S '01

March 14, 2001 - Quiz #1

problem	grade
1	
2	
3	
4	
5	
6	
total	

Name: SOLUTIONS

Recitation: ______________________

General guidelines (please read carefully before starting):

- Make sure to write your name on the space designated above.
- **Open book**: you can use any material you wish.
- All answers should be given in the space provided. Please do not turn in any extra material. If you need more space, use the back page.
- You have **120 minutes** to complete your quiz.
- Make reasonable approximations and *state them*, i.e. quasi-neutrality, depletion approximation, etc.
- Partial credit will be given for setting up problems without calculations. **NO** credit will be given for answers without reasons.
- Use the symbols utilized in class for the various physical parameters, i.e. μ_n, I_D, E, etc.
- Every numerical answer must have the proper units next to it. Points will be subtracted for answers without units or with wrong units.
- Use $\phi = 0$ at $n_o = p_o = n_i$ as potential reference.
- Use the following fundamental constants and physical parameters for silicon and silicon dioxide at room temperature:

$$n_i = 1 \times 10^{10}\ cm^{-3}$$
$$kT/q = 0.025\ V$$
$$q = 1.60 \times 10^{-19}\ C$$
$$\epsilon_s = 1.05 \times 10^{-12}\ F/cm$$
$$\epsilon_{ox} = 3.45 \times 10^{-13}\ F/cm$$

1. *(10 points)* Compute the equilibrium electron and hole concentrations, n_o and p_o, for silicon at room temperature doped with:

(1a) *(2 points)* Boron (B) concentration = $10^{17}\ cm^{-3}$.

p-type material with $N_a = 10^{17}\ cm^{-3}$

$$p_o = 10^{17}\ cm^{-3}$$
$$n_o = 10^{3}\ cm^{-3}$$

(1b) *(2 points)* Phosphorus (P) concentration = $5 \times 10^{16}\ cm^{-3}$ and Antimony (Sb) concentration = $5 \times 10^{16}\ cm^{-3}$.

Both n-type dopants, hence $N_d = 5\times 10^{16} + 5\times 10^{16} = 10^{17}\ cm^{-3}$

$$n_o = 10^{17}\ cm^{-3}$$
$$p_o = 10^{3}\ cm^{-3}$$

(1c) *(2 points)* Arsenic (As) concentration = $10^{17}\ cm^{-3}$ and Boron (B) concentration = $10^{16}\ cm^{-3}$.

Here there is some "compensation":

$N_d = 10^{17}\ cm^{-3}$, $N_a = 10^{16}\ cm^{-3}$ $\Rightarrow$ effectively $N_d - N_a = 9\times 10^{16}\ cm^{-3}$

$$n_o = 9\times 10^{16}\ cm^{-3}$$
$$p_o = 1.1\times 10^{3}\ cm^{-3}$$

(1d) *(4 points)* In (1a) above, what is the magnitude of the electric field that must be applied to the sample for the magnitude of the majority carrier drift velocity to be equal to $10^6\ cm/s$?

For p-type doping level of $N_a = 10^{17}\ cm^{-3}$, the hole mobility is $\mu_p \simeq 350\ cm^2/V.s$. Since hole velocity is given by

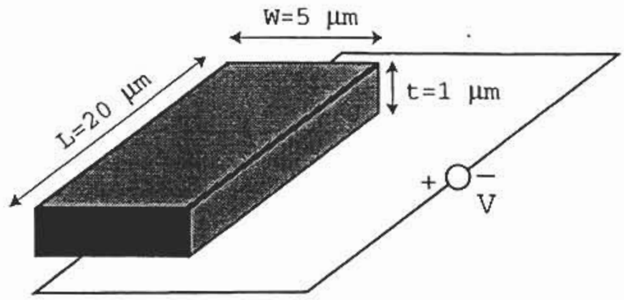
$$v_p = \mu_p E$$

Then

$$E = \frac{v_p}{\mu_p} = \frac{10^6\ cm/s}{350\ cm^2/V.s} = 2.9\times 10^{3}\ V/cm$$

