3.155J/6.152J Lecture 20: Fluids Lab Testing

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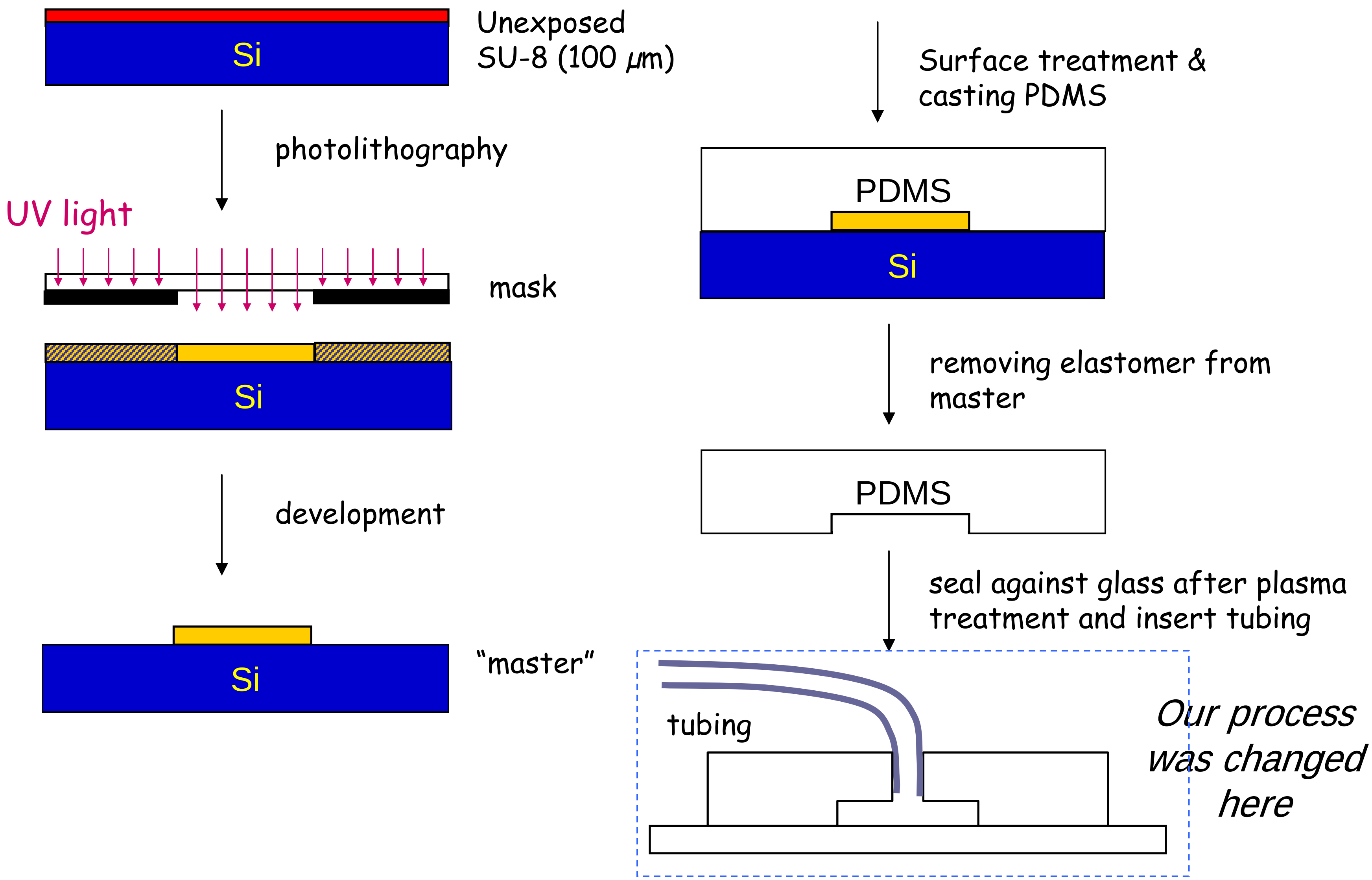
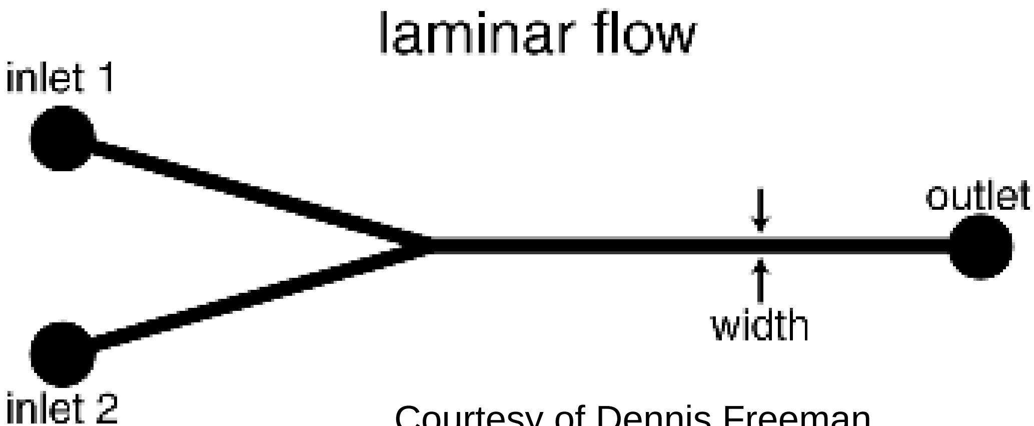
Outline

