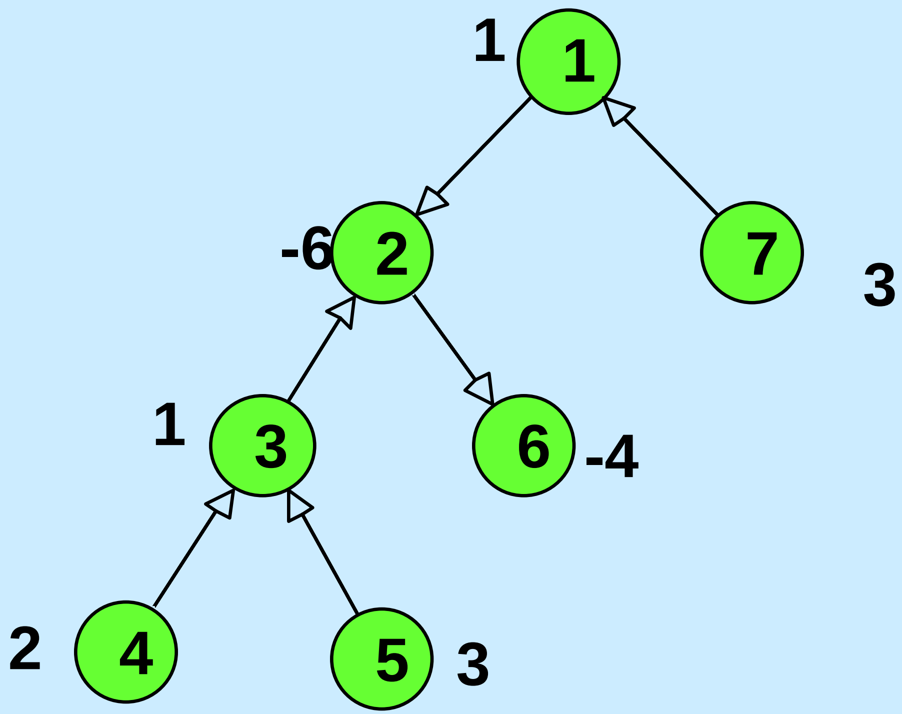
# 15.082J and 6.855J and ESD.78J

## Network Simplex Animations

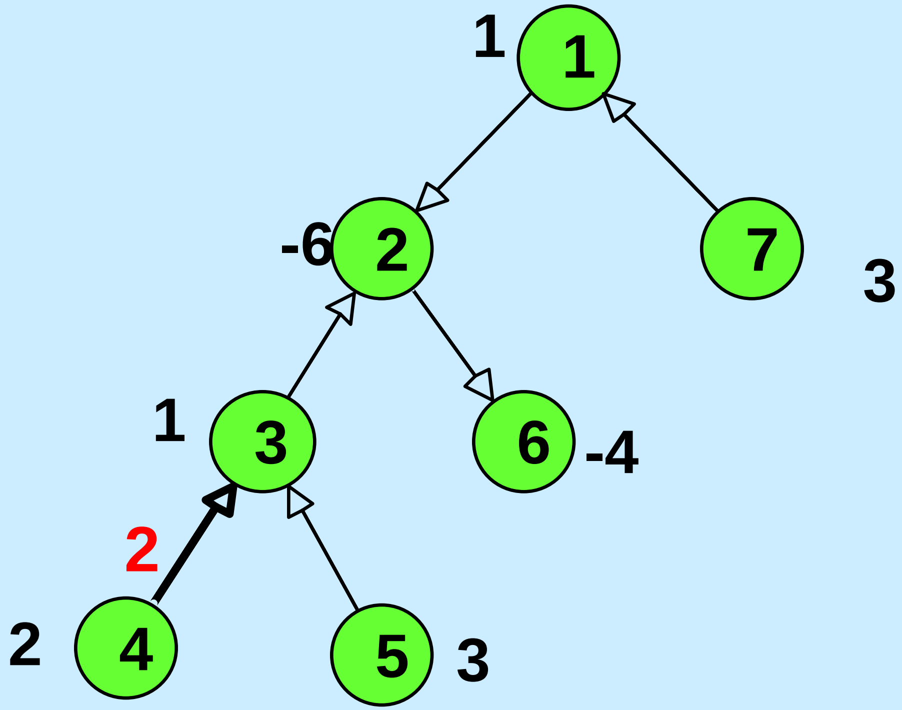
# Calculating A Spanning Tree Flow




A tree with supplies and demands. (Assume that all other arcs have a flow of 0)

What is the flow in arc (4,3)?

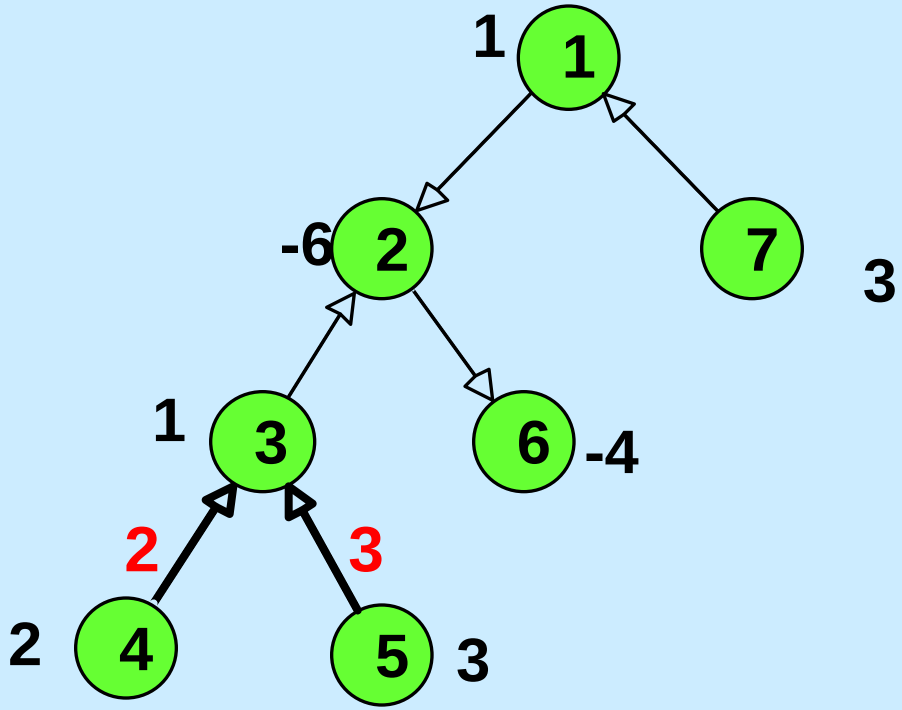
# Calculating A Spanning Tree Flow




To calculate flows, iterate up the tree, and find an arc whose flow is uniquely determined.

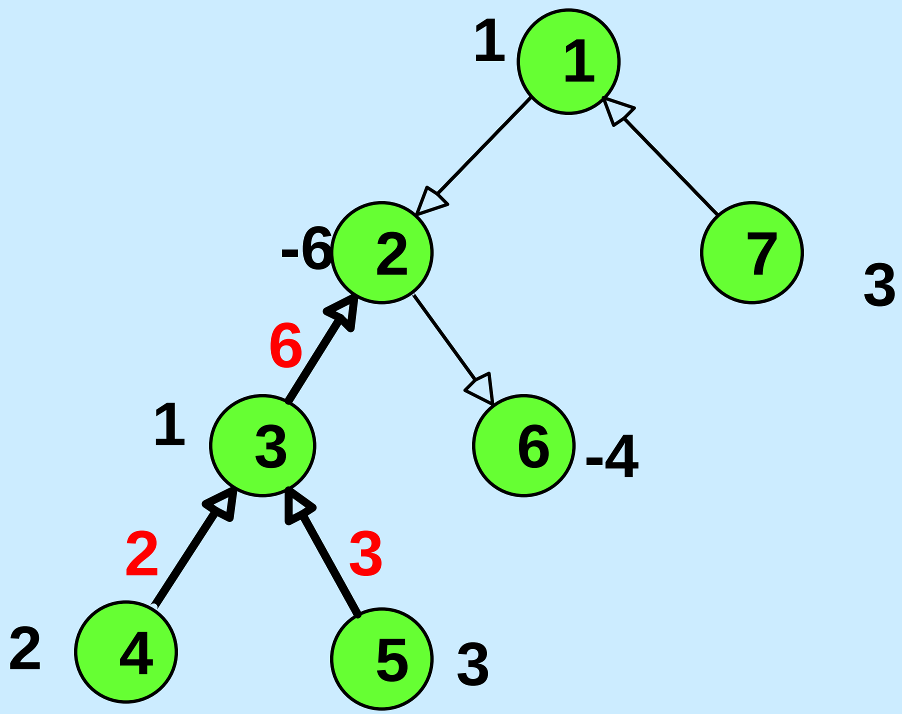
What is the flow in arc (5,3)?

# Calculating A Spanning Tree Flow



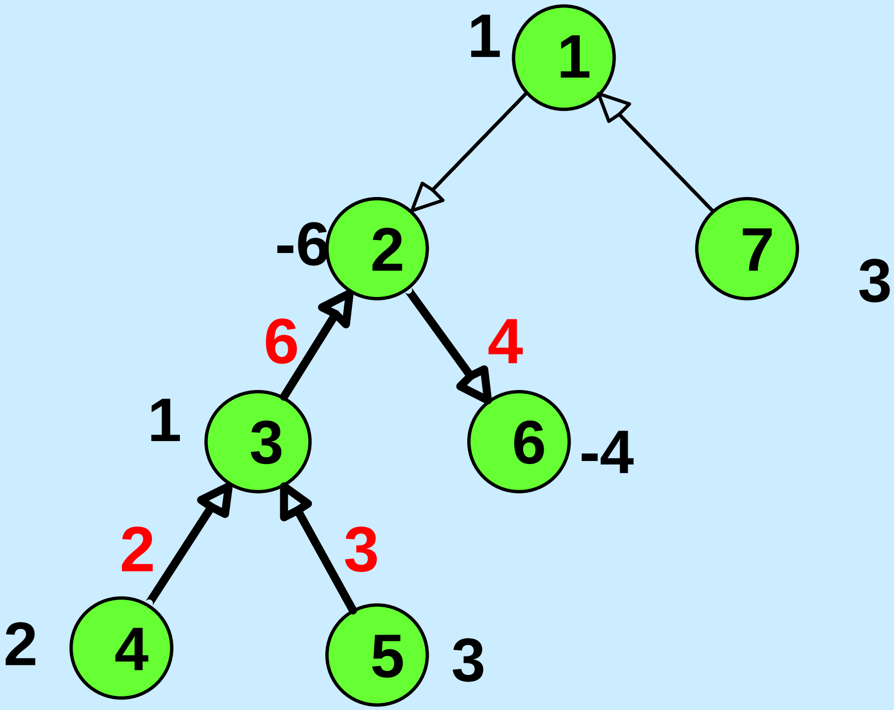

What is the flow in arc (3,2)?