2. *(10 points)* An engineer is told that a region of silicon of length 20 μm, width 5 μm and thickness 1 μm is uniformly doped with a single kind of dopant with a concentration of 10^{20} cm^{-3}. Ohmic contacts are formed at the ends of the region and she measures the I-V characteristics given in the table below. Is the sample n-type or p-type? Explain how you reach this conclusion. *[Hint: think about the sample resistance.]* Sample is at room temperature.

voltage (V)	current (A)
0	0
1	0.025
2	0.05



From the I-V characteristics we can derive the resistance of the sample:

$$R = \frac{V}{I} = \frac{2\ V}{0.05\ A} = 40\ \Omega$$

we can now derive the resistivity of the material since:

$$R = \rho \frac{L}{wt}$$

ρ is

$$\rho = R\frac{wt}{L} = 40\ \Omega\ \frac{5\times 10^{-4}\ cm \times 10^{-4}\ cm}{20\times 10^{-4}\ cm} = 10^{-3}\ \Omega\cdot cm$$

Looking at the chart of resistivity vs. doping level, we see that for $N_a = 10^{20}\ cm^{-3}$, the resistivity of Si at room temperature is precisely $10^{-3}\ \Omega\cdot cm$. Hence, this is a p-type sample.

at room temperature

3. *(10 points)* In a certain n-type region of a semiconductor in thermal equilibrium, there is a hole concentration with the following spatial distribution:

$$p_o(x) = 10^3(1 - 9 \times 10^3 x)\ cm^{-3} \qquad \text{for } 0 \le x \le 10^{-4} \text{ with } x \text{ in } cm$$

Assume that in this region, the electron mobility and hole mobilities are $\mu_n = 500\ cm^2/V \cdot s$ and $\mu_p = 200\ cm^2/V \cdot s$, respectively.

(3a.) *(5 points)* Derive an expression for and sketch the hole diffusion current density in this region.

Hole diffusion current density is given by:

$$J_p^{diff} = -q D_p \frac{dp_o}{dx}$$

D_p can be obtained using Boltzmann relation:

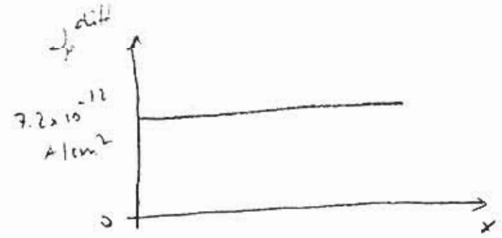
$$D_p = \frac{kT}{q} \mu_p = 0.025 \times 200 = 5\ cm^2/s$$

$\frac{dp_o}{dx}$ is given by

$$\frac{dp_o}{dx} = 10^3 (-9 \times 10^3) = -9 \times 10^6\ cm^{-4}$$

Then

$$J_p^{diff} = +1.6 \times 10^{-19}\ C \times 5\ cm^2/s \times 9 \times 10^6\ cm^{-4} = 7.2 \times 10^{-12}\ A/cm^2$$



(3b.) *(5 points)* Derive an expression for and sketch the electric field distribution in this region.

Since this is thermal equilibrium the sum of drift plus diffusion current must be zero. Then:

$$J_p = J_p^{diff} + J_p^{drift} = 0 \quad \Rightarrow \quad J_p^{drift} = -J_p^{diff}$$

The drift current is given by:

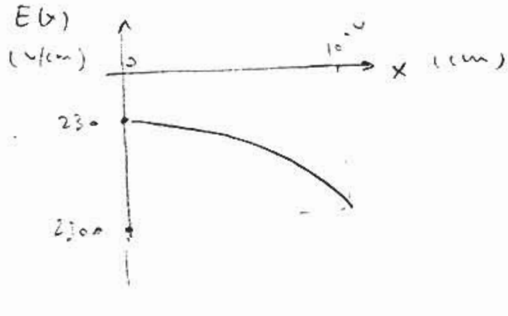
$$J_p^{drift} = q \mu_p p_0 E$$

Solving for E

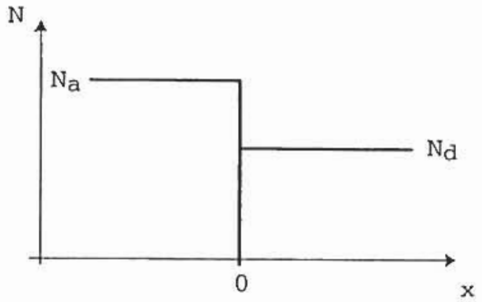
$$E = \frac{J_p^{drift}}{q \mu_p p_0} = -\frac{7.7\times10^{-12}}{1.6\times10^{-19} \times 200 \times 10^{3}\,(1-9\times10^{3}x)} = -\frac{2.3\times10^{2}}{1-9\times10^{3}x} \quad \text{V/cm}$$