- Review of the Process and Testing
- Fluidics
  - Solution of Navier-Stokes Equation
  - Solution of Diffusion Problem
- Lab Report Guidance
- References
  - Senturia, Microsystems Design, Kluwer
  - 6.021 Web Site on Microfluidics Lab
  - Plummer, Chapter 7, p.382-384

Process Flow - Overview

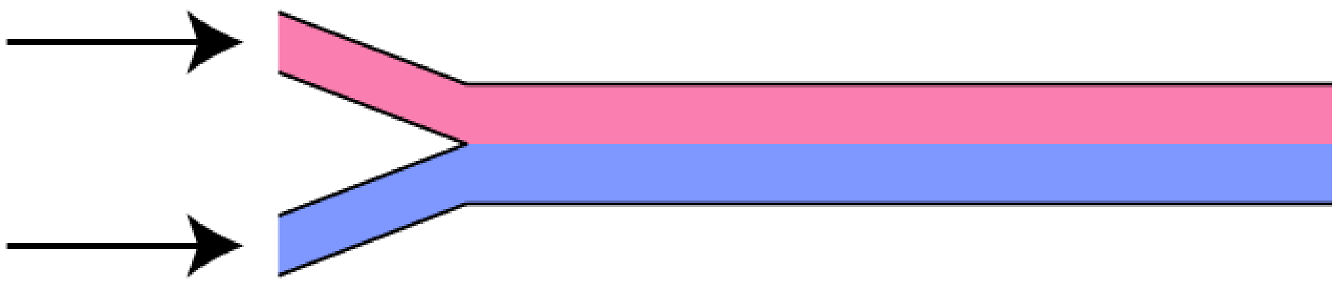


The Mixer



Courtesy of Dennis Freeman.

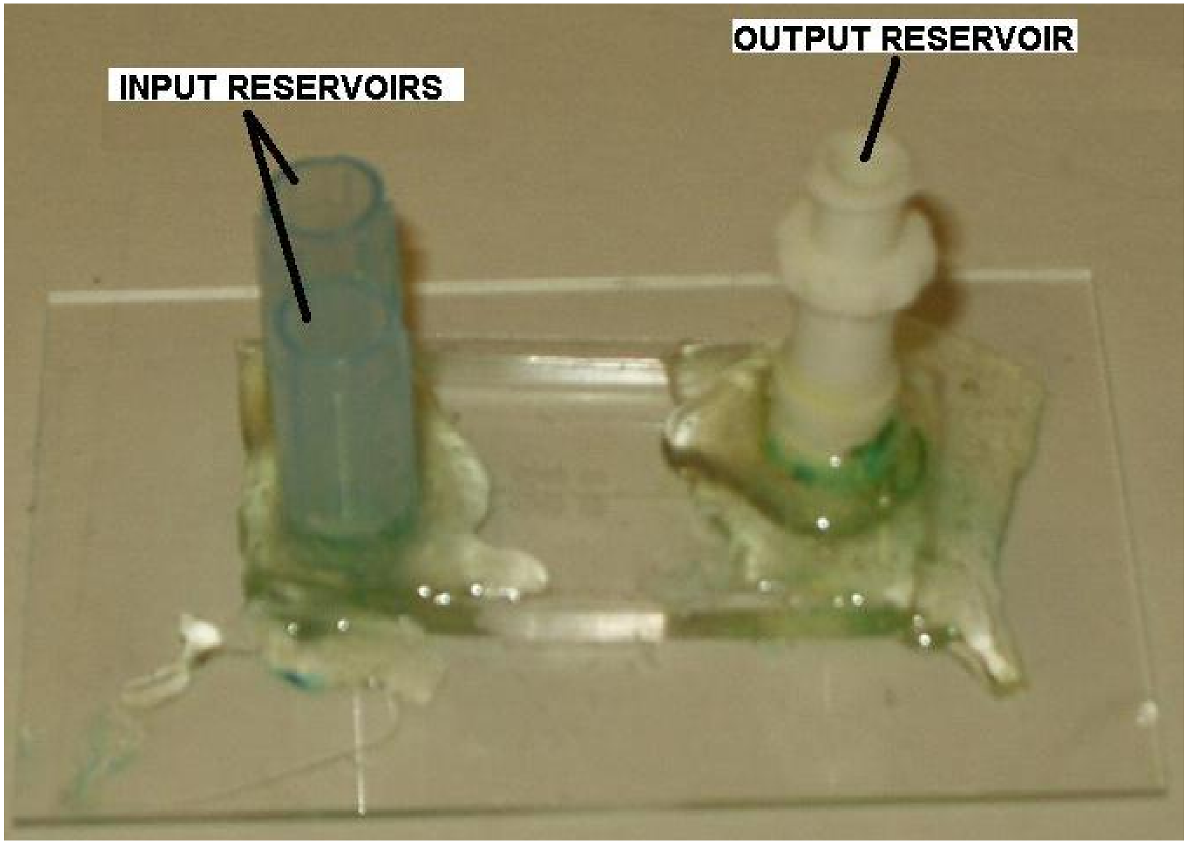
Width = 250μm, 500 μm
Depth = 100 μm
Inlet Length = 25 mm
Outlet Length = 35 mm



Courtesy of Dennis Freeman.

Images: Prof. D. Freeman

Packaging/Testing



Courtesy of Dennis Freeman.