# Calculating A Spanning Tree Flow



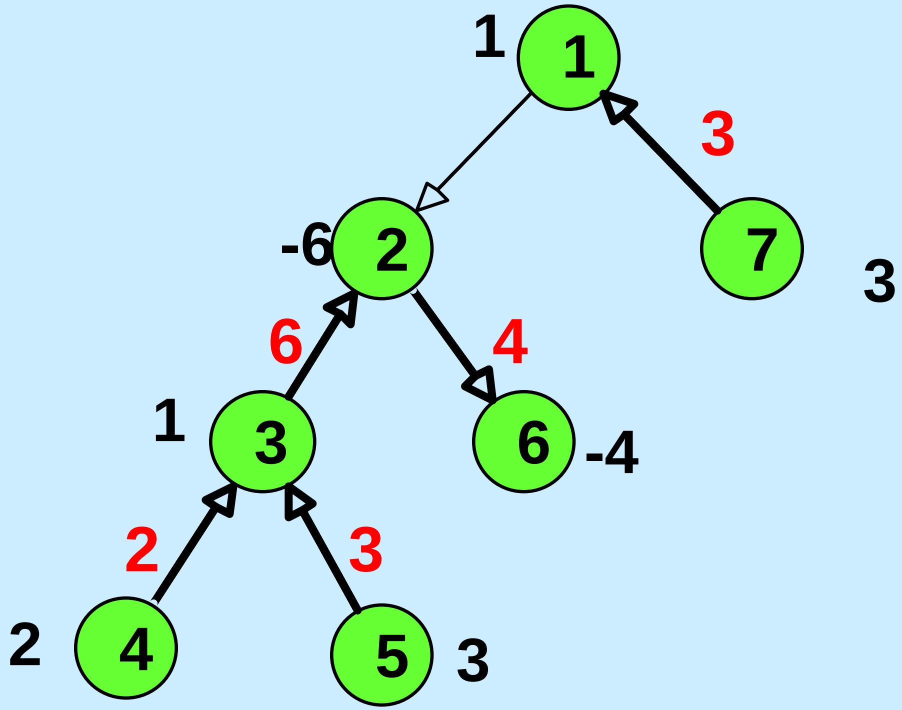

What is the flow in arc (2,6)?

# Calculating A Spanning Tree Flow



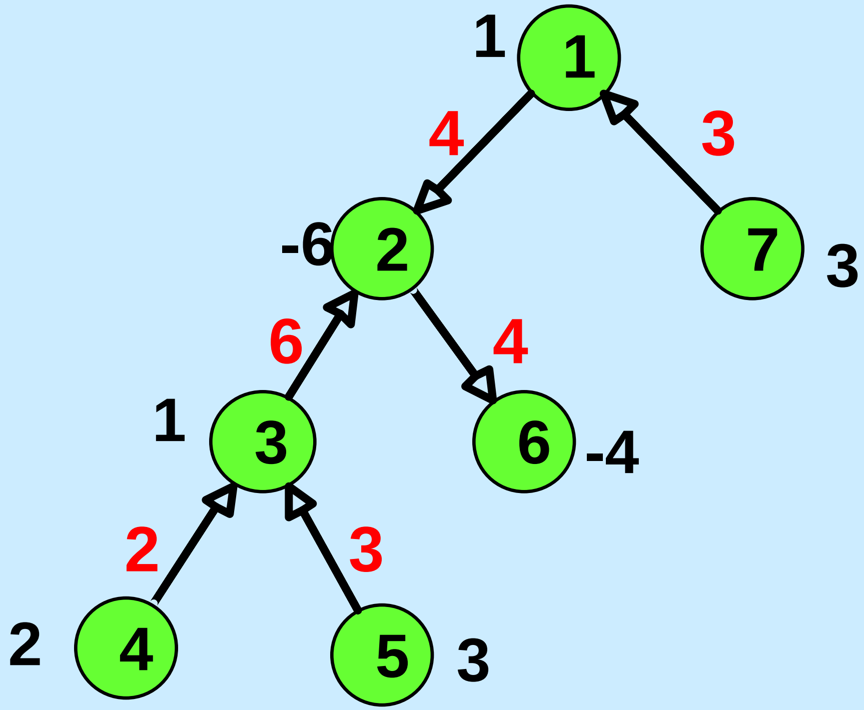

What is the flow in arc (7,1)?

# Calculating A Spanning Tree Flow



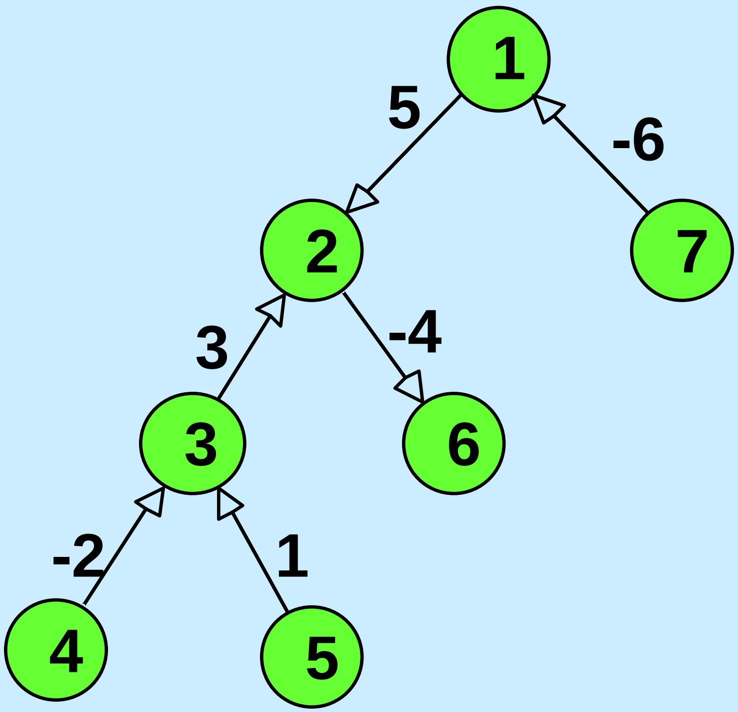

What is the flow in arc (1,6)?

# Calculating A Spanning Tree Flow




**Note**: there are two different ways of calculating the flow on (1,2), and both ways give a flow of 4. Is this a coincidence?

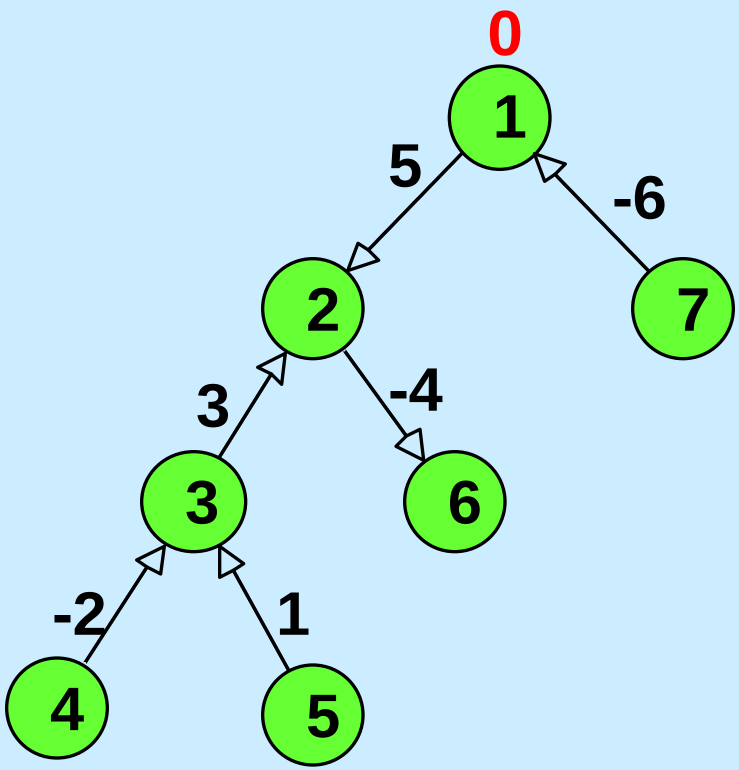
# Calculating Simplex Multipliers for a Spanning Tree




Here is a spanning tree with arc costs. How can one choose node potentials so that reduced costs of tree arcs is 0?

Recall: the reduced cost of (i,j) is
$c_{ij} - \pi_i + \pi_j$

# Calculating Simplex Multipliers for a Spanning Tree

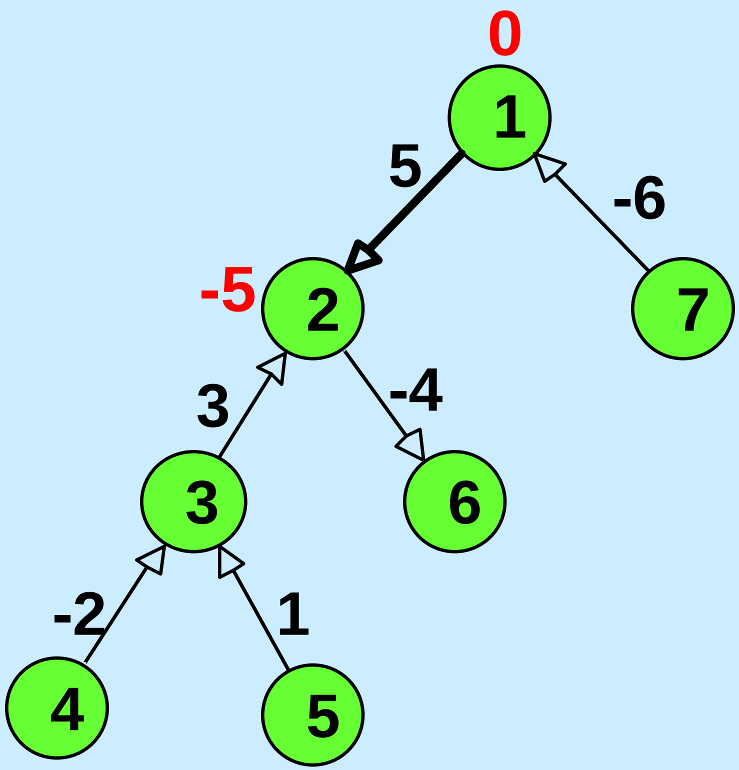



There is a redundant constraint in the minimum cost flow problem.

One can set $\pi_1$ arbitrarily. We will let $\pi_i = 0$.

What is the simplex multiplier for node 2?

# Calculating Simplex Multipliers for a Spanning Tree

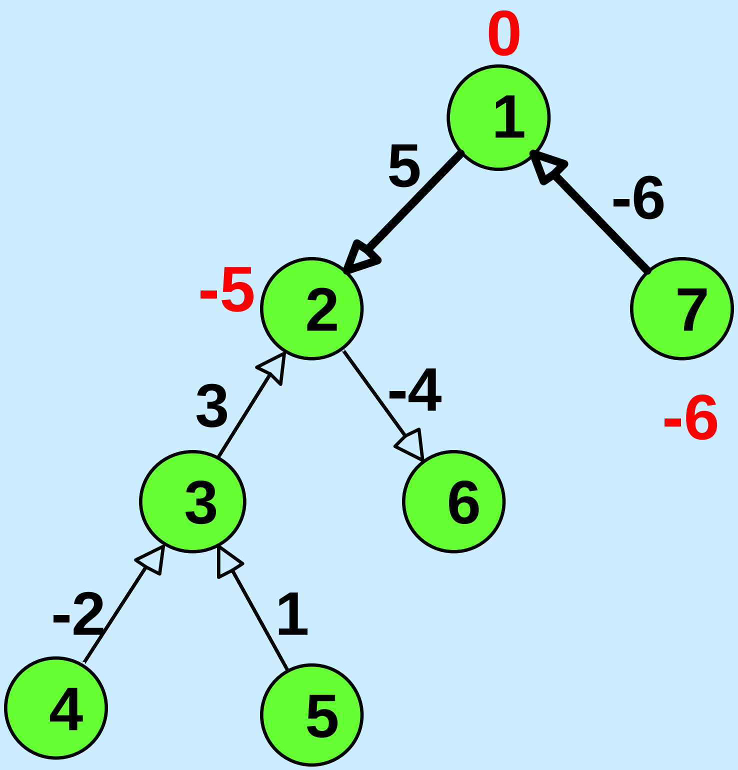



The reduced cost of (1,2) is $c_{12} - \pi_1 + \pi_2 = 0$.

