

# Lecture 6 - PN Junction and MOS Electrostatics (III)

## ELECTROSTATICS OF PN JUNCTION UNDER BIAS

September 27, 2005

### Contents:

1. electrostatics of pn junction under bias
2. depletion capacitance

### Reading assignment:

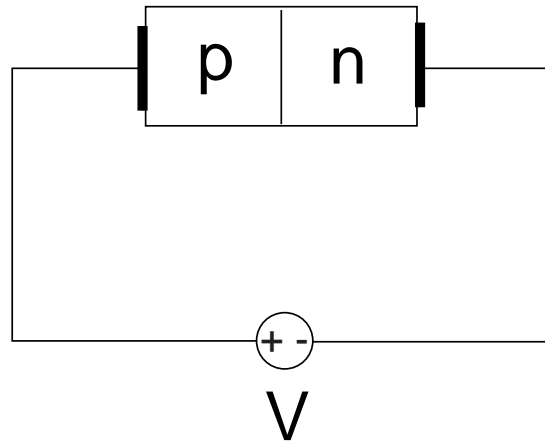
Howe and Sodini, Ch. 3, §§3.5-3.6

## Key questions

- What happens to the electrostatics of a pn junction if a voltage is applied across its terminals?
- Why does a pn junction behave in some way like a capacitor?

# 1. Electrostatics of pn junction under bias

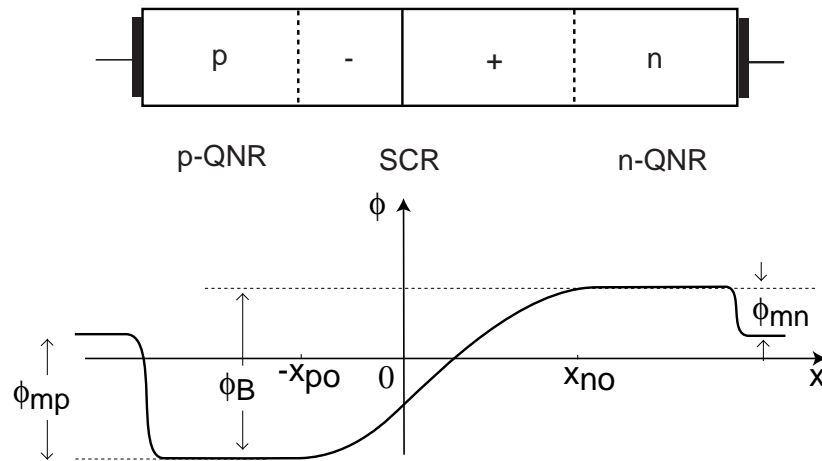
Bias convention for pn junction:



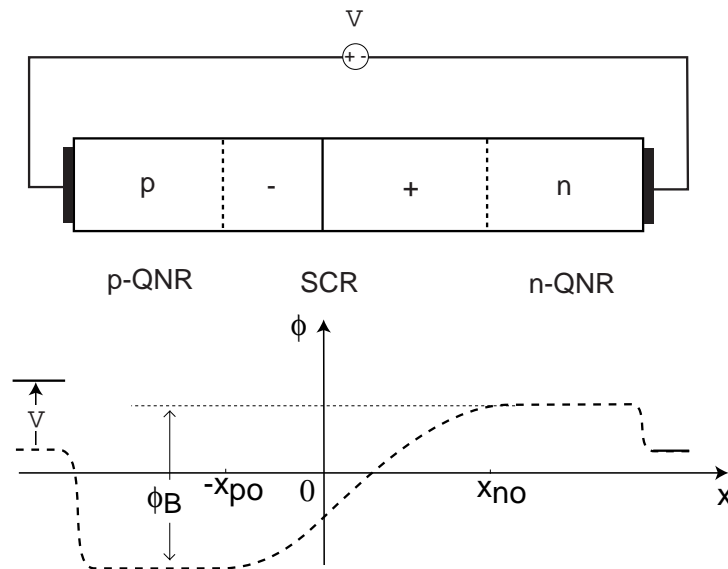
$V > 0$  *forward* bias

$V < 0$  *reverse* bias

- Potential distribution across pn junction in thermal equilibrium:

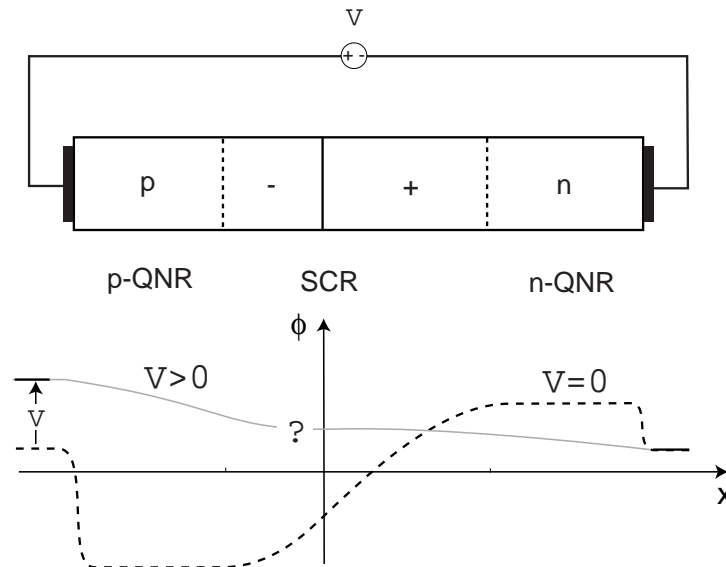


- Apply voltage to p-side with respect to n-side:



Battery imposes a potential difference across diode

How does potential distribution inside junction change as a result of bias?



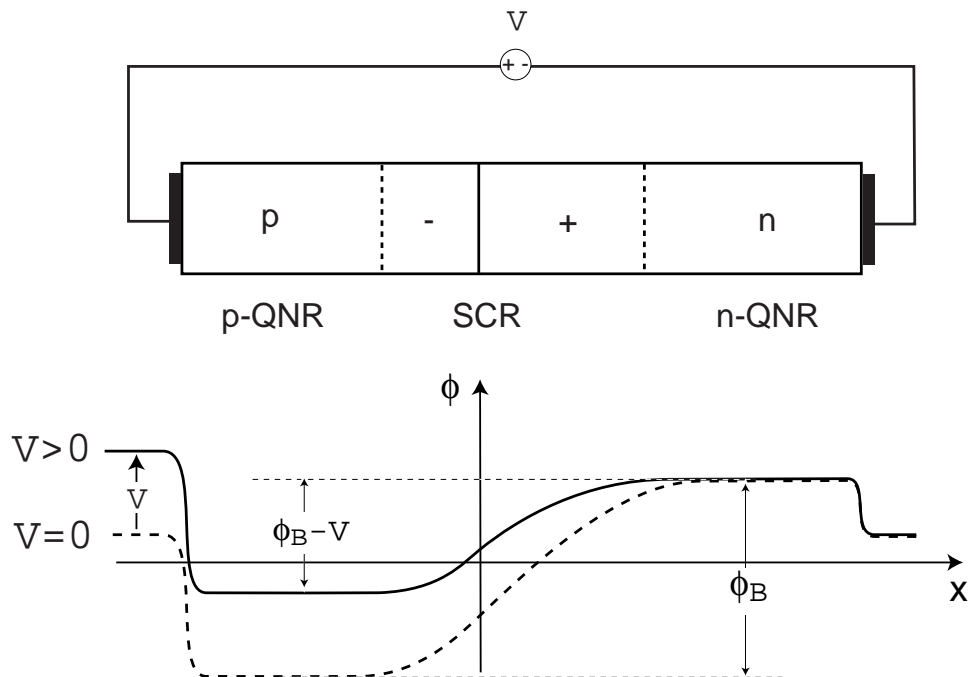
Five regions where  $V$  can drop:

- metal/p-QNR contact?
- p-QNR?
- SCR?
- n-QNR?
- metal/n-QNR contact?

In which region does  $V$  drop most?

Or, how is  $V$  distributed across diode?

Essentially, all applied voltage drops across SCR:

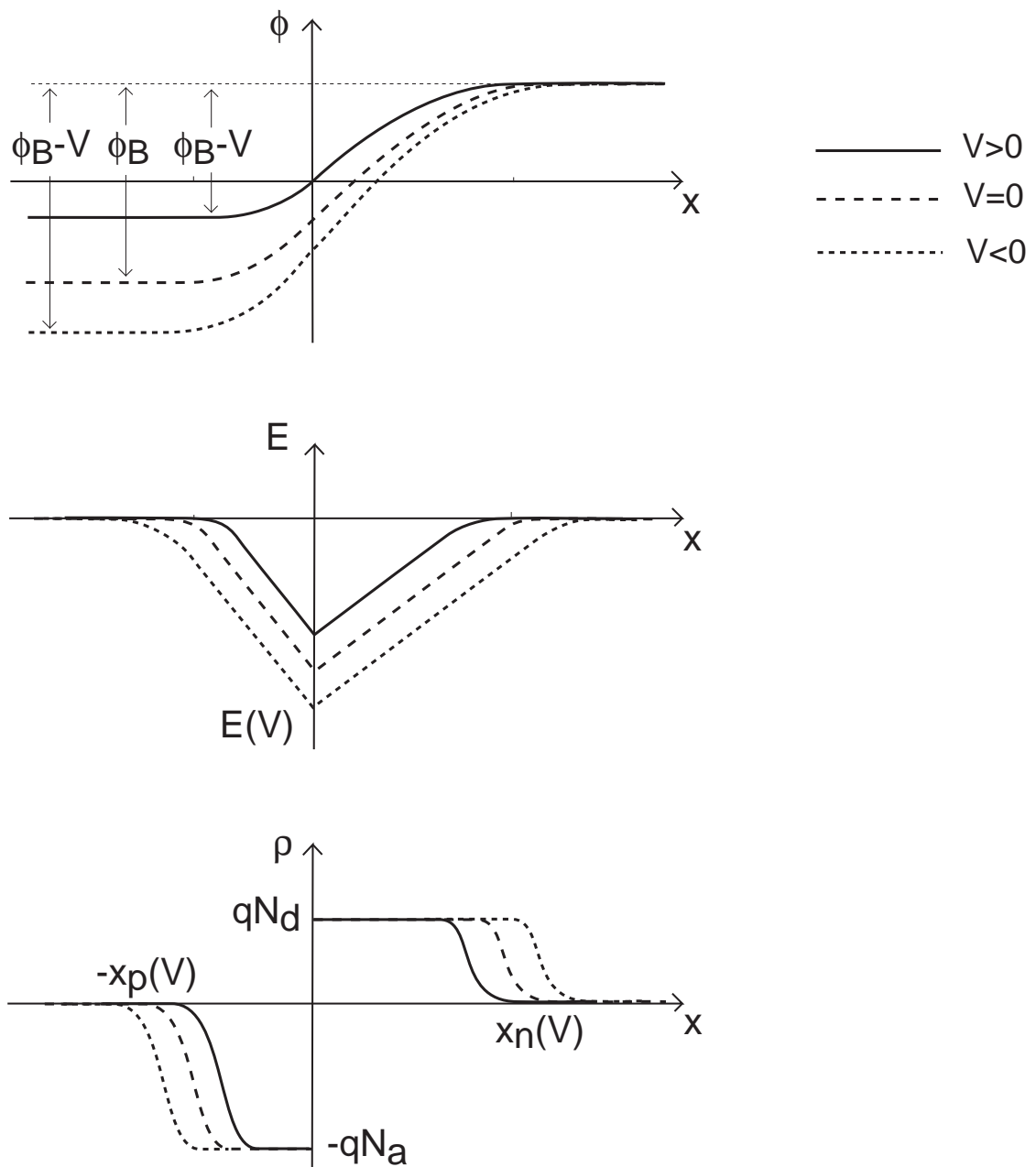


Potential difference across junction (potential "barrier"):

- in equilibrium:  $\phi_B$
- in forward bias:  $\phi_B - V < \phi_B$
- in reverse bias:  $\phi_B - V > \phi_B$  (since  $V < 0$ )

What happens to SCR electrostatics?

## SCR electrostatics under bias:



*forward bias:* built-in potential  $\downarrow \Rightarrow |E| \downarrow \Rightarrow x_d \downarrow$   
*reverse bias:* built-in potential  $\uparrow \Rightarrow |E| \uparrow \Rightarrow x_d \uparrow$

Fundamentally,

- electrostatics of SCR under bias unchanged from thermal equilibrium
- SCR dipole of charge modulated to accommodate modified potential build up across junction

Useful consequence:

- Analytical formulation of electrostatics of SCR identical to that of thermal equilibrium if:

$$\phi_B \longrightarrow \phi_B - V$$

Then, within depletion approximation:

$$x_n(V) = \sqrt{\frac{2\epsilon_s(\phi_B - V)N_a}{q(N_a + N_d)N_d}} \quad x_p(V) = \sqrt{\frac{2\epsilon_s(\phi_B - V)N_d}{q(N_a + N_d)N_a}}$$

$$x_d(V) = \sqrt{\frac{2\epsilon_s(\phi_B - V)(N_a + N_d)}{qN_aN_d}}$$

$$|E|(V) = \sqrt{\frac{2q(\phi_B - V)N_aN_d}{\epsilon_s(N_a + N_d)}}$$



Can all be rewritten as:

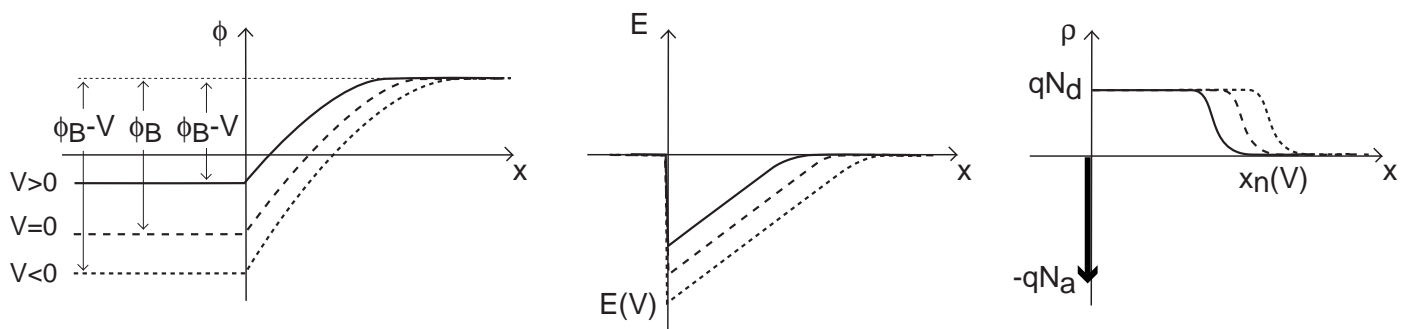
$$x_n(V) = x_{no} \sqrt{1 - \frac{V}{\phi_B}}$$