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Images: Prof. D. Freeman

Experiment

- Gravity feed of fluids
  - Requires ‘priming’ of channel
- Particles for velocity measurement
  - We will attempt this
- Dye for diffusion experiments
- Measurements
  - Particle velocity
  - Diffusion

Navier-Stokes

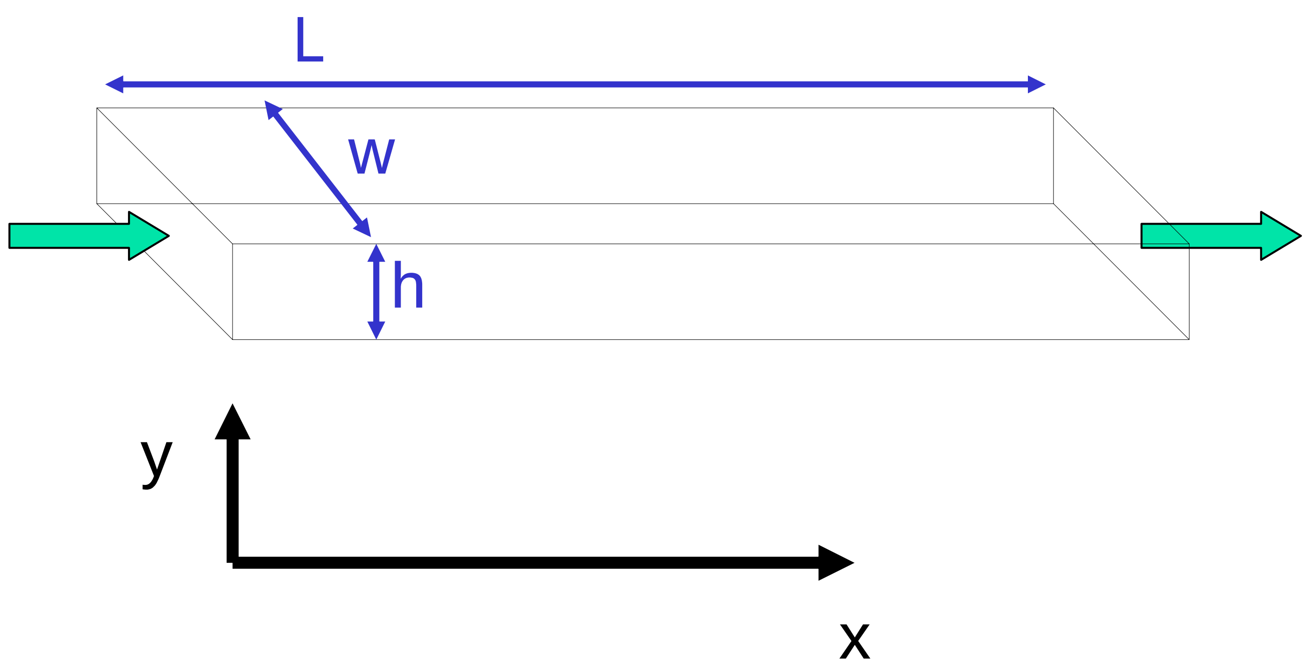
- The Navier-Stokes equation for incompressible flow:

$$\rho_m \frac{D\mathbf{U}}{Dt} = \eta \nabla^2 \mathbf{U} - \nabla P^*$$

  - U = velocity
  - P^* = pressure (*minus gravity body force*)
  - ρ_m = fluid density (10^3 kg/m^3 for water)
  - η = viscosity (10^{-3} Pa-s for water)

Poiseuille Flow

- Assume width (w) >> height (h)
- Neglect entrance effects (L >> h)



Simplify to our problem

- No time dependence
  - d/dt = 0
- Flow is constant in x-direction (and 0 in z)
  - U = f(y)
- Pressure is only a function of x
  - A linear pressure drop