Thus $5 - 0 + \pi_2 = 0$.

What is the simplex multiplier for node 7?

# Calculating Simplex Multipliers for a Spanning Tree

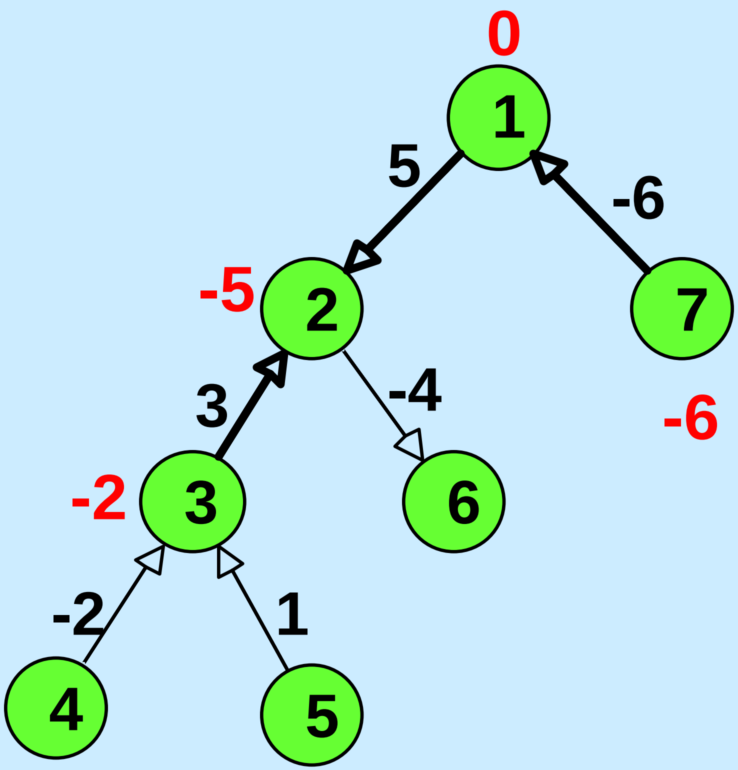



The reduced cost of (1,2) is $c_{71}$ - $\pi_7$ + $\pi_1$ = 0.

Thus -6 - $\pi_2$ +0 = 0.

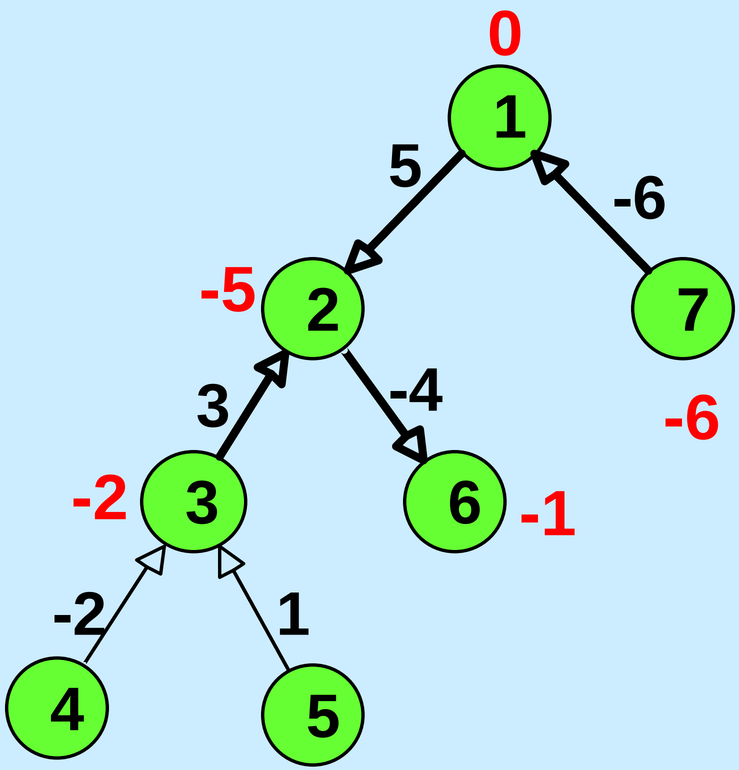
What is the simplex multiplier for node 3?

# Calculating Simplex Multipliers for a Spanning Tree



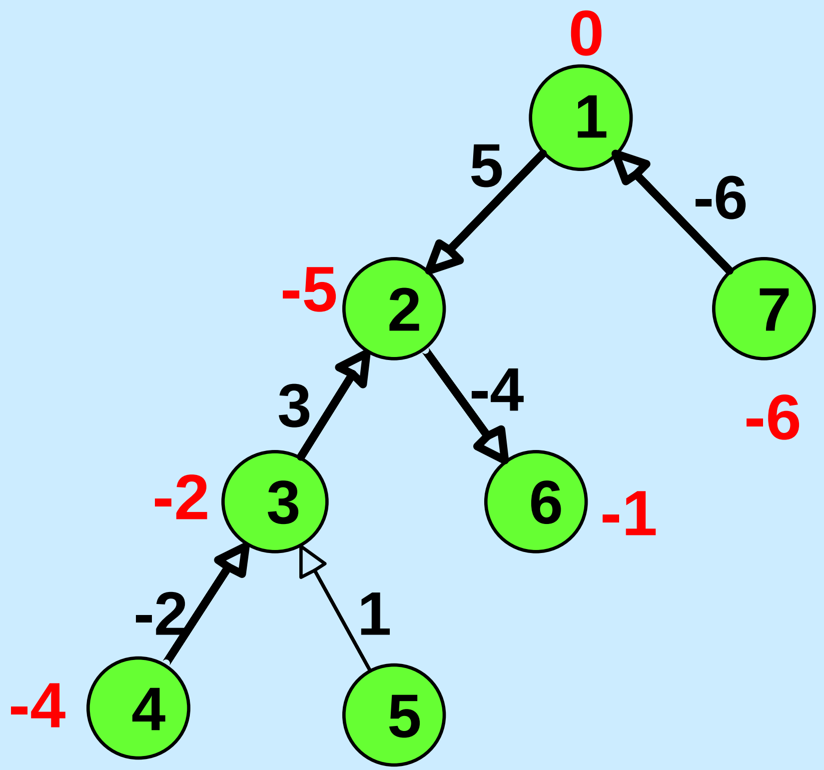

**What is the simplex multiplier for node 6?**

# Calculating Simplex Multipliers for a Spanning Tree



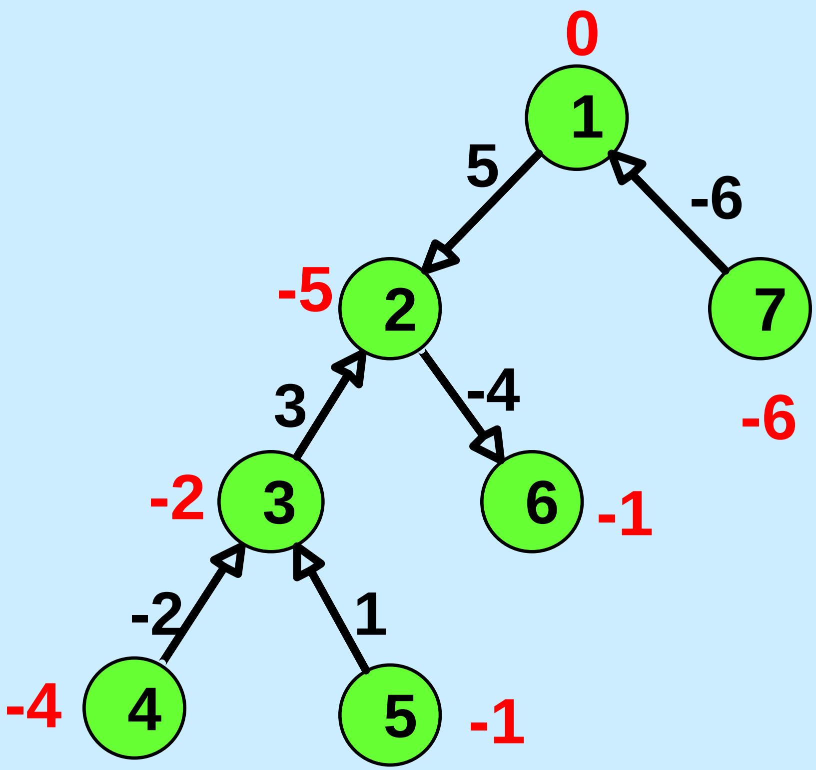

**What is the simplex multiplier for node 4?**

# Calculating Simplex Multipliers for a Spanning Tree



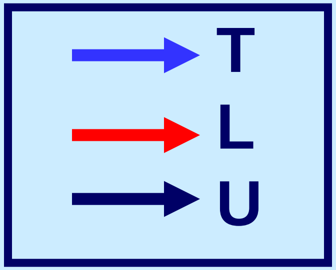

What is the simplex multiplier for node 5?

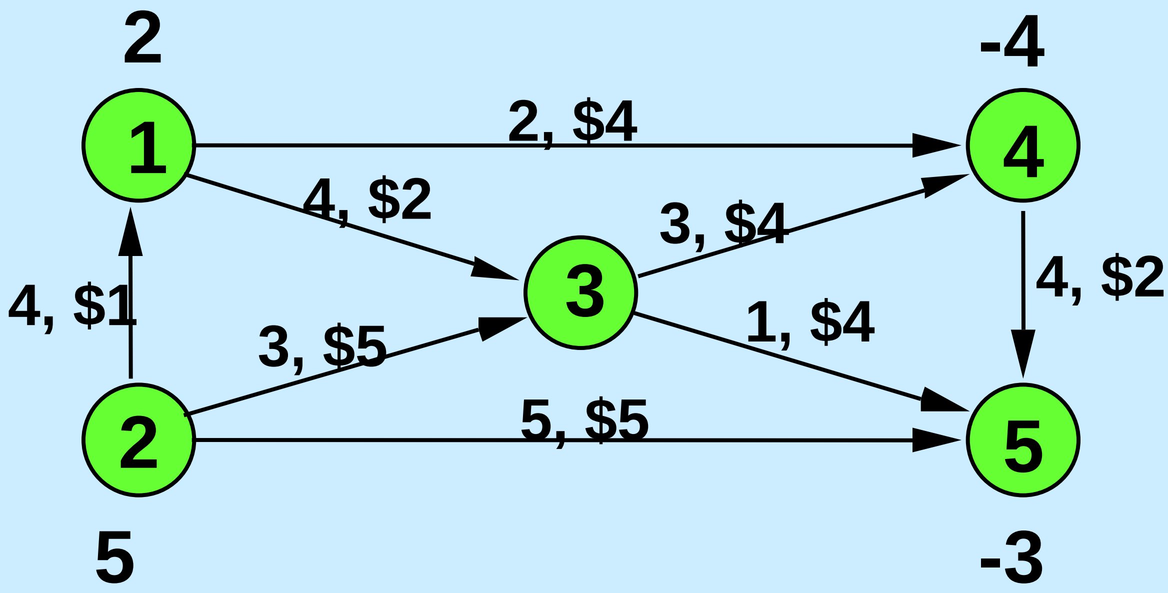
# Calculating Simplex Multipliers for a Spanning Tree




These are the simplex multipliers associated with this tree. They do not depend on arc flows, nor on costs of non-tree arcs.

