Outline : finish shape factors / aggregation
membrane mechanics · stretch, bend, shear
micropipette experiments - pure lipid bilayers
- neutrophils (white blood cells)
- red blood cells

□ Review of L # 19

- Cell structural components : membrane, cytoskeleton, nucleus & organelles, cytosol
- Geometric arguments for self assembly :
  - geometric packing with 3 parameters

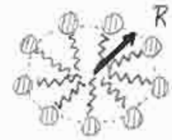
$\left\{ \begin{array}{ll} a_0 & \text{area occupied by head group} \\ v & \text{volume of tail} \\ l_c & \text{critical length* of hydrocarbon chain} \end{array} \right.$

$l_c \ll$ contour length
* over which it remains "fluid like"

| dimensionless group $\frac{v}{a_0 l_c}$ |

hence micelle  or bilayer 

- example spherical micelles $\left\{ \begin{array}{ll} \text{Radius} & R \\ \text{Area} & 4\pi R^2 \\ \text{Volume} & \frac{4}{3}\pi R^3 \end{array} \right.$, know a_0 and v for amphiphile



number of micelles $= \frac{4\pi R^2}{a_0} = \frac{4\pi R^3}{3v}$

hence $\frac{v}{a_0 R} = \frac{1}{3}$ and becouse $R \ll l_c$, we have $\boxed{\frac{v}{a_0 l_c} \ll \frac{1}{3}}$

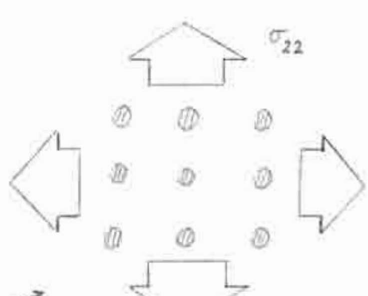
- Amphiphiles with 2 tails are a good biological design
  - low CMC
  - large residence times
  - spherical containers

energy cost for binding / closing $\ll$ hydrophobic energy at ends

□ Membrane mechanics

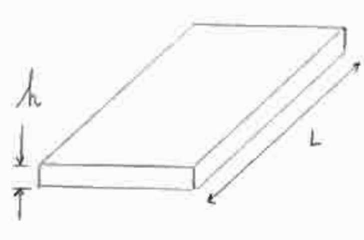
Major modes of deformation stretch / bend / shear

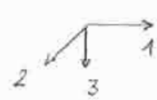
- Stretching



top view , stretching exposes hydrophobic core => resists

Model of a thin plate





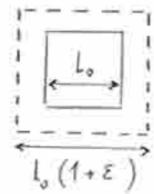
$h \ll L$
5 nm microns

- homogeneous ideal elastic . HIE hypothesis

$$\varepsilon_{11} = \frac{1-\nu}{E}\,\sigma_{11} - \frac{\nu}{E}\left(\sigma_{11} + \sigma_{22} + \cancel{\sigma_{33}}^{\,0}\right) \quad \text{and for } \varepsilon_{11} = \varepsilon_{22} = \varepsilon,\ \sigma_{11} = \sigma_{22} = \sigma$$

$$\boxed{\sigma = \frac{E}{1-\nu}\,\varepsilon}$$

- expansion of area $\dfrac{\Delta A}{A_0} = \dfrac{(L_0 + \varepsilon L_0)^2 - L_0^2}{L_0^2} \simeq 2\varepsilon$



$$\boxed{\sigma = \frac{E}{2(1-\nu)}\cdot\frac{\Delta A}{A_0}}$$

- define a surface tension N : $[N] = [\text{Force} / \text{Length}]$

$[\sigma] = [\text{Force} / \text{Length}^2]$

$N_1 = \int_{-h/2}^{h/2} \sigma_{11}\, dx_3$: integrate over width of plate , so here $\sigma h = N$

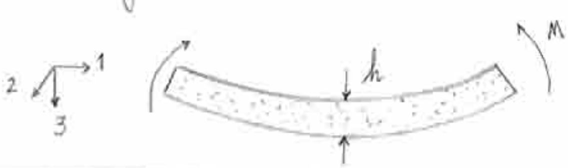
$$\boxed{N = \frac{Eh}{2(1-\nu)}\cdot\frac{\Delta A}{A_0} = K_e\,\frac{\Delta A}{A_0}}$$

with area expansion modulus for HIE $\boxed{K_e = \dfrac{Eh}{2(1-\nu)}}$

- Typical numbers for K_e : pure lipid bilayer ~ 0.1 - 0.2 N/m
  red blood cell (cortex) ~ 0.45 N/m (Evans & Waugh)
  water / air interface ~ 0.07 N/m