$$\frac{dP}{dx} = -K \quad \text{(constant)}$$

$$\rho_m \frac{D\mathbf{U}}{Dt} = \eta \nabla^2 \mathbf{U} - \nabla P^*$$

$$\frac{\partial^2 U_x}{\partial y^2} = -\frac{K}{\eta}$$

Poiseuille Flow

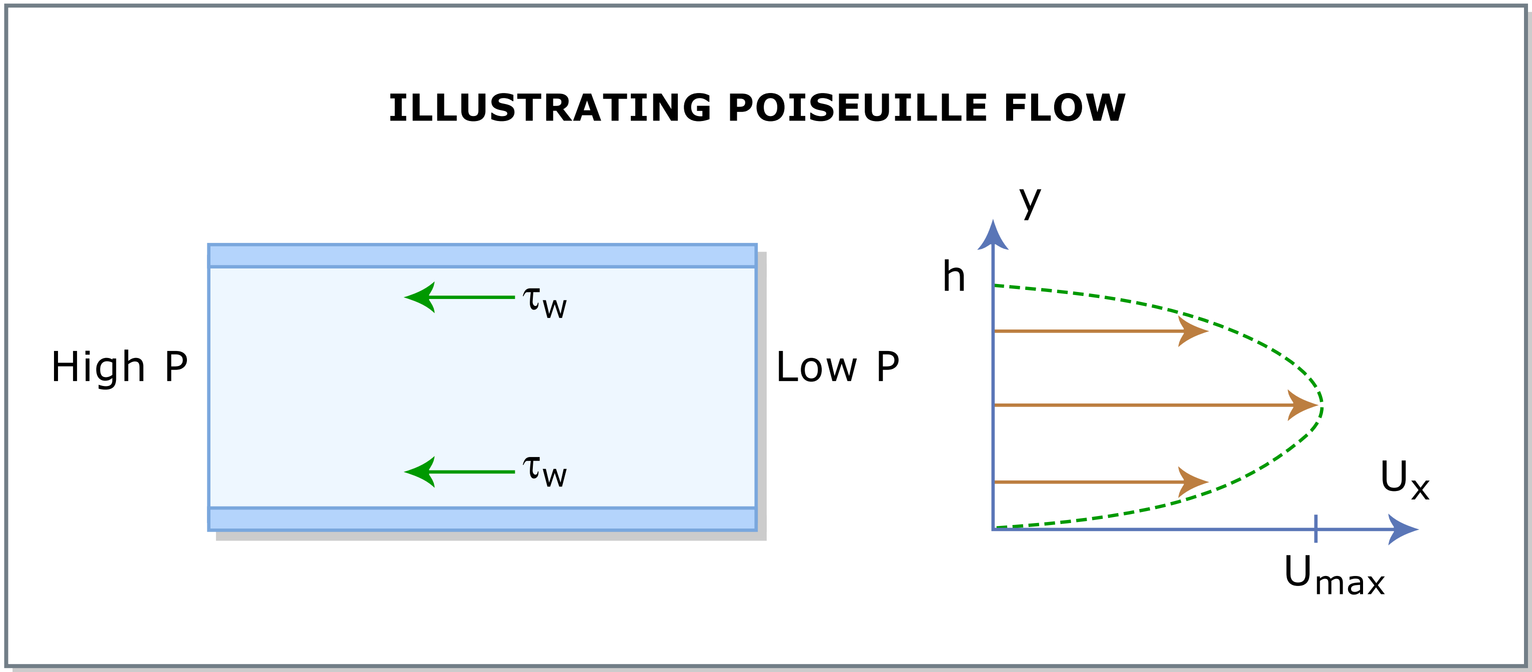


Figure by MIT OCW.

- 'No-Slip' Boundary conditions
  - $U_x(y=0) = 0$
  - $U_x(y=h) = 0$

Solution

$$\frac{\partial^2 U_x}{\partial y^2} = -\frac{K}{\eta}$$

- Solution is a quadratic polynomial
  - $U_x = a + by + cy^2$
- Using boundary conditions and substitution

$$U_x = \frac{1}{2\eta}[y(h-y)]K$$

Parabolic Flow Profile

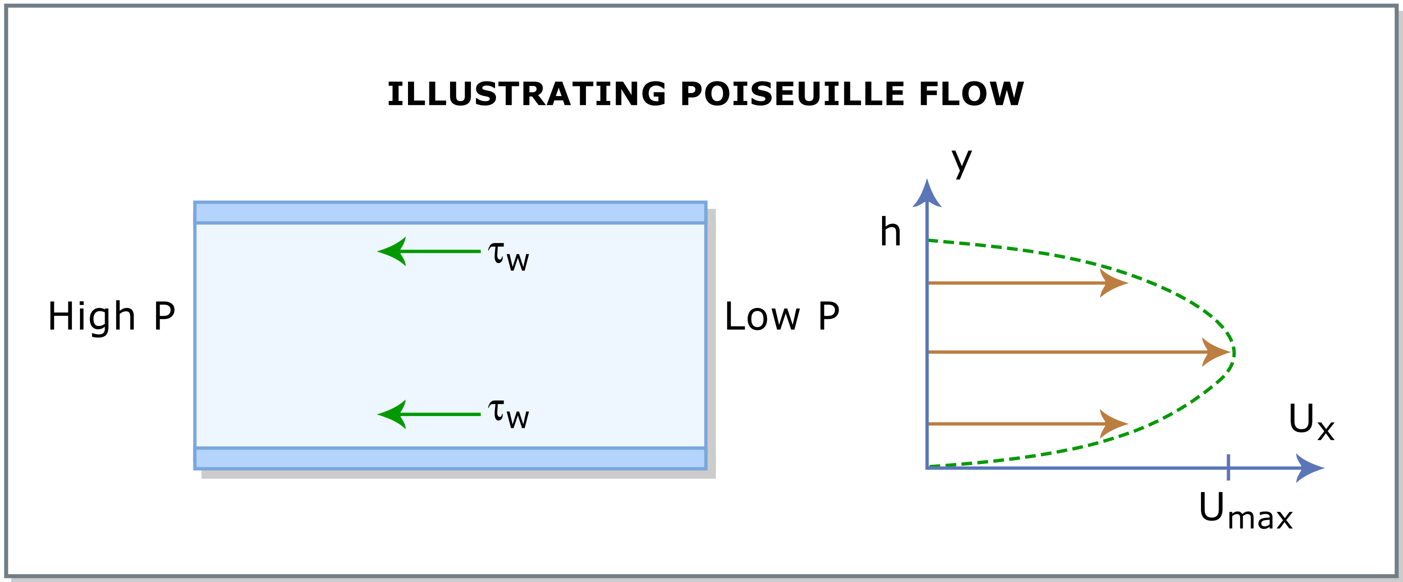


Figure by MIT OCW.