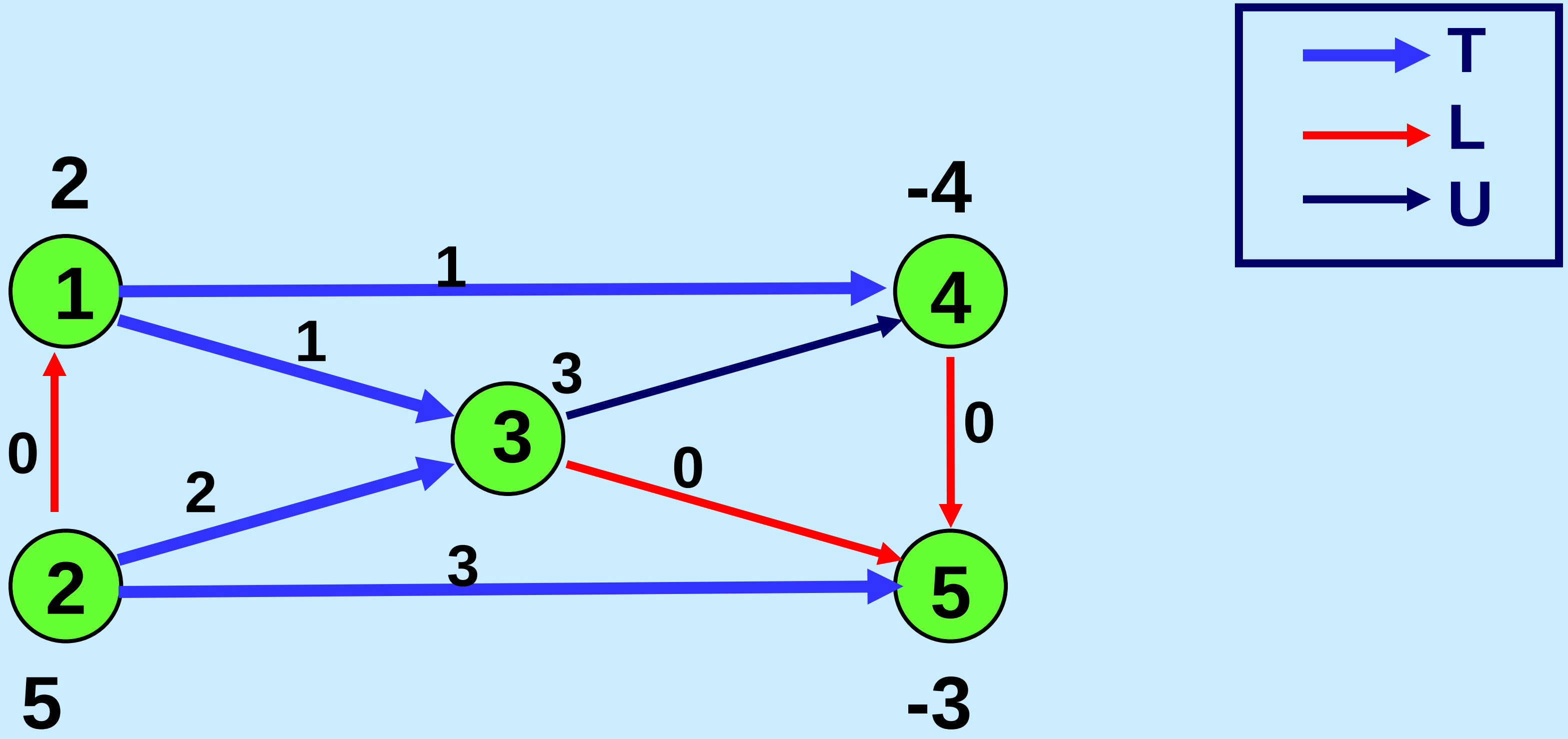
# Network Simplex Algorithm



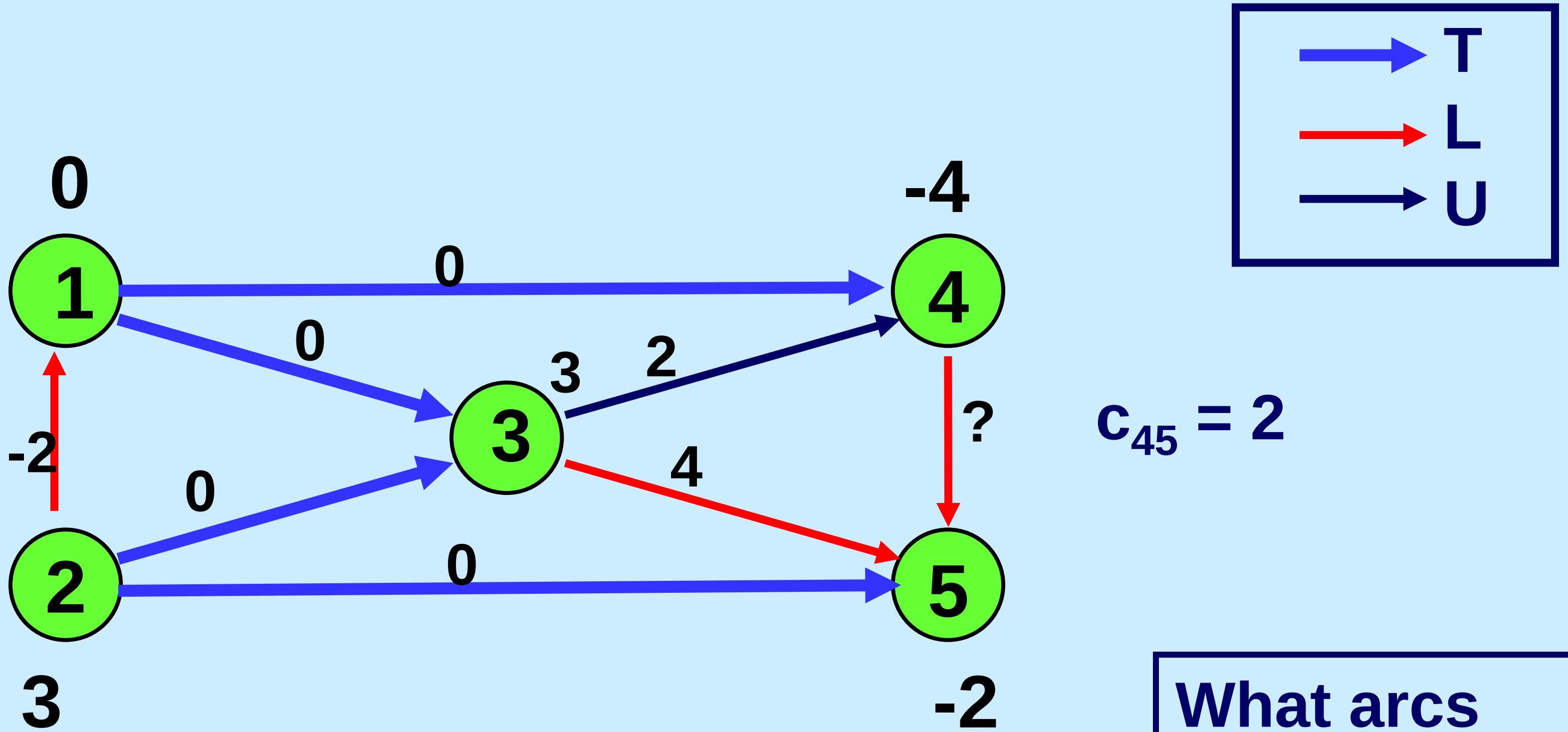

**The minimum Cost Flow Problem**

# Spanning tree flows




**An Initial Spanning Tree Solution**

# Simplex Multipliers and Reduced Costs

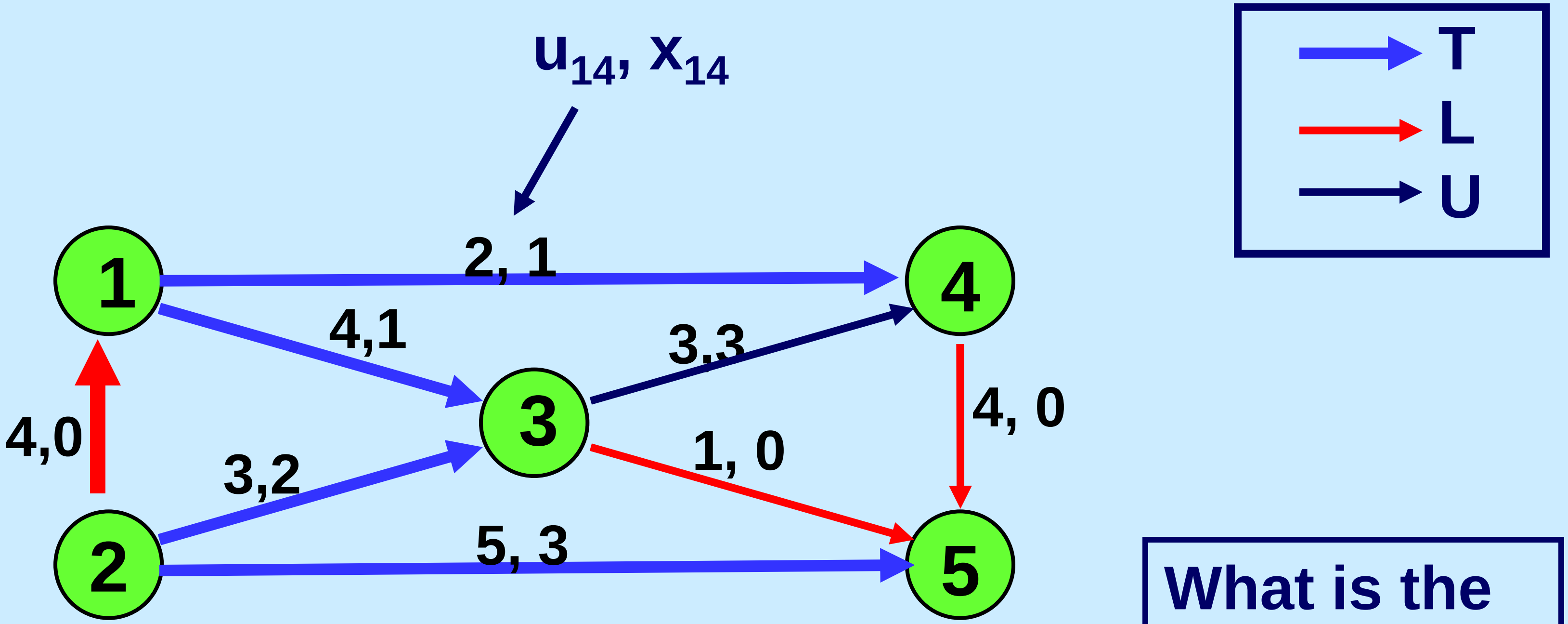



$c_{45} = 2$

What arcs are violating?

