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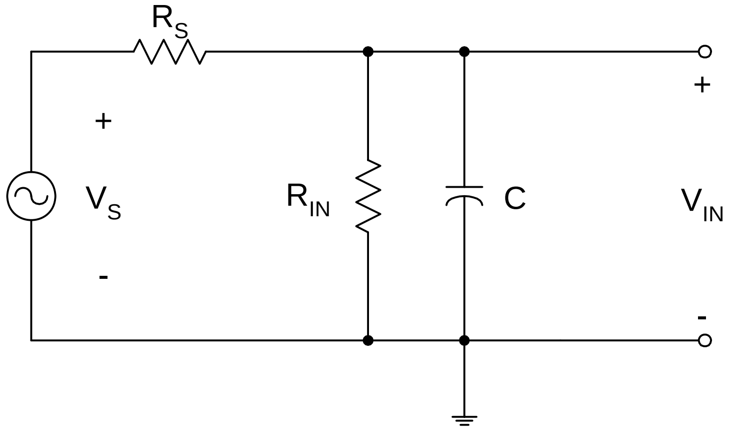
High-Frequency Cutoff Calculations

by Ron Roscoe



The schematic above represents a source driving an input to an amplifier. R_{IN} is the shunt input resistance associated with the amplifier input, and C represents the total shunt capacitance that includes the input capacitance and any associated connecting cable shunt capacitance. The equation for the voltage-divider transfer from source to input is:

$$\frac{V_{IN}}{V_S} = \frac{\dfrac{R_{IN} \times \dfrac{1}{sC}}{R_{IN} + \dfrac{1}{sC}}}{R_S + \dfrac{R_{IN} \times \dfrac{1}{sC}}{R_{IN} + \dfrac{1}{sC}}} = \frac{R_{IN} \times \dfrac{1}{sC}}{R_S \times R_{IN} + \dfrac{R_S}{sC} + \dfrac{R_{IN}}{sC}} = \frac{R_{IN}}{R_S + R_{IN} + R_S R_{IN} sC}$$

$$\frac{V_{IN}}{V_S} = \frac{1}{\dfrac{R_S}{R_{IN}} + 1 + R_S sC} \qquad s = j\omega; \qquad \frac{V_{IN}}{V_S} = \frac{1}{\dfrac{R_S}{R_{IN}} + 1 + j\omega R_S C} \qquad [1]$$

At low frequencies, the capacitive impedance is very large, and the simple resistive voltage divider determines the voltage transfer or "gain":

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$$\frac{V_{IN}}{V_S} = \frac{1}{\frac{R_S}{R_{IN}} + 1} = \frac{R_{IN}}{R_S + R_{IN}}; \qquad [2]$$

Let's take the example of the audiophile who wants to use his preamp outputs to drive another integrated amplifier in another room, using 30 feet of Belden 9264 single-conductor audio cable, which has a nominal capacitance of 34 picofarads per foot. Let's assume the preamp has a 600Ω output impedance, and that the integrated amplifier has line inputs with 100kΩ input impedance in parallel with another 50 pF of input capacitance. The source impedance is small enough to be ignored at low frequencies, giving a voltage-transfer ratio of 1 [equation 2]. The highest audible frequency for the average young human is 20kHz; what is the relative frequency response of the cable-source resistance-input resistance combination?

Solution: Because the ratio R_S/R_{IN} is small compared to 1, equation 1 simplifies to:

$$\frac{V_{IN}}{V_S} = \frac{1}{1 + j\omega R_S C};$$

When $\omega R_S C$ = 1, the magnitude of the denominator will equal 1.414, and the expression will have a magnitude of 0.707 which is -3dB relative to the value 1 for the purely resistive divider when R_S can be ignored. For this example, the -3dB frequency is:

$$1 = 2\pi f R_S C; f = \frac{1}{2\pi R_S C} = \frac{1}{2\pi 600 \times 1070 pF} = \frac{1}{3.77 \times 10^3 \times 1070 \times 10^{-12}} = 248 kHz$$

which is ten times higher than it needs to be! Thus, this connection is virtually flat to 20kHz.

However, suppose the preamp had a source impedance of 10,000Ω. This is still small enough so that we can ignore the ratio R_S/R_{IN} in equation 1, but now the -3dB point has dropped to about 15kHz, enough for many listeners to notice a difference.