$$U_x = \frac{1}{2\eta}[y(h-y)]K$$

- Maximum velocity

$$U_{\max} = \frac{h^2}{8\eta}K$$

- Flow rate

$$Q = W\int_0^h U_x\,dy = \frac{Wh^3}{12\eta}K$$

- Average velocity

$$\bar{U} = \frac{Q}{Wh} = \frac{h^2}{12\eta}K = \frac{2}{3}U_{\max}$$

Pressure drop over length

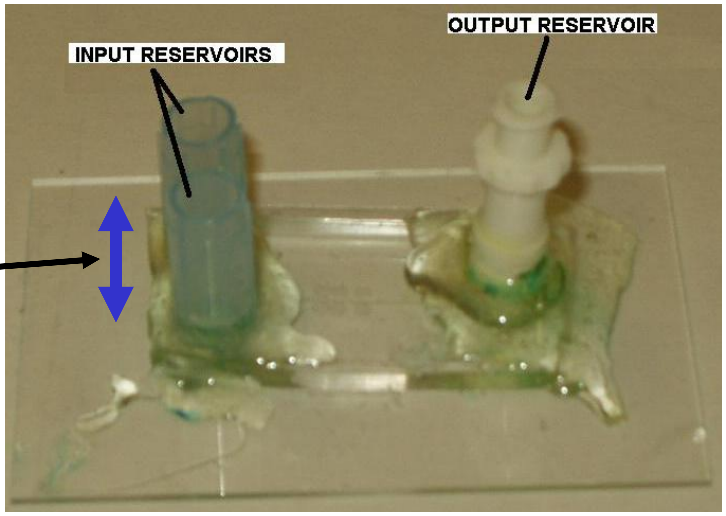
$$K = \frac{\Delta P}{L}$$

$$\Delta P = \frac{12\eta L}{Wh^3}Q$$

$\Delta P = \rho g H$

H = height of water

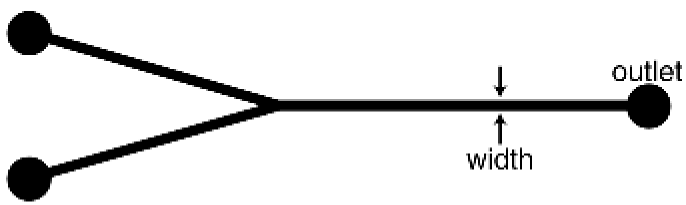
g = gravity



Courtesy of Dennis Freeman.

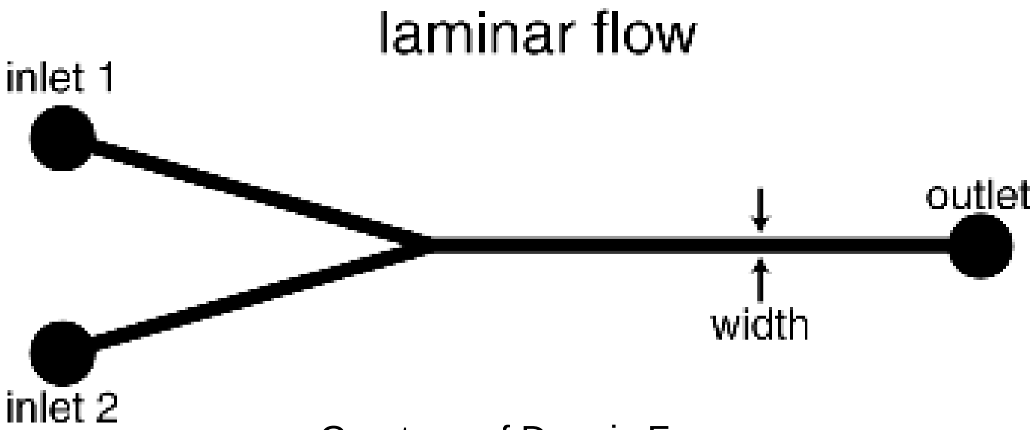
Flow Issues

- Edge effects
  - Flow rate
- Particle location in channel
- Dimensions
- Merging of channels
  - How to model



Courtesy of Dennis Freeman.

The Mixer – Mixing by diffusion



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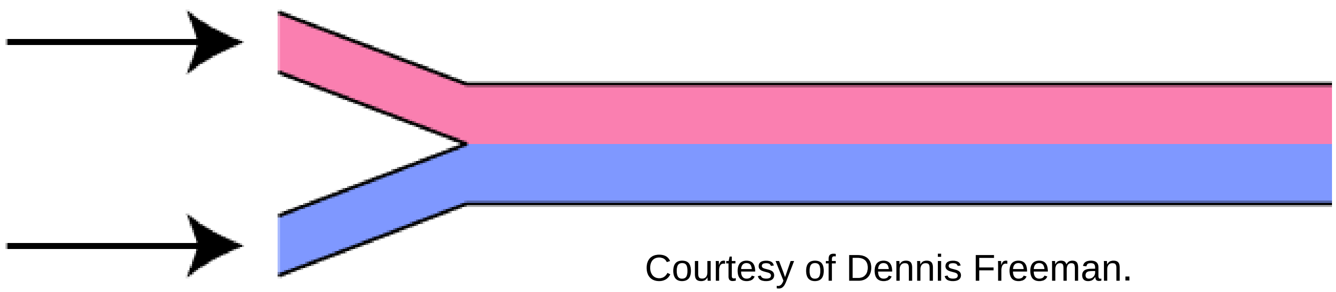