At $x = 0$ this gives $E(0) = 230$ V/cm. At $x = 10^{-4}$ cm, $E(10^{-4}\,\text{cm}) = 2300$ V/cm

The electric field distribution looks like:



4. *(20 points)* Consider an abrupt pn junction with $N_a = 10^{17}\ cm^{-3}$ and $N_d = 10^{16}\ cm^{-3}$, as sketched below.



4a) *(6 points)* Compute the value of the electrostatic potential at $x = 0$ in thermal equilibrium *(numerical answer expected)*.

We can calculate this by integrating electric field starting from either side of the junction. Starting from the quasi-neutral n-side we have:

$$\phi(0) = \phi_n - \frac{qN_d x_{no}^2}{2\epsilon_s}$$

where x_{no} is the extent of the depletion region on the n side. This is given by:

$$x_{no} = \sqrt{\frac{2\epsilon_s \phi_B N_a}{q(N_a+N_d)N_d}}$$

We need:

$$\phi_n = 60 \times 6 = 360 \text{ mV}$$

$$\phi_B = \phi_n - \phi_p = 360 + 60 \times 7 = 0.78 \text{ V}$$

Then

$$\phi(0) = \phi_n - \frac{\cancel{q}\cancel{N_d}}{\cancel{2\epsilon_s}} \frac{\cancel{2\epsilon_s}\phi_B N_a}{\cancel{q}(N_a+N_d)\cancel{N_d}} = \phi_n - \frac{\phi_B N_a}{N_a+N_d} = 0.36 - \frac{0.78 \times 10^{17}}{1.1 \times 10^{17}} = -0.35 \text{ V}$$

4b) *(4 points)* Compute n_o and p_o at $x = 0$ in thermal equilibrium *(numerical answer expected)*.

Using the Boltzmann relation now:

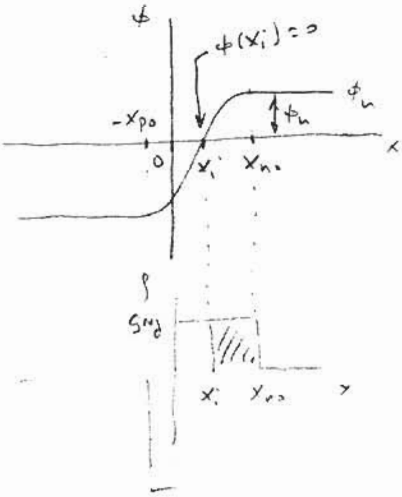
$$n_o(0) = n_i \exp \frac{q\phi(0)}{kT} = 10^{10} \exp - \frac{0.35}{0.025} = 8.3\times10^{3}\ cm^{-3}$$

$$p_o(0) = \frac{n_i^2}{n_o(0)} = \frac{10^{20}}{8.3\times10^{3}} = 1.2\times10^{16}\ cm^{-3}$$

call it x_i

4c) *(5 points)* Compute the value of x for which $n_o = p_o = n_i$ in thermal equilibrium *(numerical answer expected)*.

Since $N_a > N_d$, this point is to the right of the metallurgical junction, that is, $x_i > 0$. At x_i, $\phi(x_i) = 0$ by choice of potential reference. So we have to compute the extent of depletion region on the n side needed to drop a potential equal to ϕ_n. Using simple electrostatics:



$$\frac{qN_a(x_{no}-x_i)^2}{2\epsilon_s} = \phi_n$$

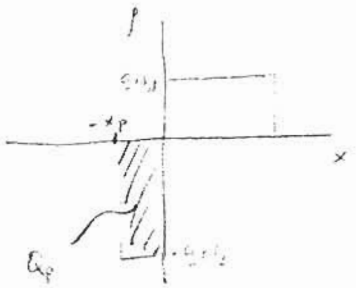
Solving for x_i:

$$x_i = x_{no} - \sqrt{\frac{2\epsilon_s\phi_n}{qN_d}} = \sqrt{\frac{2\epsilon_s\phi_B N_a}{q(N_a+N_d)N_d}} - \sqrt{\frac{2\epsilon_s\phi_n}{qN_d}} =$$

$$= \sqrt{\frac{2\epsilon_s}{qN_d}}\left(\sqrt{\frac{\phi_B N_a}{N_a+N_d}} - \sqrt{\phi_n}\right) = \sqrt{\frac{2\times1.05\times10^{-12}}{1.6\times10^{-19}\times10^{16}}}\left(\sqrt{\frac{0.78\times10^{17}}{1.1\times10^{17}}} - \sqrt{0.36}\right) =$$

$$= 8.8\times10^{-6}\ cm = 0.088\ \mu m$$

4d) *(5 points)* Compute the total amount of charge per unit area on the p side of the junction when a reverse bias voltage of 5 V is applied to the diode *(numerical answer with appropriate sign expected)*.



The total charge on the p-side of the diode is negative and is given by

$$Q_p = -qN_a x_p$$

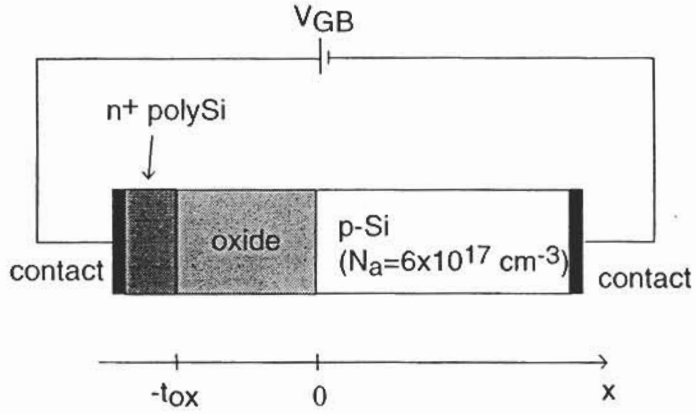
x_p is given by:

$$x_p = \sqrt{\frac{2\epsilon_s(\phi_B - V)N_d}{q(N_a+N_d)N_a}}$$

Then

$$Q_p = -\sqrt{\frac{2q\epsilon_s(\phi_B - V)N_d N_a}{N_a+N_d}} = -\sqrt{\frac{2\times1.6\times10^{-19}\times \overbrace{\quad}^{1.05\times10^{-12}} \times (0.78+5)\times10^{16}\times10^{17}}{1.1\times10^{17}}} = -1.3\times10^{-7}\ C/cm^2$$

5. *(30 points)* Consider the following MOS structure:



The oxide thickness is $t_{ox} = 5\ nm = 5 \times 10^{-7}\ cm$. To save you time, for this structure:

$$\gamma = \frac{1}{C_{ox}}\sqrt{2\epsilon_s q N_a} = 0.65\ V^{1/2}$$

(5a) *(5 points)* Compute the threshold voltage of the structure *(numerical answer with appropriate sign expected).*

$$V_T = V_{FB} - 2\phi_p + \gamma\sqrt{-2\phi_p}$$

$$V_{FB} = -\phi_B = -\phi_{n+} + \phi_p = -0.55 - 0.47 = -1.02\ V$$

$$\phi_p = -\frac{kT}{q}\ln\frac{N_a}{n_i} = -0.026\ln\frac{6\times10^{17}}{10^{10}} = -0.47\ V$$

Then

$$V_T = -1.02 + 2\times0.47 + 0.65\sqrt{2\times0.47} = 0.55\ V$$

(5b) *(5 points)* What is the value of V_{GB} that leads to a sheet charge density in the inversion layer of $Q_n = -10^{-6}\ C/cm^2$? *(numerical answer expected).*

Use MOS charge control relation:

$$Q_n = -C_{ox}(V_{GB} - V_T)$$

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.45 \times 10^{-13}}{5 \times 10^{-7}} = 6.9 \times 10^{-7}\ F/cm^2$$

Then

$$V_{GB} = V_T - \frac{Q_n}{C_{ox}} = 0.55 + \frac{10^{-6}}{6.9 \times 10^{-7}} = 2\ V$$