Now let's look at a classical example of high frequency losses due to shunt capacitance, the 10:1 oscilloscope probe. Most of you know that the standard oscilloscope vertical amplifier input impedance is 1 Megohm in parallel with about 20 pF shunt input capacitance [it's printed on the scope faceplate]. If we used a 1:1 probe on high impedance circuits, we might load down high-impedance circuits with the input impedance of the 'scope, or affect the high frequency response of the circuit under test with the extra 20 pF of input capacitance, so the 10:1 probe has become the defacto standard. This requires that R_S becomes 9 Megohms. Now equation [1] becomes:

$$\frac{V_{IN}}{V_S} = \frac{1}{\frac{R_S}{R_{IN}} + 1 + j\omega R_S C} = \frac{1}{9 + 1 + j\omega R_S C} = \frac{1}{10 + j2\pi f\, 9 \times 10^6 \times 20 \times 10^{-12}} \qquad [4]$$

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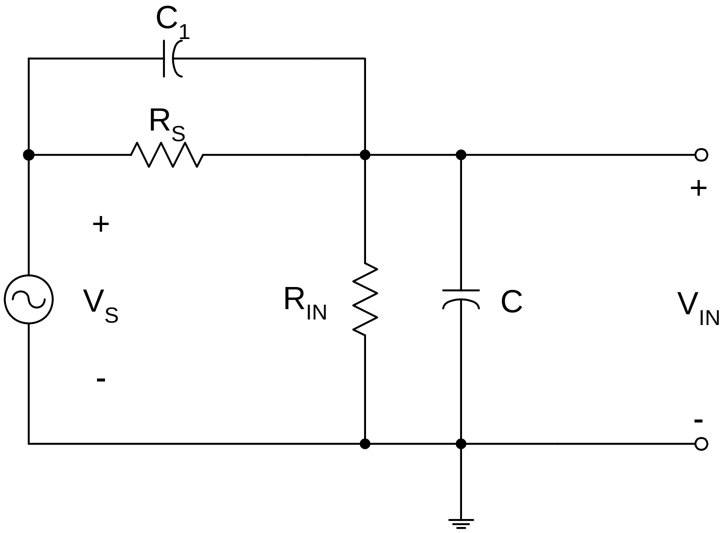
Again, setting the real part equal to the imaginary part and solving for f:

$$10 = 2\pi f\, 9\times10^{6}\times20\times10^{-12}; \qquad f = \frac{10}{2\pi 9\times10^{6}\times20\times10^{-12}} = 8.84kHz$$

Thus, the 10:1 probe in combination with the input capacitance of the scope is down 3 dB at 8.84 k Hz!! Since we hope to use our oscilloscopes to view frequencies up to 150 MHz, this is a depressing development!

However, we can compensate for this situation by adding a compensating capacitor C_1 in parallel with R_S, as shown in the modified figure below.



The equation for this voltage divider is:

$$\frac{V_{IN}}{V_S} = \frac{\dfrac{R_{IN}\times\dfrac{1}{sC}}{R_{IN}+\dfrac{1}{sC}}}{\dfrac{R_S\times\dfrac{1}{sC_1}}{R_S+\dfrac{1}{sC_1}}+\dfrac{R_{IN}\times\dfrac{1}{sC}}{R_{IN}+\dfrac{1}{sC}}} \qquad [5]$$

Equation [5] expands to:

$$\frac{V_{IN}}{V_S} = \frac{R_{IN}\left(sC_1R_S+1\right)}{R_S\left(sCR_{IN}+1\right)+R_{IN}\left(sC_1R_S+1\right)} \qquad [6]$$

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If we choose: $C_1 R_S = C R_{IN}$; $or\ C_1 = C\dfrac{R_{IN}}{R_S}$; $C_1 = 20\,pF\dfrac{1M\Omega}{9\,M\Omega} = 2.22\,pF$ [7]

then equation [6] reduces to:

$$\frac{V_{IN}}{V_S} = \frac{R_{IN}}{R_S + R_{IN}} \qquad [8]$$

which is independent of frequency. In real life, C_1 is usually made adjustable so that it may be adjusted to exactly compensate for the input capacitance C. The scope probe is attached to a calibration square wave signal generated by the oscilloscope and the adjustable capacitor in the probe is adjusted to produce a flat top on the square wave. [See figure below.]