rupture for modest strains : $\dfrac{\Delta A}{A_0} \sim 2 - 4\ \%$

• Pure bending



by definition $M = -K_B \dfrac{\partial^2 u_3}{\partial x_1^2}$

M moment
K_B bending modulus
curvature $\sim \dfrac{1}{R}$

$$\boxed{K_B = \frac{Eh^3}{12(1-\nu^2)}}$$ bending modulus for HIE

- Typical values of K_B (energy) : pure lipid bilayer ~ 10 - 30 kT
  red blood cell ~ 10 - 30 kT

• Shear top view

- define membrane shear modulus $\sigma_{21} = 2G\,\varepsilon_{21}$

shear tension : $N_{21} = 2h\,G\,\varepsilon_{21}$ by integrating over the thickness

$\boxed{K_S = 2hG}$ membrane shear modulus

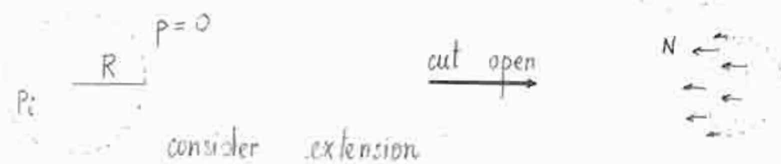
- experiments very easy to shear . pure lipid bilayers $K_S \ll 1$
  red blood cell ~ 2.5 $\mu N/m$ (Hérion 1999)

if HIE $G = \dfrac{E}{2(1-\nu)}$ => $\begin{cases} G \simeq E \\ \text{shear modulus \& Young's modulus have the same order of magnitude} \end{cases}$

because $\left.\begin{array}{l} K_S \simeq h\,G \\ K_E \simeq h\,E \end{array}\right\}$ would expect $K_S \simeq K_E$, but very far off! HIE not perfect

- Osmotic swelling
- create a vesicle

Radii by light scattering

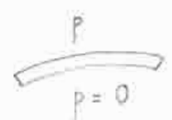


consider extension only => surface tension N

$2\pi R N = p_i \pi R^2$
(projection onto cut plane
surface tension = pressure)

$$\boxed{P_i = \frac{2N}{R}}$$ Laplace's equation

- Scaling analysis of bending & tension (details in Roger Kamm's manuscript Ch. 2.1 p. 13)



$$P + \underbrace{N \underbrace{\frac{\partial^2 u_3}{\partial x_1^2}}_{\text{curvature}}}_{\text{surface tension}} - \underbrace{K_B \frac{\partial^4 u_3}{\partial x_1^4}}_{\text{bending}} = 0$$

- order of magnitude of these two effects : - bending $K_B \dfrac{\partial^4 u_3}{\partial x_1^4} \sim K_B \dfrac{u_3}{L^4}$

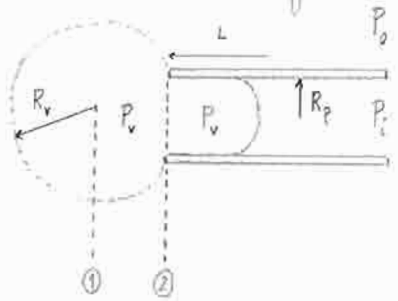
  - tension $N \dfrac{\partial^2 u_3}{\partial x_1^2} \sim N \dfrac{u_3}{L^2}$

L ≡ characteristic dimension over which u_3 varies

$$\boxed{\frac{\text{bending}}{\text{tension}} \sim \frac{K_B}{N L^2}}$$

$\begin{cases} \text{if} \ll 1 & \text{get rid of bending term in balance equation} \\ \text{if} \gg 1 & \text{" tension " "} \end{cases}$

- Micropipette aspiration only consider N



- force balance

① $P_v - P_o = \dfrac{2N}{R_v}$

② $P_v - P_i = \dfrac{2N}{R_p}$

$$\boxed{N = \frac{1}{2}\left(P_o - P_i\right)\left(\frac{1}{R_p} - \frac{1}{R_v}\right)^{-1}}$$

- get $\dfrac{\Delta A}{A_o}$ from geometric arguments

$$\frac{\Delta A}{A_o} = \frac{R_p^2}{2 R_v^2}\left(\frac{1}{R_p} - \frac{1}{R_v}\right) L$$

lipid bilayer N ↗ with strain
neutrophil . constant N
ruffled membrane