$$x_p(V) = x_{po} \sqrt{1 - \frac{V}{\phi_B}}$$

$$x_d(V) = x_{do} \sqrt{1 - \frac{V}{\phi_B}}$$

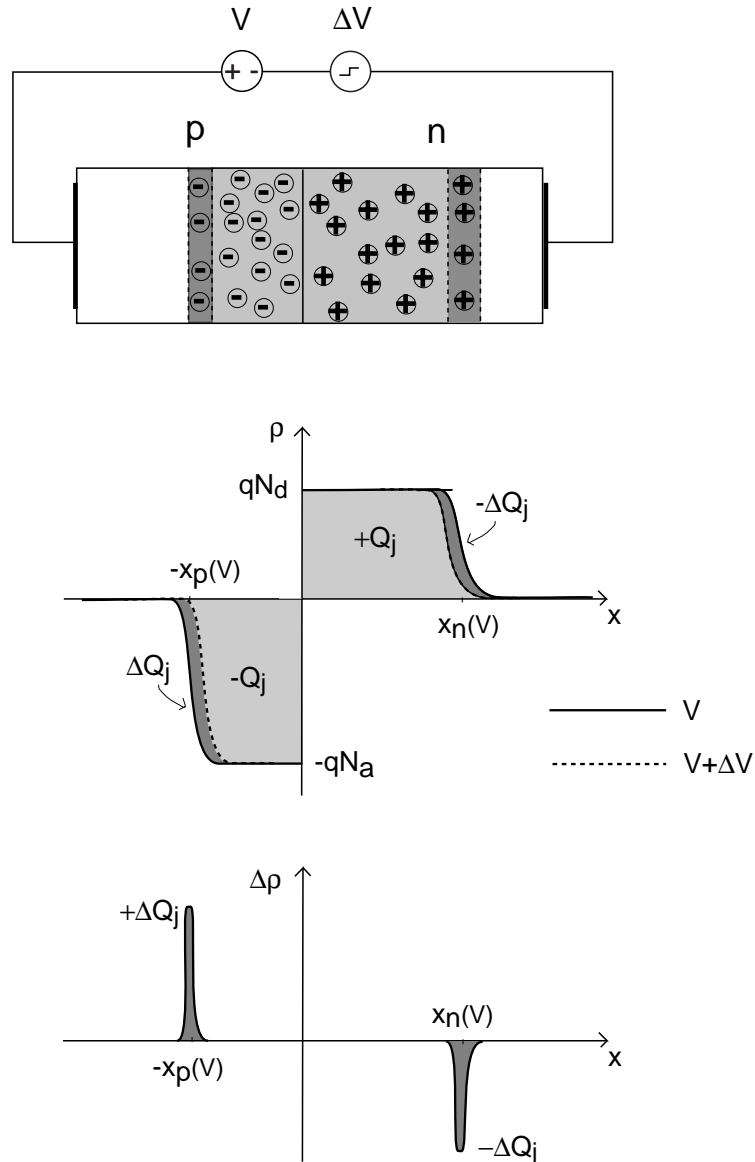
$$|E|(V) = |E_o| \sqrt{1 - \frac{V}{\phi_B}}$$

In strongly asymmetric junction, all changes take place in lowly doped side:



## 2. Depletion capacitance

Apply *small signal* on top of bias:

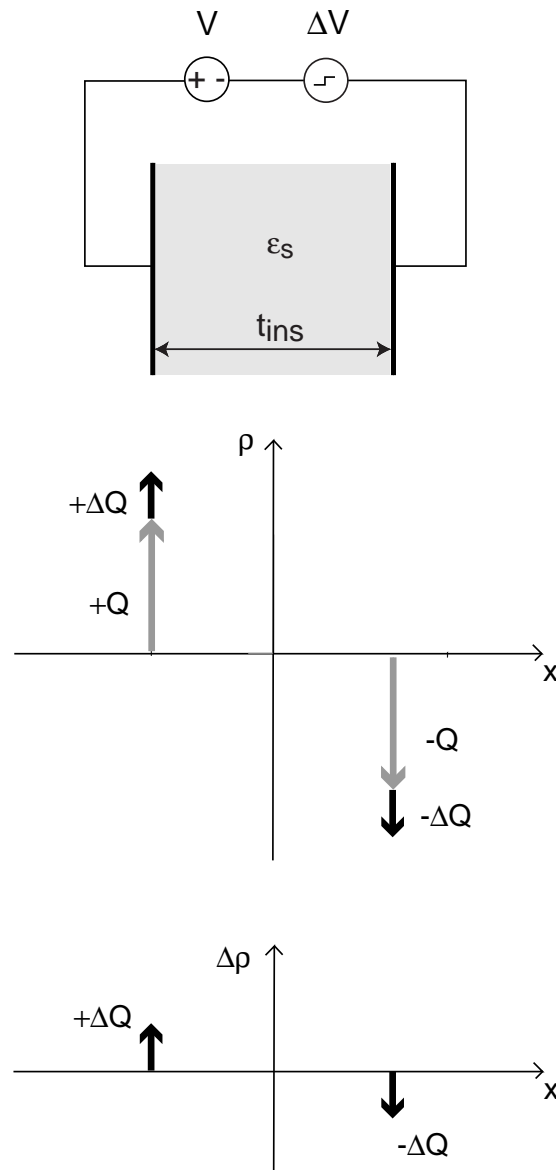


Change in  $\Delta V$  across diode:

$\Rightarrow$  change of  $\Delta Q_j$  at  $-x_p$

$\Rightarrow$  change of  $-\Delta Q_j$  at  $x_n$

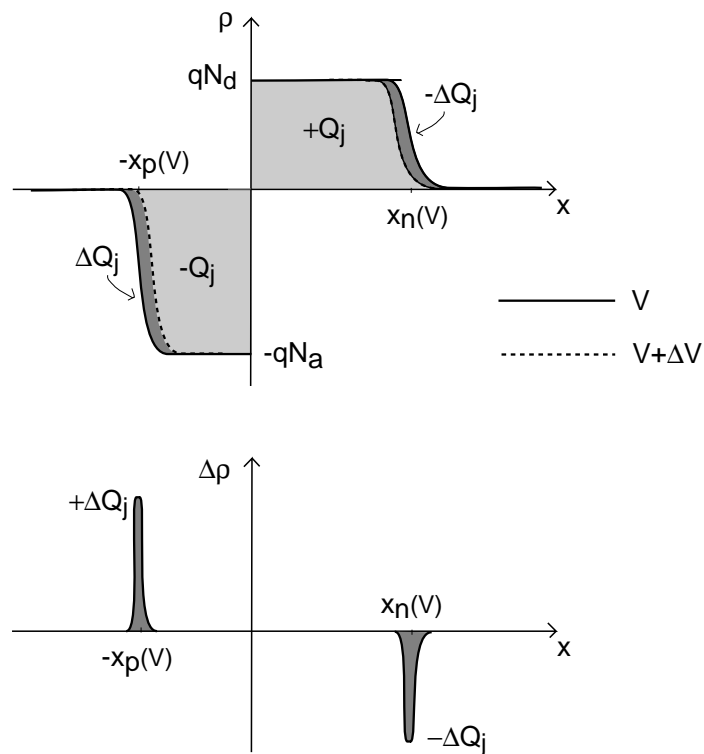
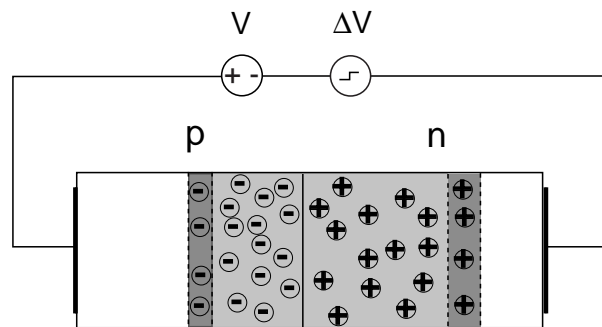
Looks like parallel-plate capacitor:



Capacitance per unit area:

$$C = \frac{\epsilon_s}{t_{ins}}$$

In analogy, in pn junction:



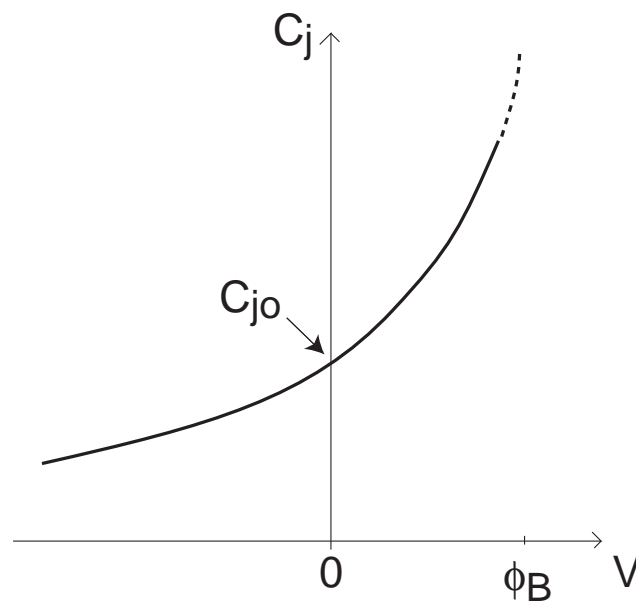
*Depletion capacitance* per unit area (depletion approx.):