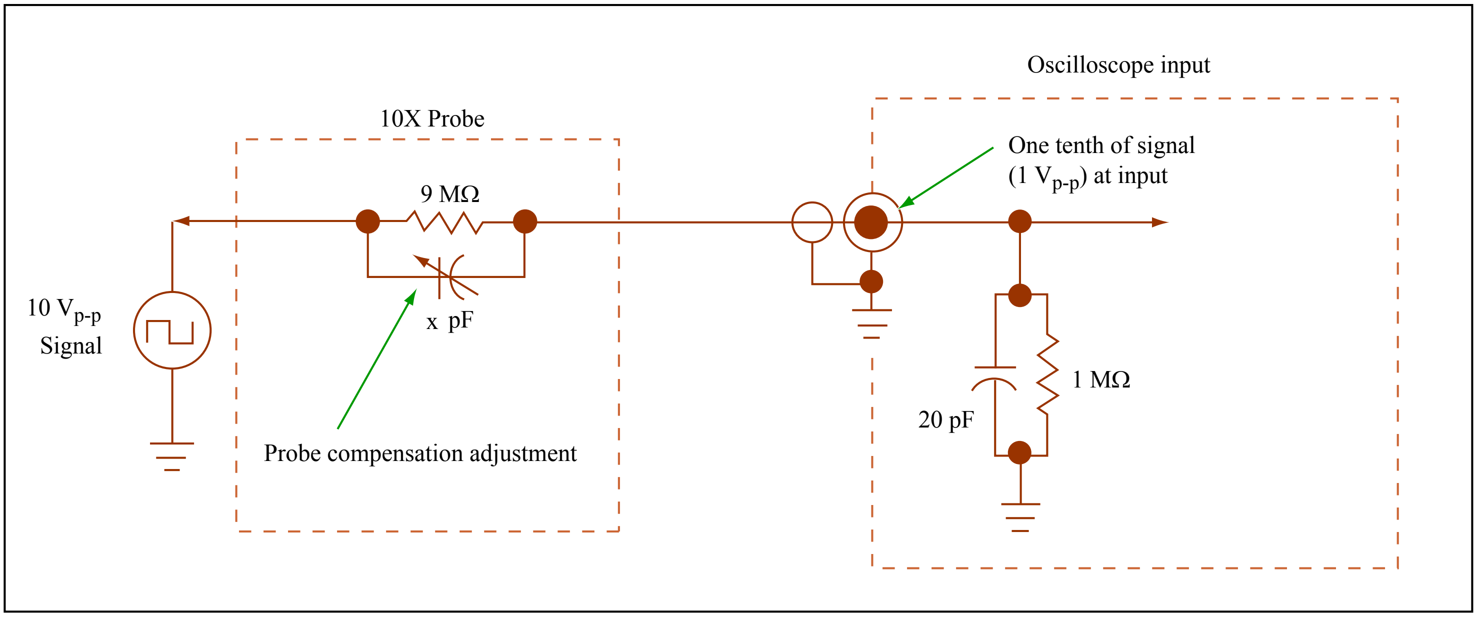

Figure by MIT OpenCourseWare.

Typical Probe/Oscilloscope 10-to-1 Divider Network [from Tektronix]

Speaking of real life, there is still some capacitance at the probe tip, in spite of this compensation scheme; anywhere from 2 to 5pF or more. This may not be a problem when probing lower frequency circuits, but when you get up into the lower VHF frequencies, say 50 MHz and above, this extra capacitance can be a problem, causing oscillations or resulting in reading errors. This capacitance can be somewhat reduced by removing the “witch’s hat” on the probe tip, and using the sharp point that’s under the hat to directly probe the circuit. In addition, the inductance of the “long” [relatively] ground lead and its clip can cause oscillations and reading errors as well. This inductance can be reduced by removing the ground lead from the probe, and replacing it with a heavy-duty paper clip wrapped around the probe at the same place that the ground lead was clipped on. Leave enough of the paper clip sticking out straight so that it can touch the ground bus or ground plane of the circuit you are working on, at the same time the sharp point on the probe tip contacts the high side of the circuit you wish to probe.

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Please check the user manual of your oscilloscope for information on probe compensation.

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