Mathematics for Computer Science

MIT 6.042J/18.062J

Team Problem 1

Mathematics for Computer Science

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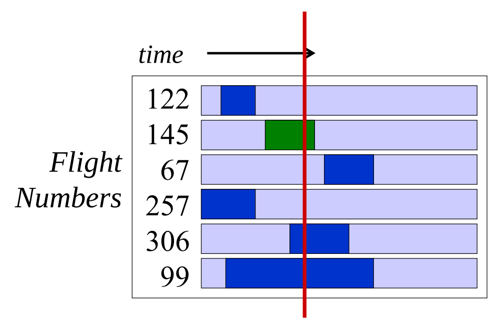
Graph Coloring

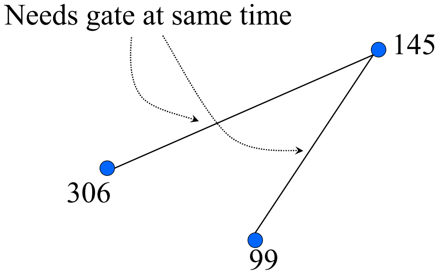
Airline Gate Allocation

Given a set of airline flights needing gates at overlapping times,
how many different gates do I need in order to accommodate them?

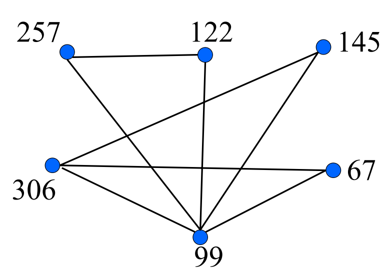
Airline Schedule



Model as a Graph



Model as a Graph

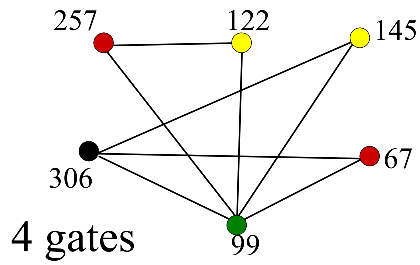


Color vertices

so that adjacent vertices have different colors.

colors = # gates needed

Coloring the Vertices

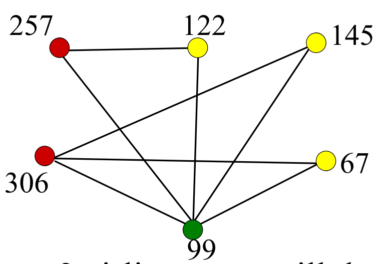


assign Gates:

- 257, 67
- 122,145
- 99
- 306

4 gates

Better: 3 Colors

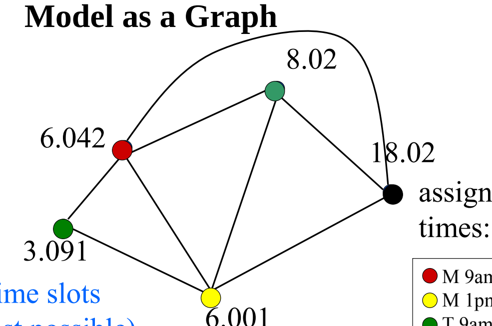


so 3 airline gates will do

Final Exam Time Slots

Two subjects conflict if a student is taking both. Assign conflicting subjects to different time slots, keeping exam period short.

Model as a Graph



assign times:

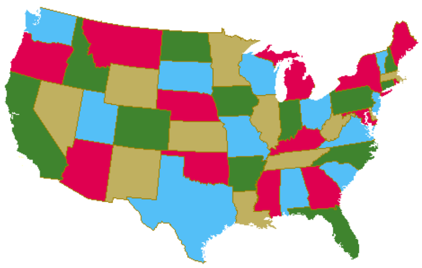
- M 9am
- M 1pm
- T 9am
- T 1pm

4 time slots (best possible)

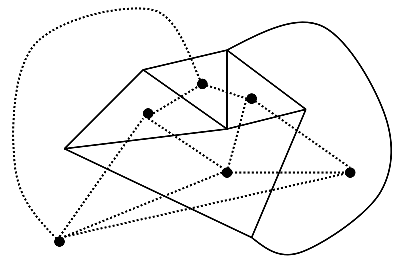
More Conflicting Allocation Problems

- # separate habitats to house different species of animals, some incompatible with others?
- # different frequencies for radio stations that interfere with each other?
- # different colors to color a map?

Map Coloring



Countries are the Vertices



Four Color Theorem

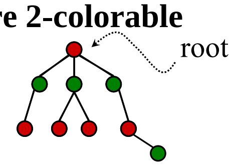
Any planar map is 4-colorable.
False proof published 1850's
(was correct for 5 colors).
Proof with computer
calculations: 1970's.
Much improved: 1990's

Chromatic Number

$\chi(G)$ = Chromatic Number of G

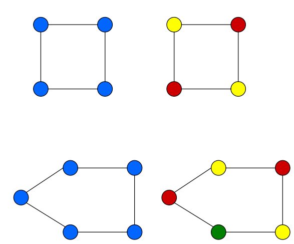
:= minimum #colors for G

Trees are 2-colorable



Pick any vertex as "root."
If (unique) path from root to w is
odd length: w even length: w

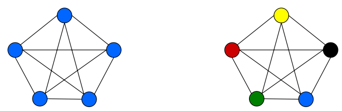
Simple Cycles



$\chi(C_{\text{even}}) = 2$

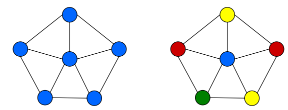
$\chi(C_{\text{odd}}) = 3$

Complete Graph K_5



$$\chi(K_n) = n$$

The Wheel W_n



W_5

$$\chi(W_{odd}) = 4 \qquad \chi(W_{even}) = 3$$

Bounded Degree

If deg(v) $\leq$ k for all vertices v of G, then $\chi(G) \leq k + 1$

A simple recursive coloring procedure achieves this.

Arbitrary Graphs

2-colorable? --easy to check

3-colorable? --hard to check (even if planar)

$\chi(G)$? --harder still

Team Problems

Problems 2 & 3