The initial simplex multipliers and reduced costs

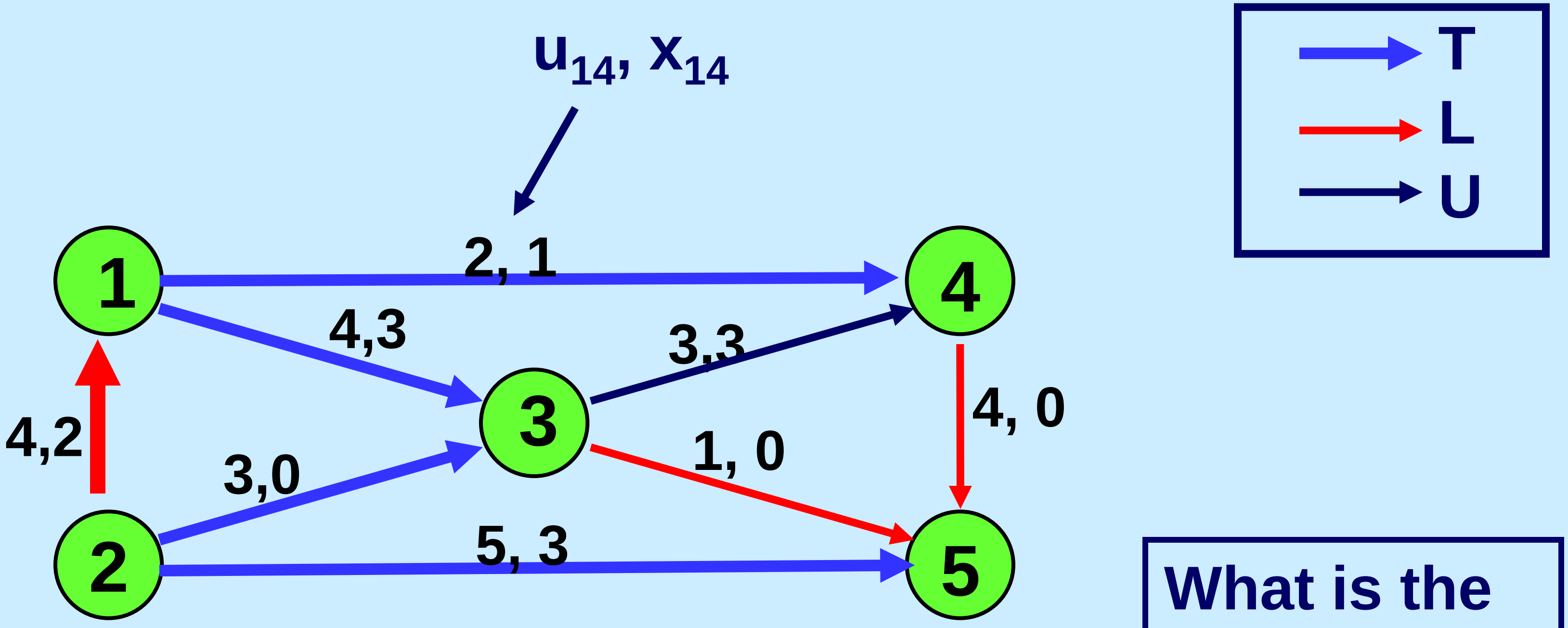
# Add a violating arc to the spanning tree, creating a cycle




**Arc (2,1) is added to the tree**

**What is the cycle, and how much flow can be sent?**

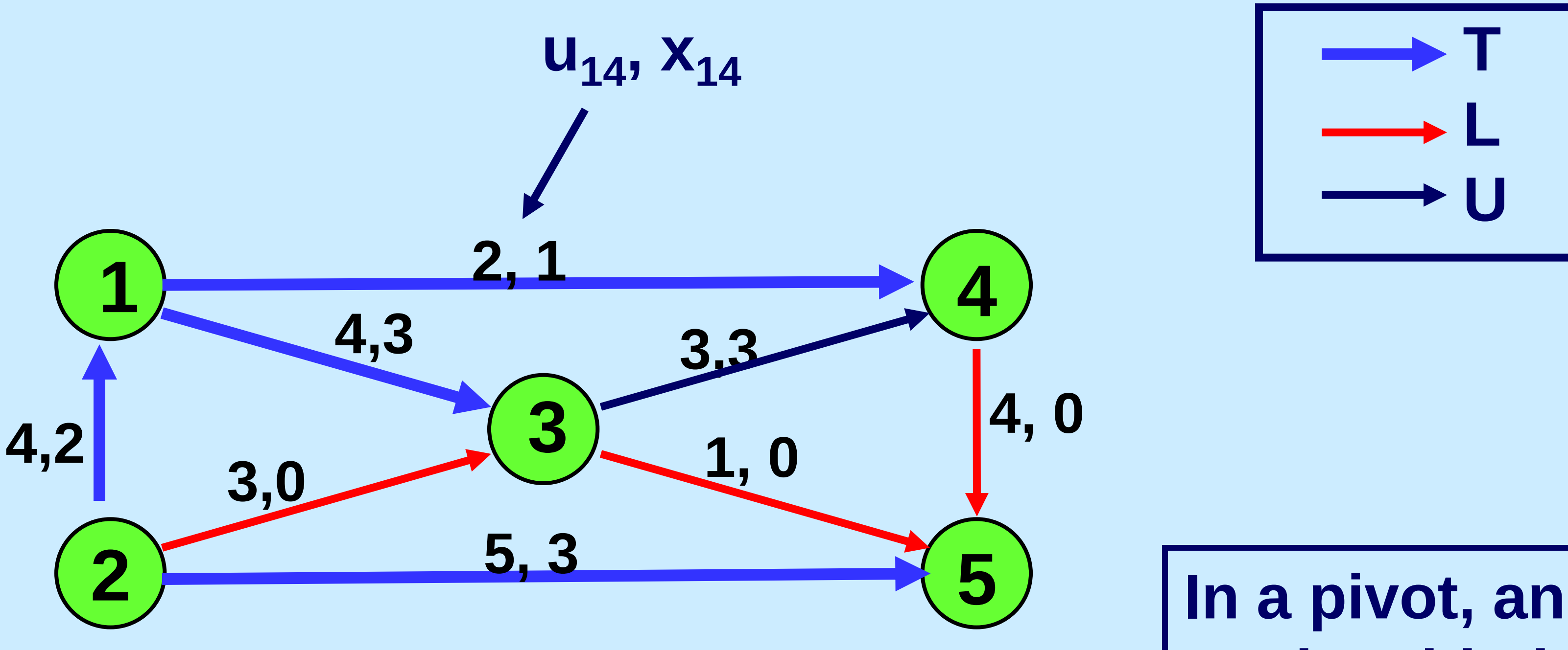
# Send Flow Around the Cycle




What is the next spanning tree?

2 units of flow were sent along the cycle.

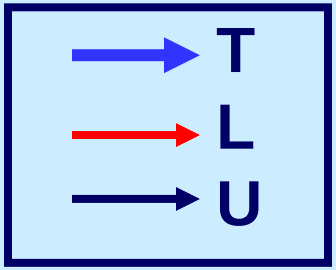
# After a pivot

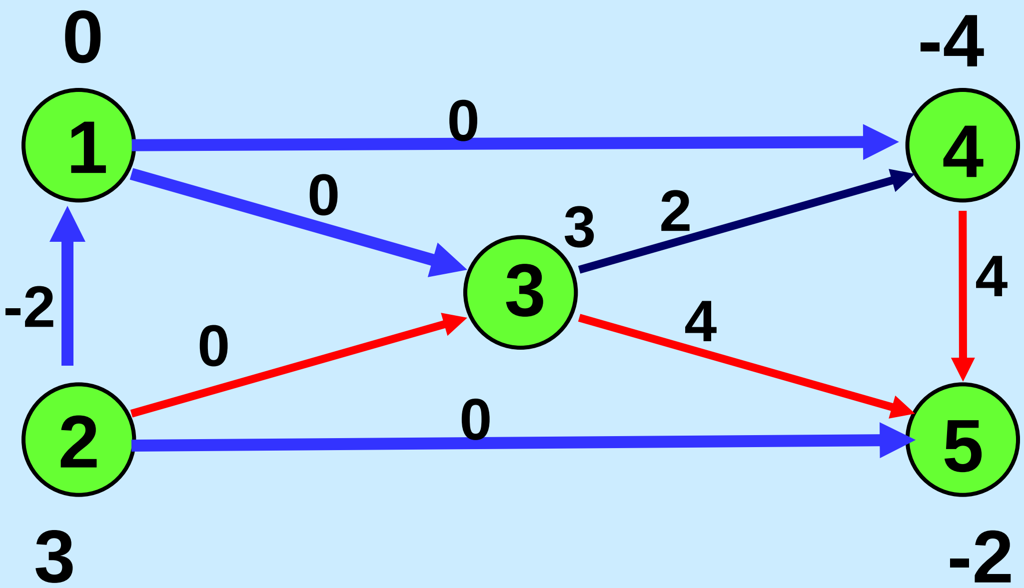



The Updated Spanning Tree

In a pivot, an arc is added to T and an arc is dropped from T.

# Updating the Multipliers

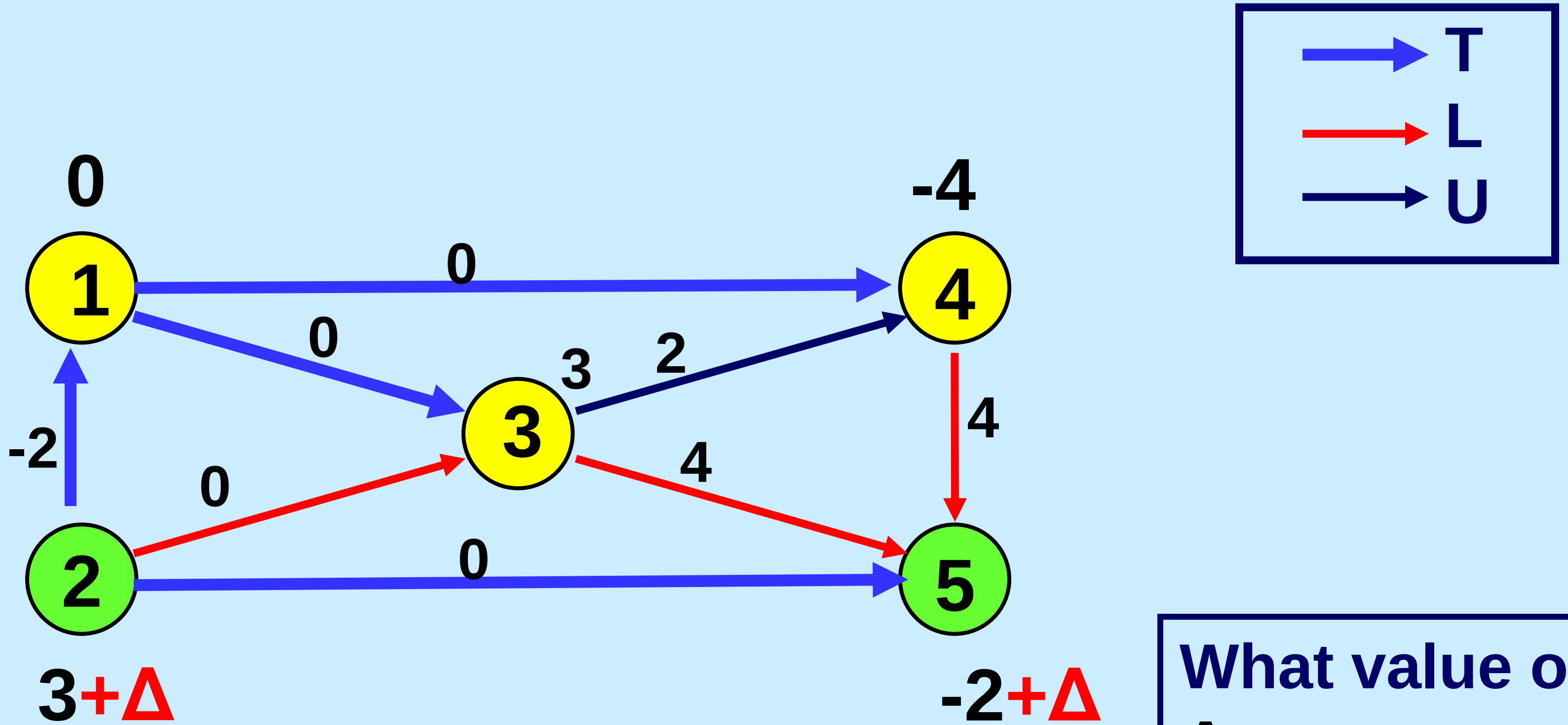



The current multipliers and reduced costs

How can we make $c^{\square}_{21} = 0$ and have other tree arcs have a 0 reduced cost?