$$C_j(V) = \frac{\epsilon_s}{x_d(V)} = \sqrt{\frac{q\epsilon_s N_a N_d}{2(\phi_B - V)(N_a + N_d)}} = \frac{C_{j0}}{\sqrt{1 - \frac{V}{\phi_B}}}$$

$$C_j(V) = \frac{\epsilon_s}{x_d(V)} = \sqrt{\frac{q\epsilon_s N_a N_d}{2(\phi_B - V)(N_a + N_d)}} = \frac{C_{j0}}{\sqrt{1 - \frac{V}{\phi_B}}}$$

Key dependencies of  $C_j$ :

- $C_j$  depends on bias (because  $x_d$  depends on bias)



- $C_j$  depends on doping:  $N_a, N_d \uparrow \Rightarrow C_j \uparrow$
- In strongly asymmetric junction (*i.e.* p<sup>+</sup>-n junction):

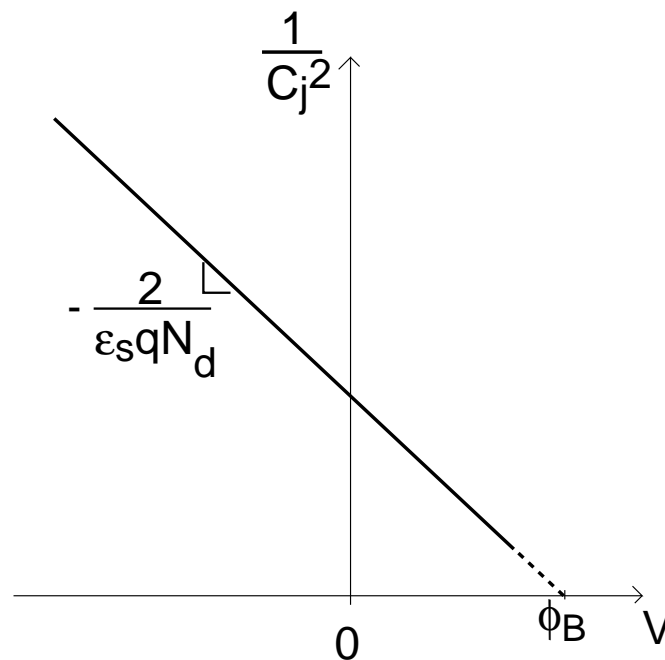
$$C_j(V) \simeq \sqrt{\frac{q\epsilon_s N_d}{2(\phi_B - V)}}$$

capacitance dominated by lowly-doped side.

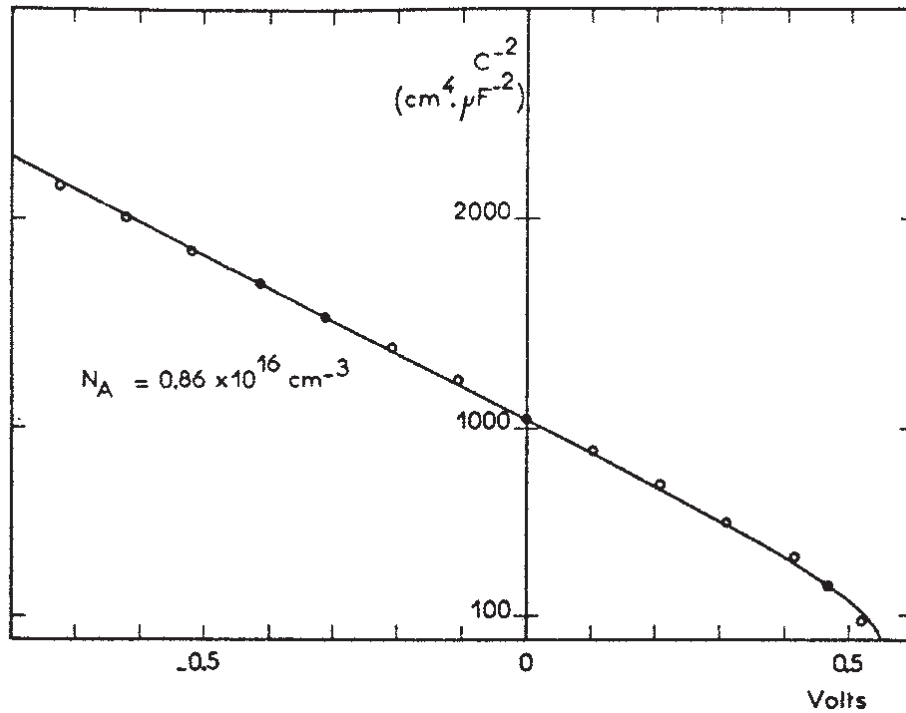
Relevance of capacitance-voltage characteristics of diode:

1. pn diode = variable capacitor (*varactor*):  
 $\Rightarrow$  useful for *voltage-controlled oscillators* (VCO)
2.  $C_j$ : important consideration in dynamics of pn diode
3. powerful characterization technique:  
*i.e.*  $1/C_j^2$  vs.  $V$  yields  $\phi_B$  and  $N_d$  in strongly asymmetric  $p^+$ -n junction:

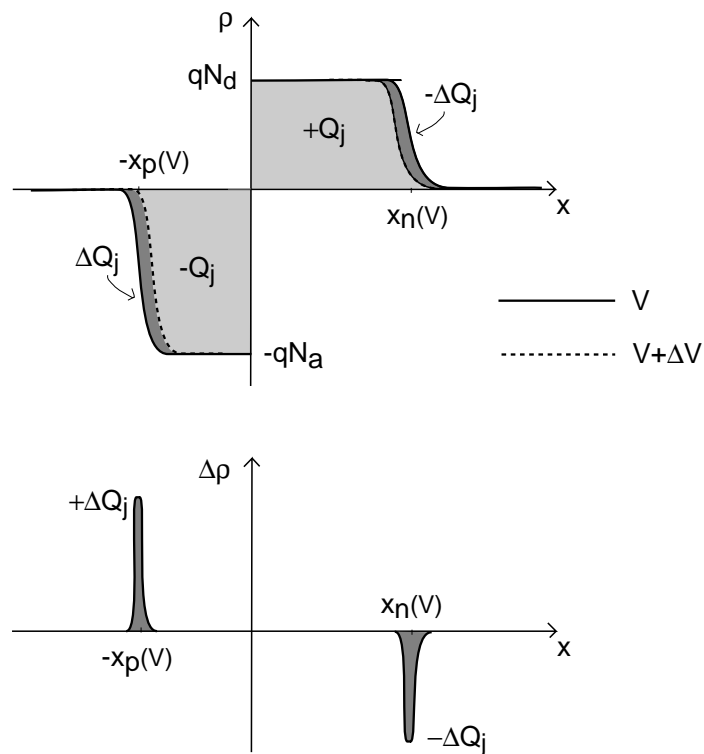
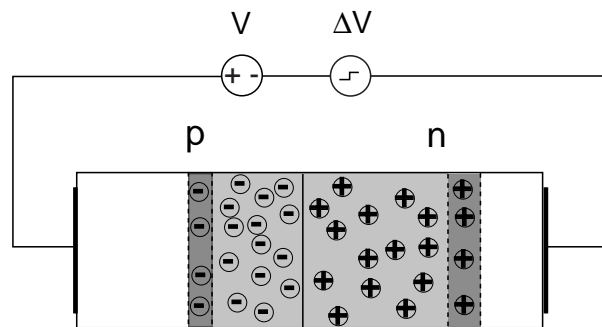
$$\frac{1}{C_j^2} \simeq \frac{2(\phi_B - V)}{q\epsilon_s N_d}$$



Experimental data [from *Fortini et al.*, *IEEE Trans. Electron Dev.* ED-29, 1604 (1982)]:



Appears in Fortini, A., A. Hairie, and M. Gomina. "Analysis and capacitive measurement of the built-in-field parameter in highly doped emitters." *IEEE Trans on Electron Devices* 29, no. 10 (1982): 1604 (© 1982 IEEE). Used with permission.

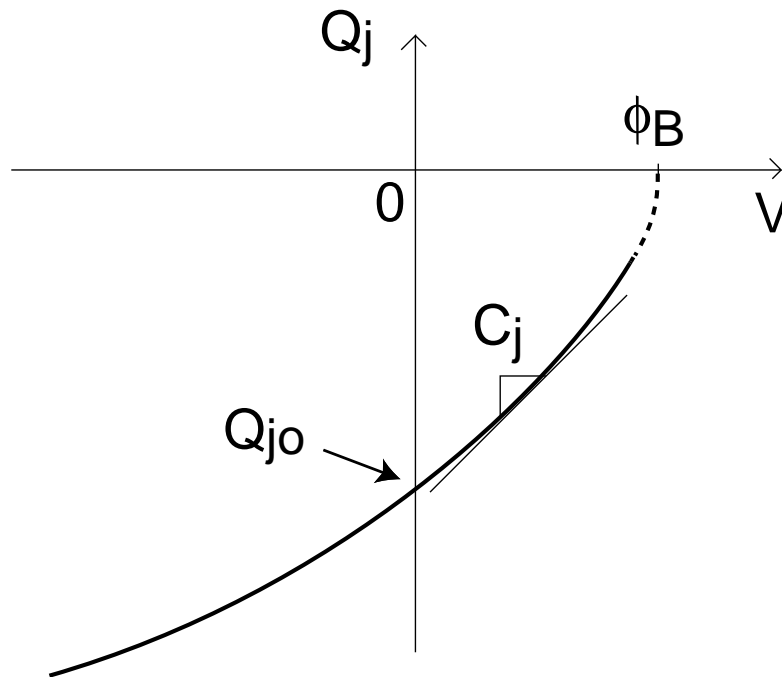
Alternative view of capacitance: *depletion charge*