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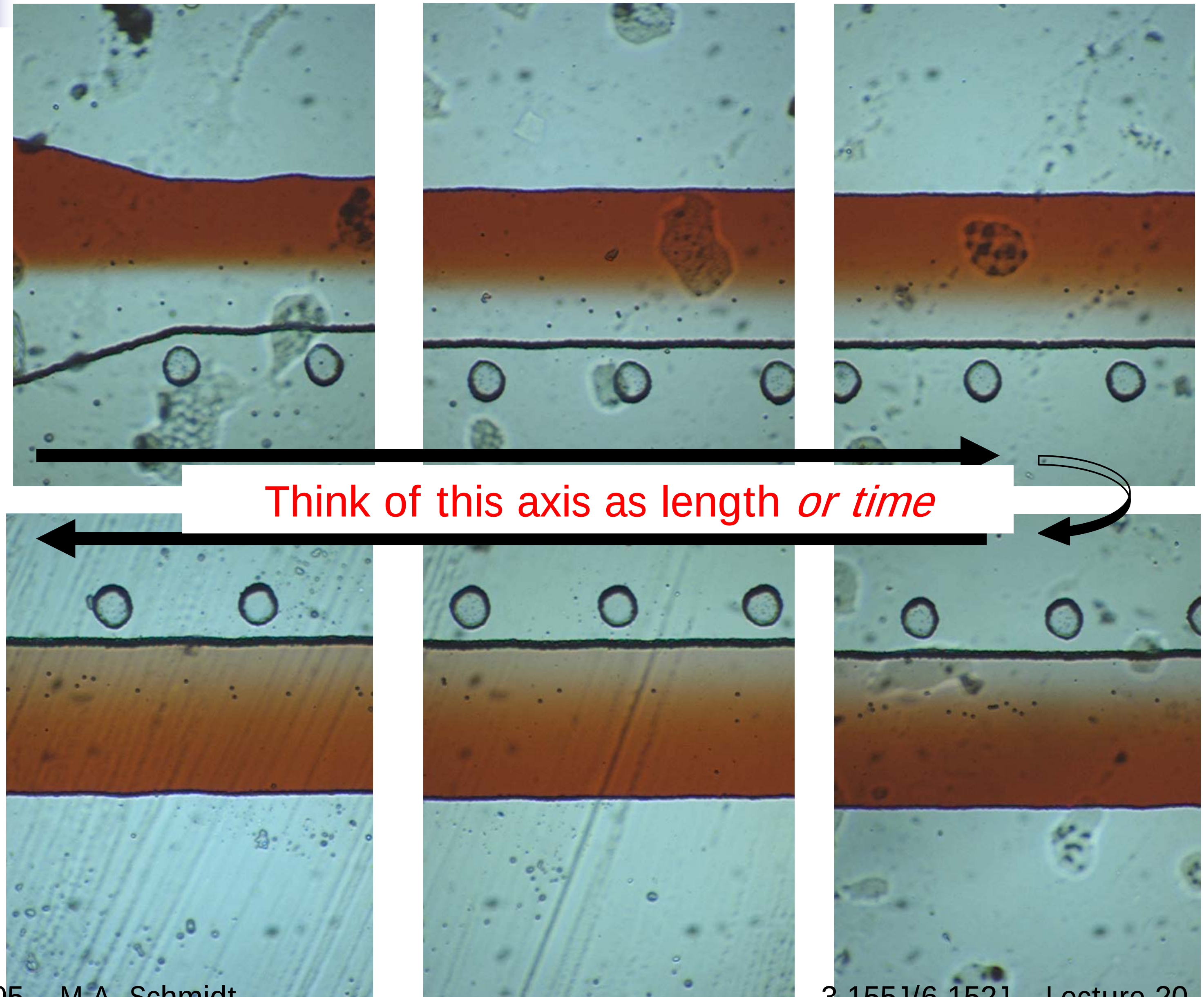
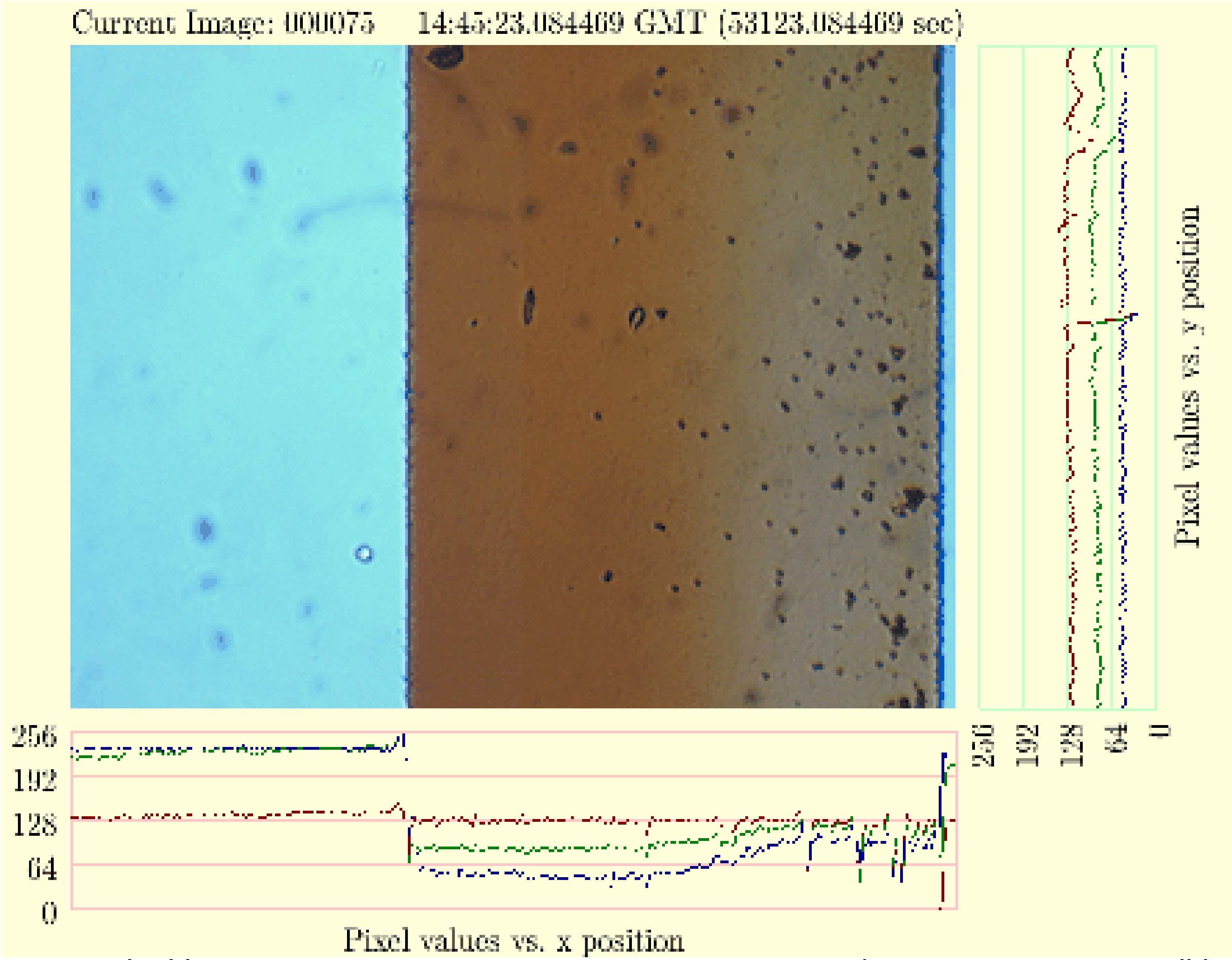
Images: Prof. D. Freeman