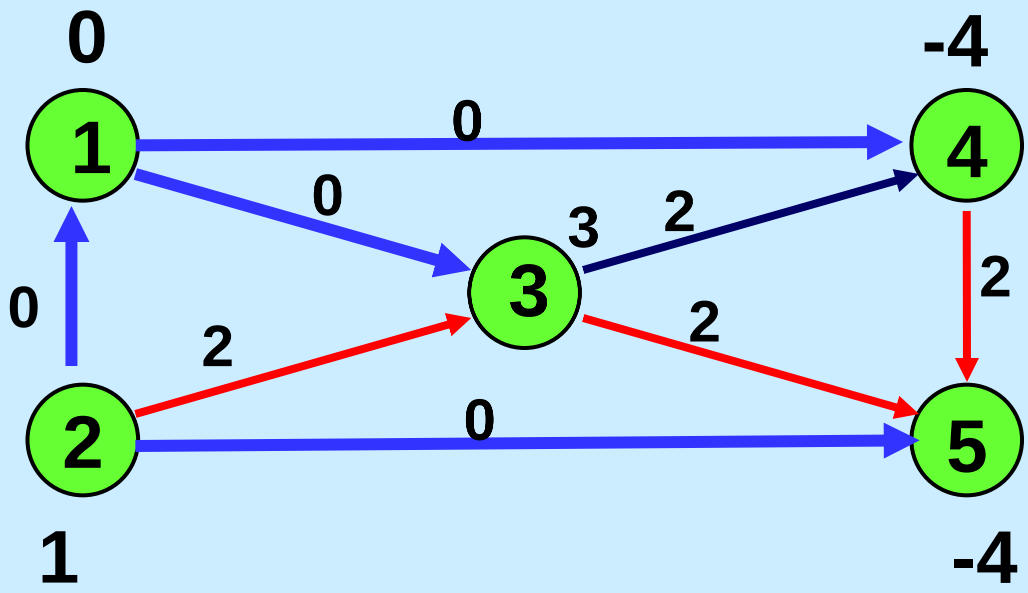
# Deleting (2,1) from T splits T into two parts

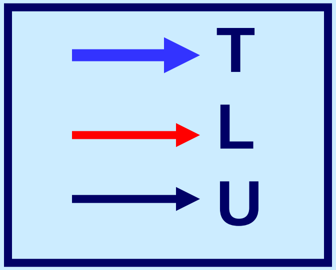



Adding Δ to multipliers on one side of the tree does not effect the reduced costs of any tree arc except (2,1). Why?

What value of Δ should be chosen to make the reduced cost of (2,1) = 0?