(5c) *(5 points)* What is the magnitude of E_{ox} (electric field across the oxide) for a condition in which $Q_G = -2 \times 10^{-7}\ C/cm^2$? *(numerical answer expected).*

Use Gauss' law:

$$|E_{ox}| = \frac{Q_G}{\epsilon_{ox}} = +\frac{2 \times 10^{-7}}{3.45 \times 10^{-13}} = 5.8 \times 10^{5}\ V/cm$$

(5d) *(5 points)* What is the magnitude of $E_s = E(x = 0^+)$ (electric field on semiconductor side of oxide-semiconductor interface) at threshold? *(numerical answer expected).*

At threshold

$$E_s = \frac{q N_a X_{dmax}}{\epsilon_s}$$

with X_{dmax} given by:

$$X_{dmax} = \sqrt{\frac{2\epsilon_s(-2\phi_p)}{qN_a}}$$

Then

$$E_s = \sqrt{\frac{qN_a 2 \ (-2\phi_p)}{\epsilon_s}} = \sqrt{\frac{1.6\times10^{-19} \times 6\times10^{17} \times 2 \times 2\times0.47}{1.05\times10^{-12}}} = 4.2\times10^{5} \ V/cm$$

(5e) *(10 points)* What is the capacitance of the MOS structure at a bias point for which the total charge in the semiconductor is equal to -2×10^{-7} C/cm^2? *(numerical answer expected).*

To check regime, we need to compute $Q_{Bmax} = qN_a X_{dmax}$. That is given by

$$|Q_{Bmax}| = \epsilon_s E_s(0)\Big|_{threshold} = 1.05\times10^{-12} \times 4.2\times10^{5} = 4.4\times10^{-7} \ C/cm^2$$

Since this is bigger than -2×10^{-7}, structure is in depletion

Thickness of depletion region is:

$$X_d = \frac{|Q_B|}{qN_a} = \frac{2\times10^{-7}}{1.6\times10^{-19} \times 6\times10^{17}} = 2.1\times10^{-6} \ cm$$

Capacitance is then:

$$C = \frac{1}{\frac{t_{ox}}{\epsilon_{ox}} + \frac{X_d}{\epsilon_s}} = \frac{1}{\frac{5\times10^{-7}}{3.45\times10^{-13}} + \frac{2.1\times10^{-6}}{1.05\times10^{-12}}} = 2.9\times10^{-7} \ F/cm^2$$

6. *(20 points)* Consider a MOSFET made out of the MOS structure of problem 5. The gate length is $L = 1\ \mu m$. The gate width is $W = 10\ \mu m$. The electron mobility in the channel is $200\ cm^2/V \cdot s$.

The MOSFET is biased with $V_{DS} = 0.1\ V$, $V_{GS} = 1\ V$ and $V_{BS} = 0\ V$. If you did not compute the threshold voltage of this structure in section (5a), assume it to be $0.5\ V$.

(6a) *(10 points)* Compute the magnitude of the inversion layer charge density at the source-end of the channel: $|Q_n(y = 0)|$ *(numerical answer expected)*.

This MOSFET is in the linear regime.

At the source, the inversion layer charge is simply given by:

$$|Q_n(y=0)| = C_{ox}(V_{GS} - V_T) = 6.9 \times 10^{-7}(1 - 0.55) =$$

$$= 3.1 \times 10^{-7}\ C/cm^2$$

(6b) *(10 points)* Compute the magnitude of the electron velocity at the source-end of the channel: $|v_y(y=0)|$ *(numerical answer expected)*.

Compute current first:

$$I_D = \frac{W}{L} \mu_n C_{ox} \left(V_{GS} - \frac{V_{DS}}{2} - V_T \right) V_{DS} =$$

$$= \frac{10}{1} \times 200 \times 6.9 \times 10^{-7} \left(1 - 0.05 - 0.55 \right) \times 0.1 = 55 \mu A$$

Now, the current is equal to the charge times the velocity times the width at the source:

$$I_D = W |Q_n(y=0)| |v_y(y=0)|$$

Then

$$v_y(y=0) = \frac{I_D}{W |Q_n(y=0)|} = \frac{55 \times 10^{-6}}{10 \times 10^{-4} \times 3.1 \times 10^{-7}} = 1.8 \times 10^{5} \text{ cm/s}$$