Diffusion Image Sequence

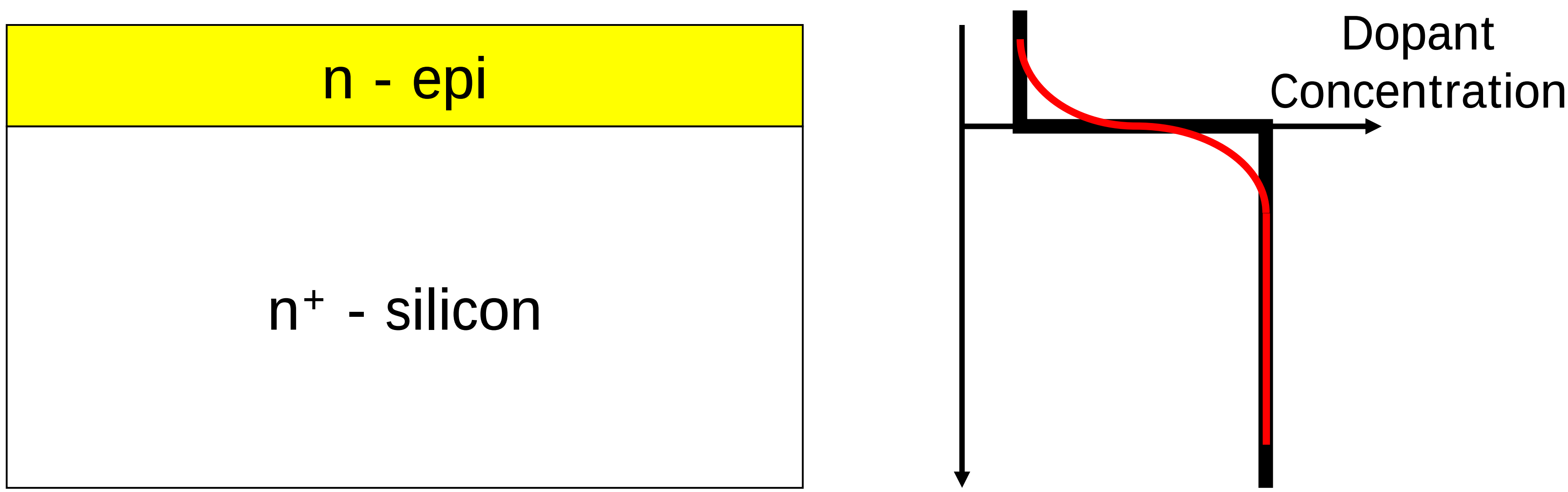


Imaging System Output



Diffusion

- Same problem as diffusion in an epi layer
  - As in the case of the design problem



Solution in Plummer, Chapter 7, p.382

Solution

- Initial Conditions

$$C = 0 \quad \text{at} \quad t = 0 \quad \text{for} \quad x > 0$$
$$C = C \quad \text{at} \quad t = 0 \quad \text{for} \quad x < 0$$

- Identical to Infinite Source Problem:

$$C(x,t) = \frac{C}{2}\left[1 - \operatorname{erf}\left(\frac{x}{2\sqrt{Dt}}\right)\right]$$