# The updated multipliers and reduced costs

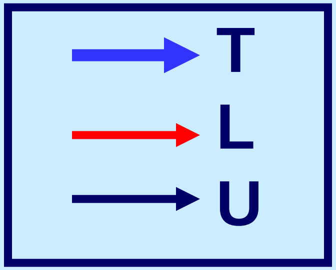


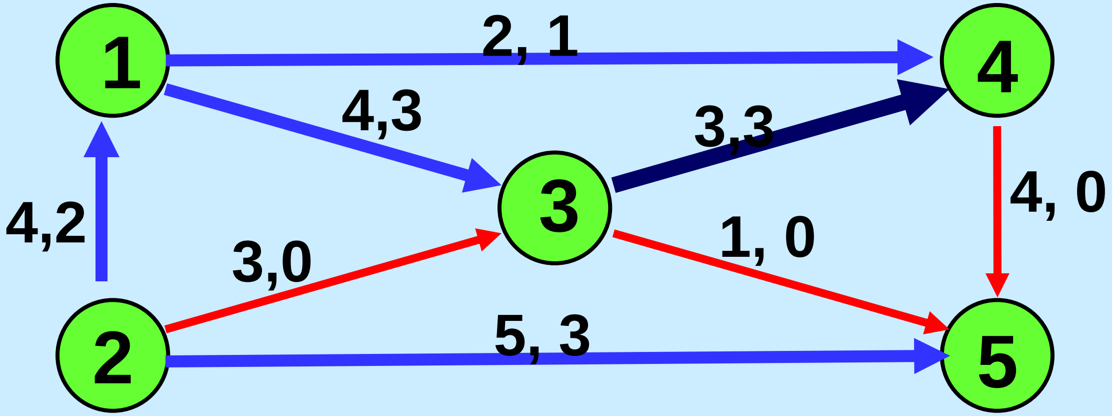




The updated multipliers and reduced costs

**Is this tree solution optimal?**

# Add a violating arc to the spanning tree, creating a cycle

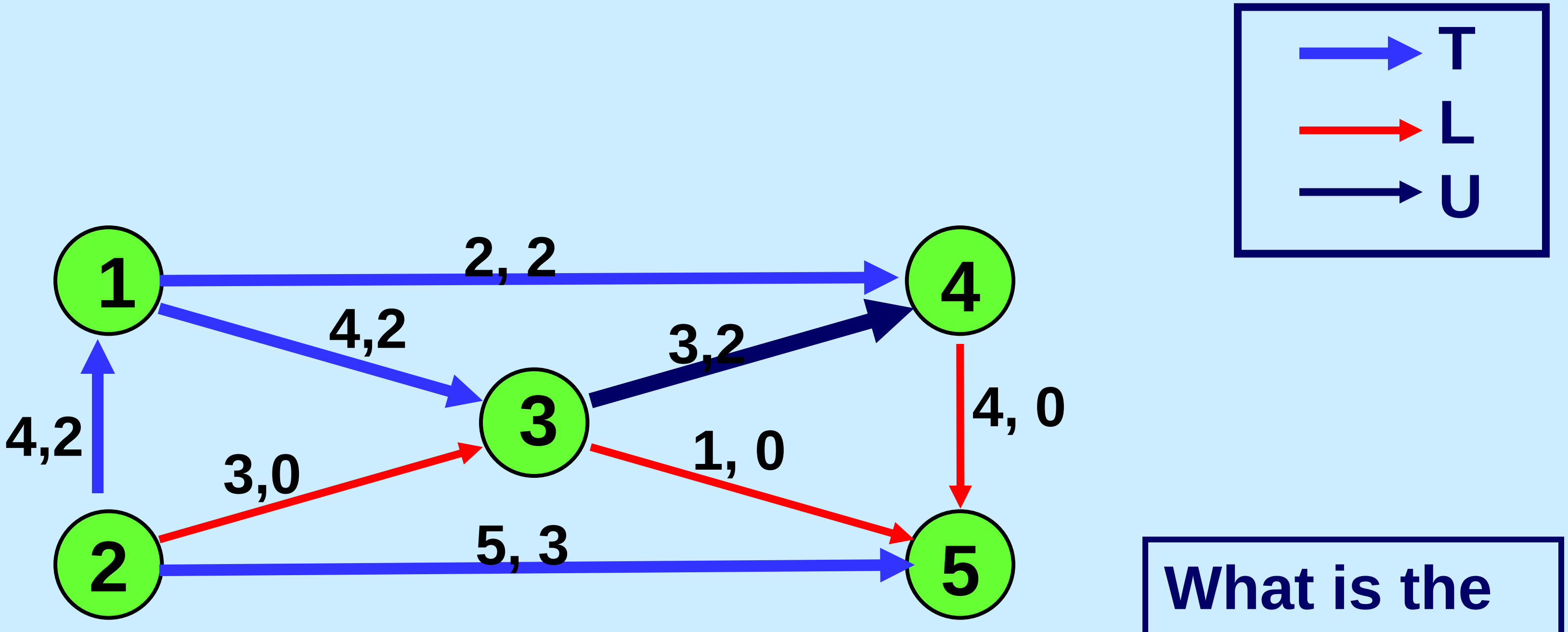



**What is the cycle, and how much flow can be sent?**

**Add arc (3,4) to the spanning tree**

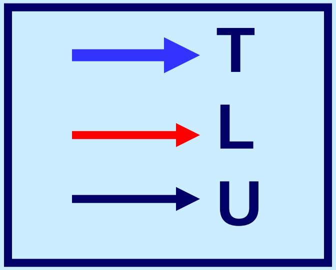
# Send Flow Around the Cycle

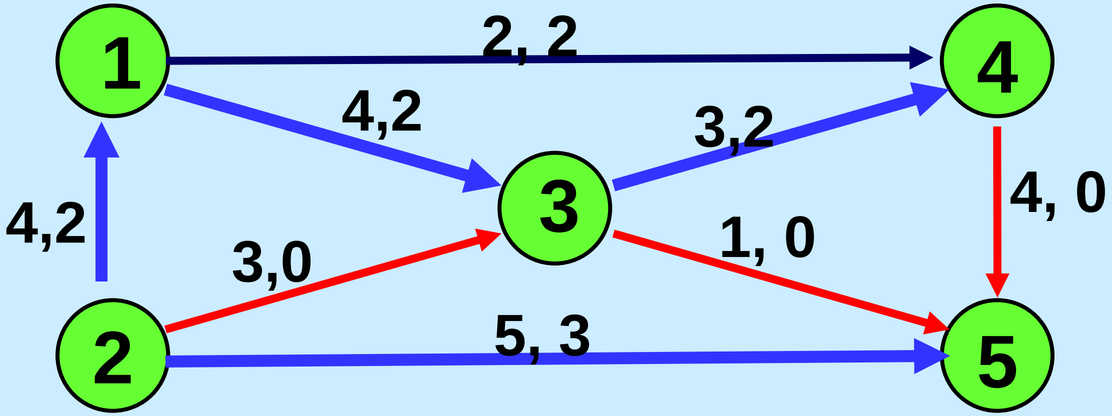



**1 unit of flow was sent around the cycle.**

**What is the next spanning tree solution?**

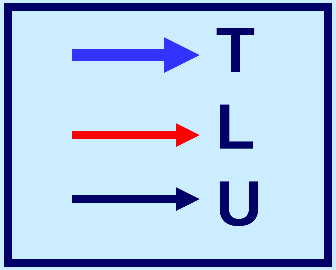
# The next spanning tree solution

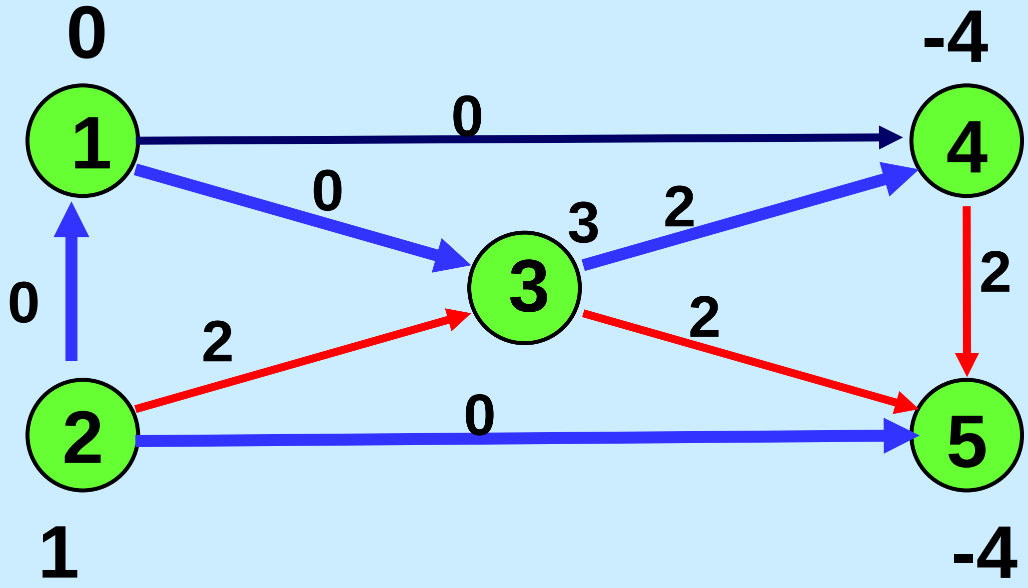



**Here is the updated spanning tree solution**

# Updated the multipliers

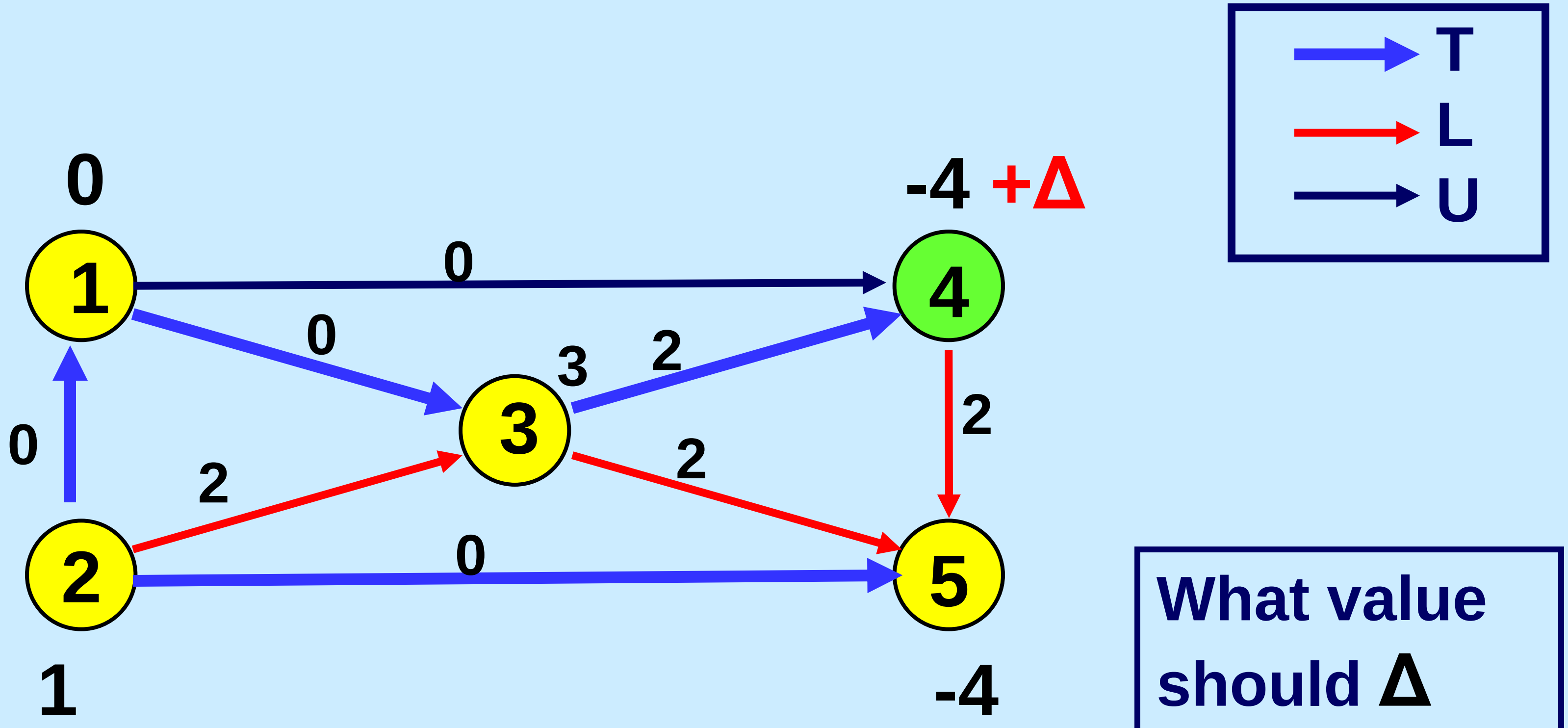



How should we modify the multipliers?

Here are the current multipliers

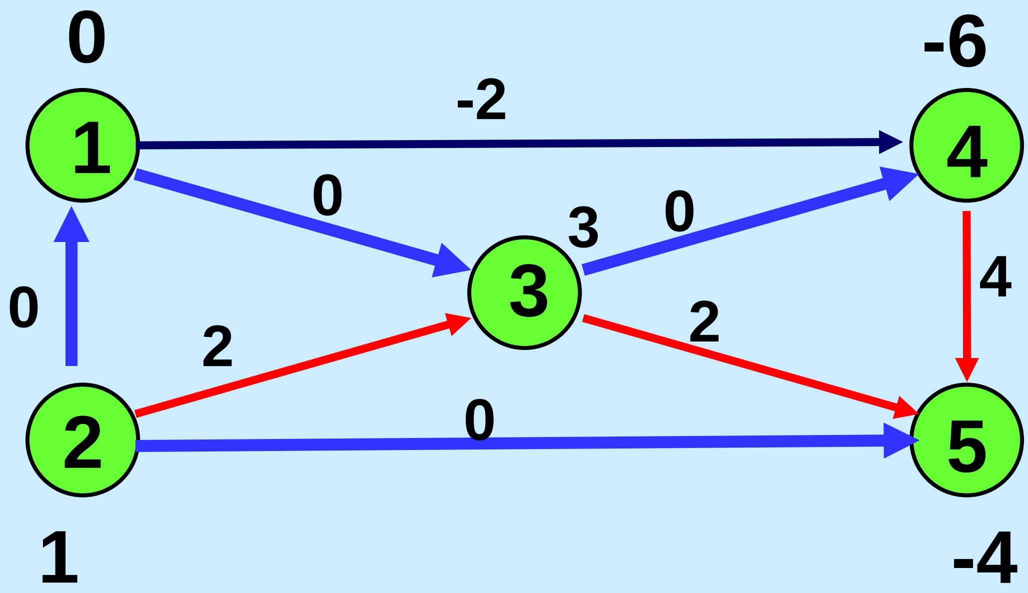
# Updated the multipliers

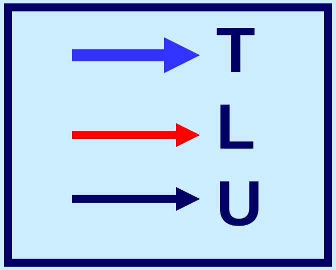



What value should Δ be?

Here are the current multipliers

# The updated multipliers

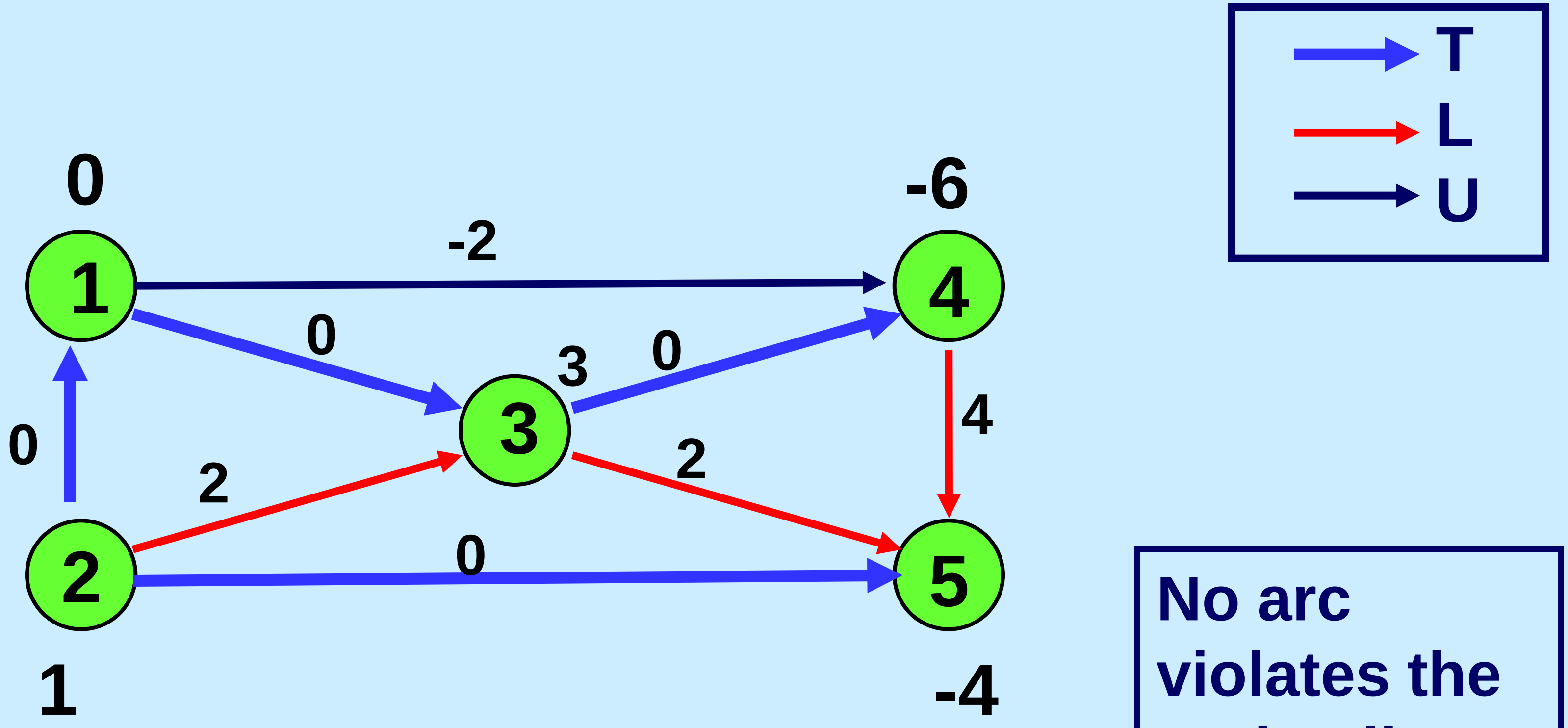



Here are the updated multipliers.

Is the current spanning tree solution optimal?

