b) **(4 points)** Write down the wave energy for these modes.

   (c) **(6 points)** Calculate the *cutoff* frequency, and describe what happens to waves driven at frequencies below the cutoff frequency.

3. **(16 points)** Write the formula for the total Coulomb cross section of charged particle, 1, colliding with charged particle, 2, *after* the cutoffs at large and small impact parameter have been applied. Estimate the momentum collision frequency for electron-ion collisions in a plasma with temperature, T_e. *(if you don't know the Coulomb cross section, just assume some value for the purposes of doing the rest of this problem)* Use this result to write down the steady state momentum equation for electrons to determine their drift velocity in the presence of an applied electric field. Derive an expression for the electrical conductivity in this plasma.

4. (**16 points**) Consider a plasma inside a toroidal vessel filled with a toroidal magnetic field as indicated in the figure below.
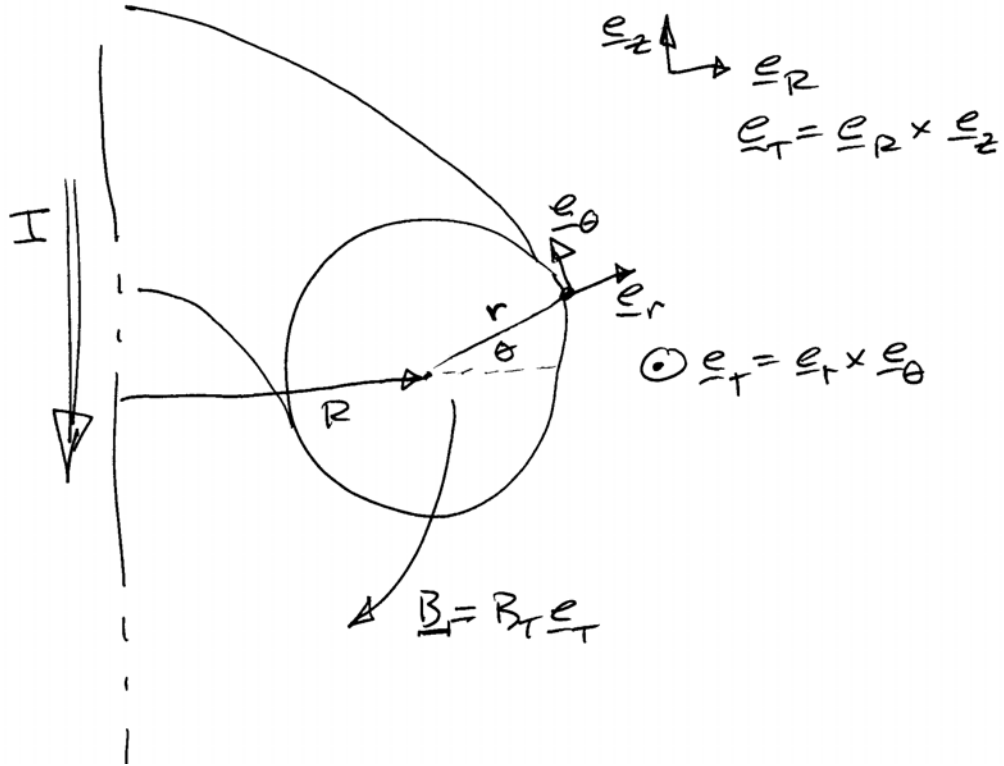

Figure for #4: Toroidal geometry showing vertical current for magnetic field generation and various unit vector conventions

The magnetic field is pure toroidal (pointing out of the plane of the paper). Analyze this system using guiding center orbits. Show that the sequence of charge separation, followed by an electric field gives a resulting flow of the entire plasma that implies *a complete lack of any kind of confinement.* Be sure to indicate all the drifts involved, their magnitudes and the resulting directions for both electron and ion flows. *EXTRA CREDIT: if you finish early you might want to actually compute the flow in quantitative terms, including any possible POLARIZATION of the plasma.*

5. (**15 points**) A theta pinch plasma has θ directed currents that produce an axial magnetic field with the radial profile,

$$B_z(r) = \begin{Bmatrix} \frac{r}{a}B_0 \; ; \; r < a \\ B_0 \qquad ; \; r > a \end{Bmatrix}$$

Compute the radial pressure profile subject to the boundary condition, $p(r = a) \rightarrow 0$. How does the result change if the edge boundary condition becomes, $p(r = a) = p_e$?

6. (**16 points**) Consider a plasma with the ions (protons) drifting in the z-direction with respect to the electrons at speed, V_0. Show that the dispersion relation for longitudinal waves, assuming a cold-two fluid model, is,

$$D = 1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pi}^2}{(\omega - kV_0)^2}$$

Hint: Try to make a simple argument for this dispersion without going through a lengthy algebraic calculation of the linearized response.

Plot the dispersion relation for the ion modes assuming the electron response was frozen ($\omega_{pe} \to 0$), and calculate the wave energy of these modes. Now allow the electrons to respond and show that the system will go unstable. *Hint #2: Make a simple argument for instability first and go into more detail with calculations and graphs if time permits. EXTRA CREDIT if you can actually calculate the growth rate by taking advantage of, $m_e/m_i \ll 1$.*