Within depletion approximation:

$$Q_j(V) = \sqrt{\frac{2q\epsilon_s N_a N_d (\phi_B - V)}{N_a + N_d}} = Q_{jo} \sqrt{1 - \frac{V}{\phi_B}}$$



$$Q_j(V) = \sqrt{\frac{2q\epsilon_s N_a N_d (\phi_B - V)}{N_a + N_d}} = Q_{jo} \sqrt{1 - \frac{V}{\phi_B}}$$



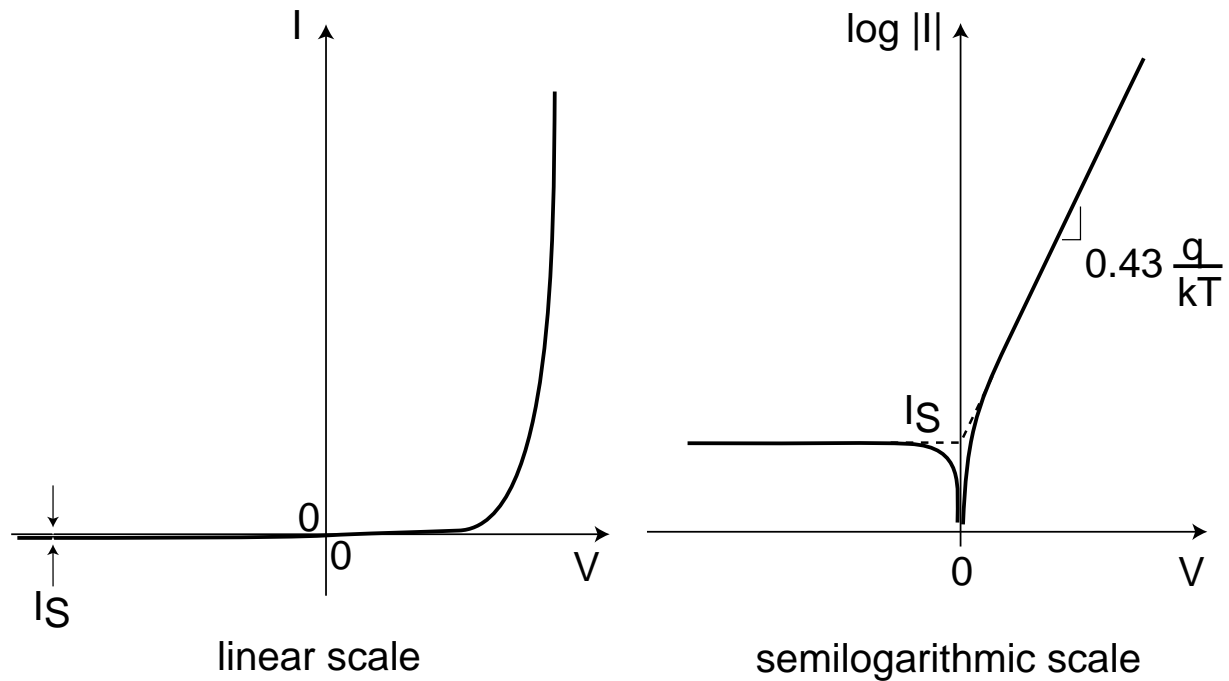
$C_j$  is slope of  $Q_j$  vs.  $V$  characteristics:

$$C_j = \frac{dQ_j}{dV}$$

**but not:**

$$C_j = \frac{Q_j}{V}$$

Application of voltage to pn junction also results in current: pn diode.



Will study after MOSFET and CMOS digital circuits.

## Key conclusions

- Voltage applied to pn junction drops across SCR:
  - ⇒ SCR electrostatics modified
    - in forward bias:  $x_d \downarrow$ ,  $|E| \downarrow$
    - in reverse bias:  $x_d \uparrow$ ,  $|E| \uparrow$
- Analytical formulation for SCR electrostatics in thermal equilibrium valid under bias if:

$$\phi_B \longrightarrow \phi_B - V$$

- As  $V$  changes, SCR charge changes too:
  - ⇒ *depletion capacitance*
- pn junction depletion capacitance depends on bias