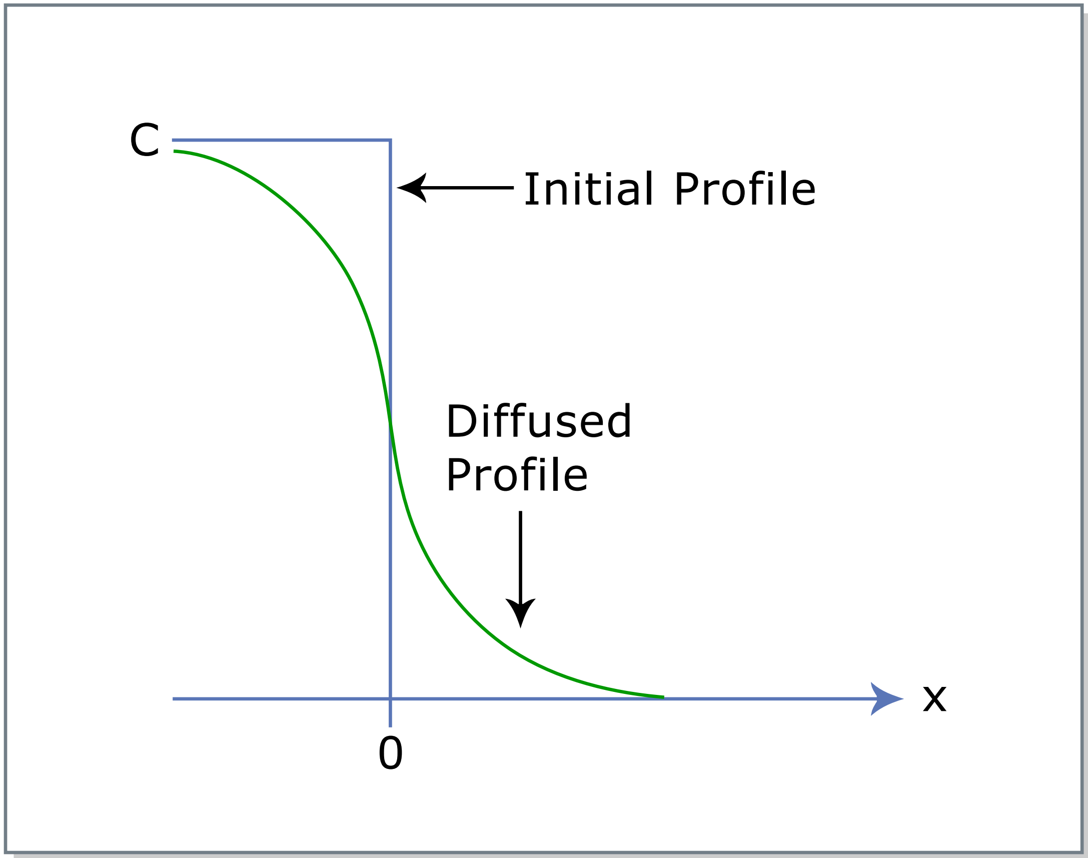


Figure by MIT OCW.

Reference: Plummer, J., M. Deal, and P. Griffin. *Silicon VLSI Technology: Fundamentals, Practice, and Modeling*. Upper Saddle River, NJ: Prentice Hall, 2000. ISBN: 0130850373.

An 'Intuitive' way to look at it...

- Think of the uniform concentration as a sum of dopant 'pulses'
- Each 'pulse' has a Gaussian diffusion profile
  - Dose = C Δx
- Apply superposition since diffusion is linear

Figure removed for copyright reasons.

Refer to Plummer, J., M. Deal, and P. Griffin. *Silicon VLSI Technology: Fundamentals, Practice, and Modeling*. Upper Saddle River, NJ: Prentice Hall, 2000. ISBN: 0130850373.

$$C(x,t) = \frac{C}{2\sqrt{\pi D t}} \sum_{i=1}^{n} \Delta x_i \exp - \frac{(x - x_i)^2}{4Dt}$$

Solution

- Taking the limit of Δx

$$C(x,t) = \frac{C}{2\sqrt{\pi D t}} \sum_{i=1}^{n} \Delta x_i \exp - \frac{(x - x_i)^2}{4Dt}$$

$$C(x,t) = \frac{C}{2\sqrt{\pi D t}} \int_0^{\infty} \exp - \frac{(x - \alpha)^2}{4Dt} d\alpha$$

$$\frac{(x - \alpha)}{2\sqrt{Dt}} = \eta$$

$$C(x,t) = \frac{'C}{\sqrt{\pi}} \int_{-\infty}^{x/2\sqrt{Dt}} \exp(-\eta^2) d\eta$$

Solution

$$C(x,t) = \frac{C}{\sqrt{\pi}} \int_{-\infty}^{x/2\sqrt{Dt}} \exp(-\eta^2)\, d\eta$$

$$C(x,t) = \frac{C}{2}\left[1 - \operatorname{erf}\left(\frac{x}{2\sqrt{Dt}}\right)\right]$$

$$\operatorname{erfc}(x) = 1 - \operatorname{erf}(x)$$

$$C(x,t) = \frac{C}{2}\left[\operatorname{erfc}\left(\frac{x}{2\sqrt{Dt}}\right)\right]$$

Error Function Solution

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Refer to Plummer, J., M. Deal, and P. Griffin. *Silicon VLSI Technology: Fundamentals, Practice, and Modeling*. Upper Saddle River, NJ: Prentice Hall, 2000. ISBN: 0130850373.

$$C(x,t) = \frac{C}{2}\left[\operatorname{erfc}\left(\frac{x}{2\sqrt{Dt}}\right)\right]$$

Diffusion Issues

- Fitting ideal curve to measured profiles
- Scaling time to position
- Choice of velocity
- Non-ideal flow profiles

Fluids Lab Report

- Follow the Letters format
  - Purpose: Characterization of a Liquid Micromixer
- Report Flow Velocity
- Compare to calculated
  - Estimate errors
- Extract an effective diffusion coefficient
  - Utilize ‘best estimate’ for flow velocity
- Compare to expected ($D \sim 2 \times 10^{-6}\ cm^2/s$)
  - Identify relevant non-idealities