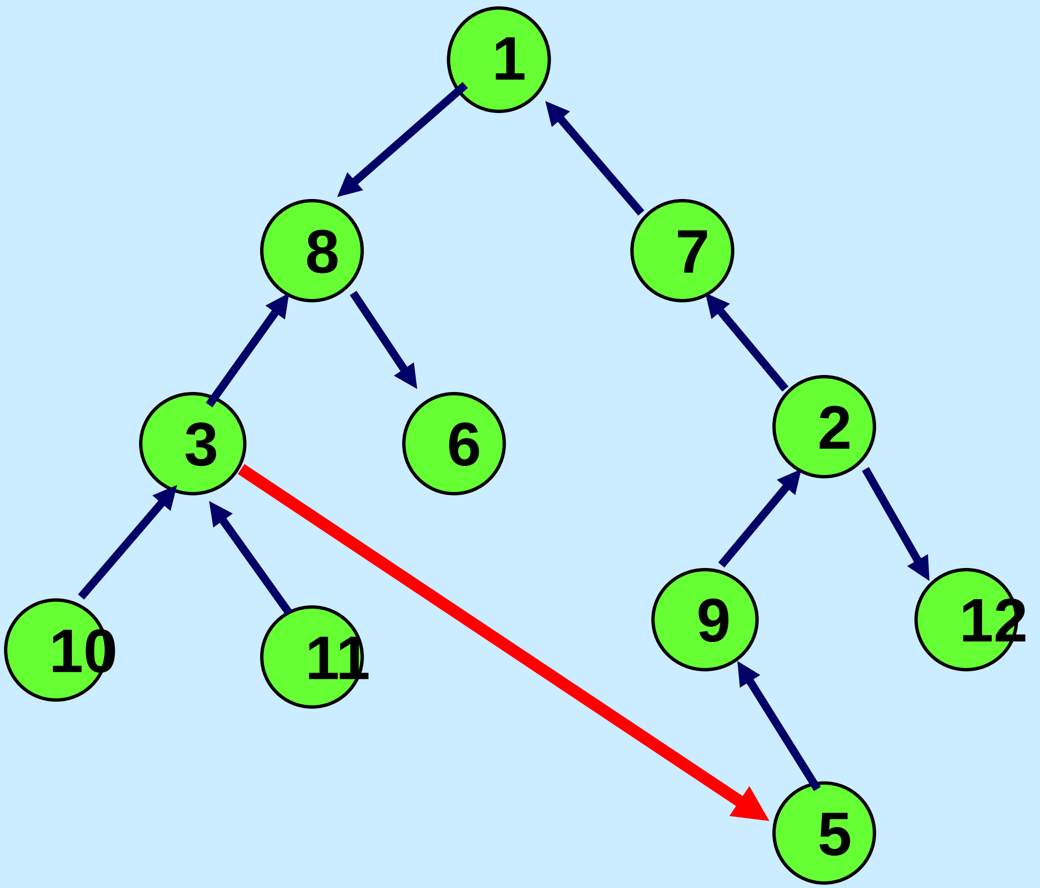
# The Optimal Solution



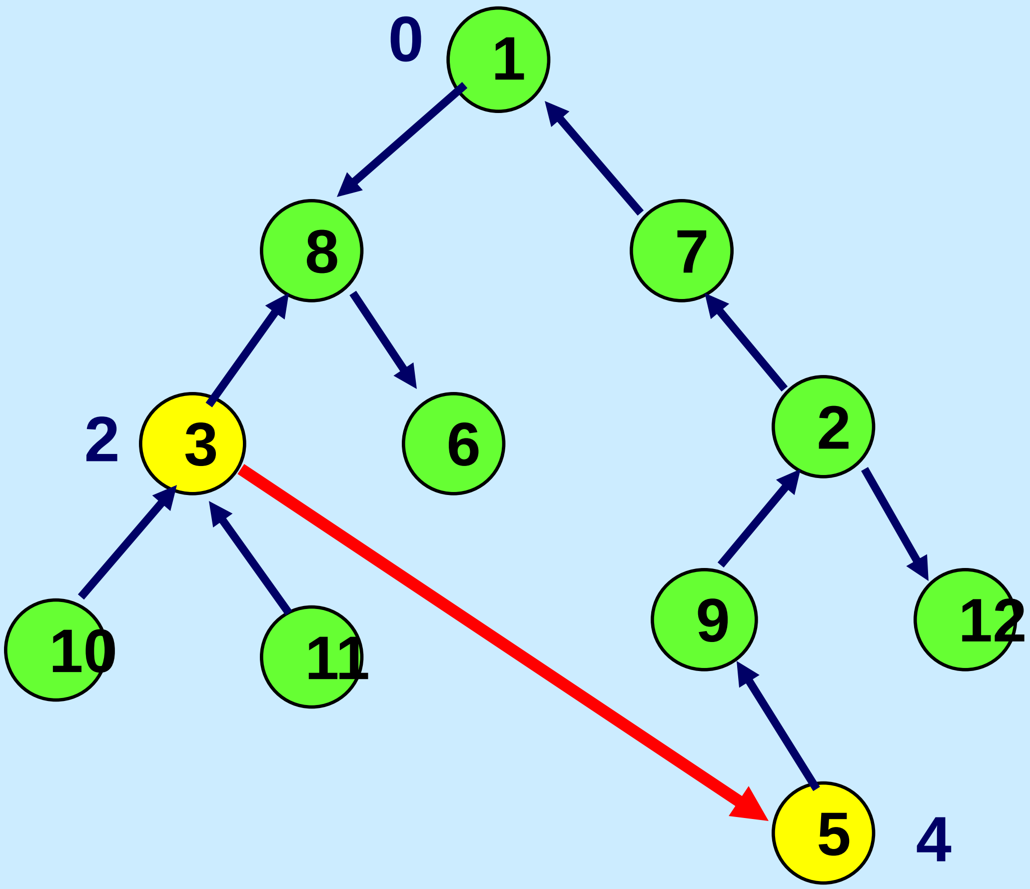

No arc violates the optimality conditions.

Here is the optimal solution.

# Finding the Cycle



# Use Depth and Predecessor

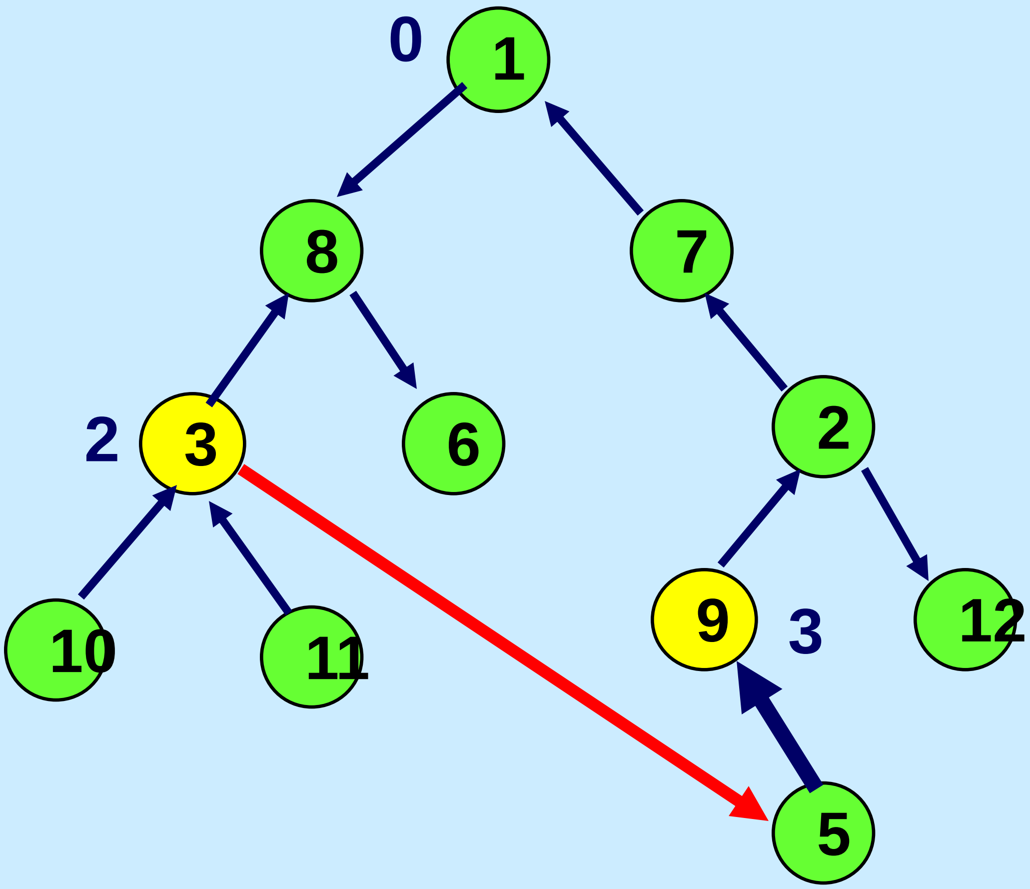



depth(5) = 4;

depth(3) = 2;

replace node 5 by pred(5)

# Use Depth and Predecessor

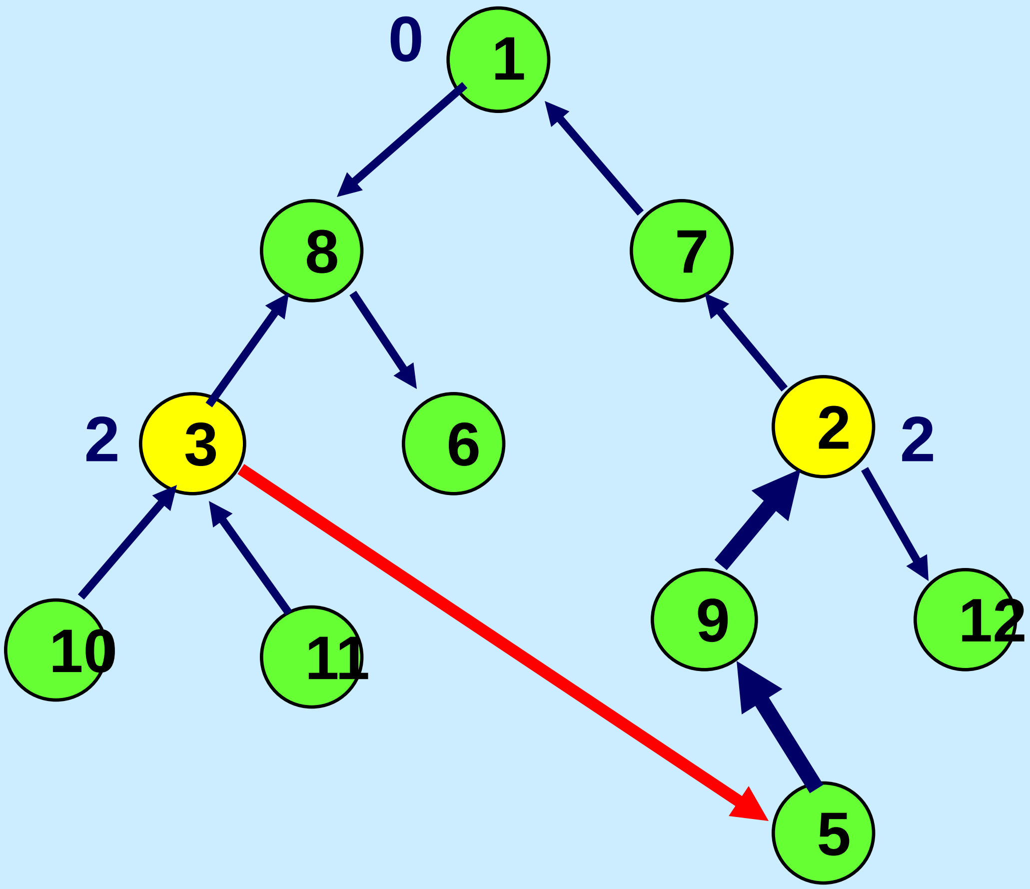



depth(9) = 3;

depth(3) = 2;

replace node 9 by pred(9)

# Use Depth and Predecessor



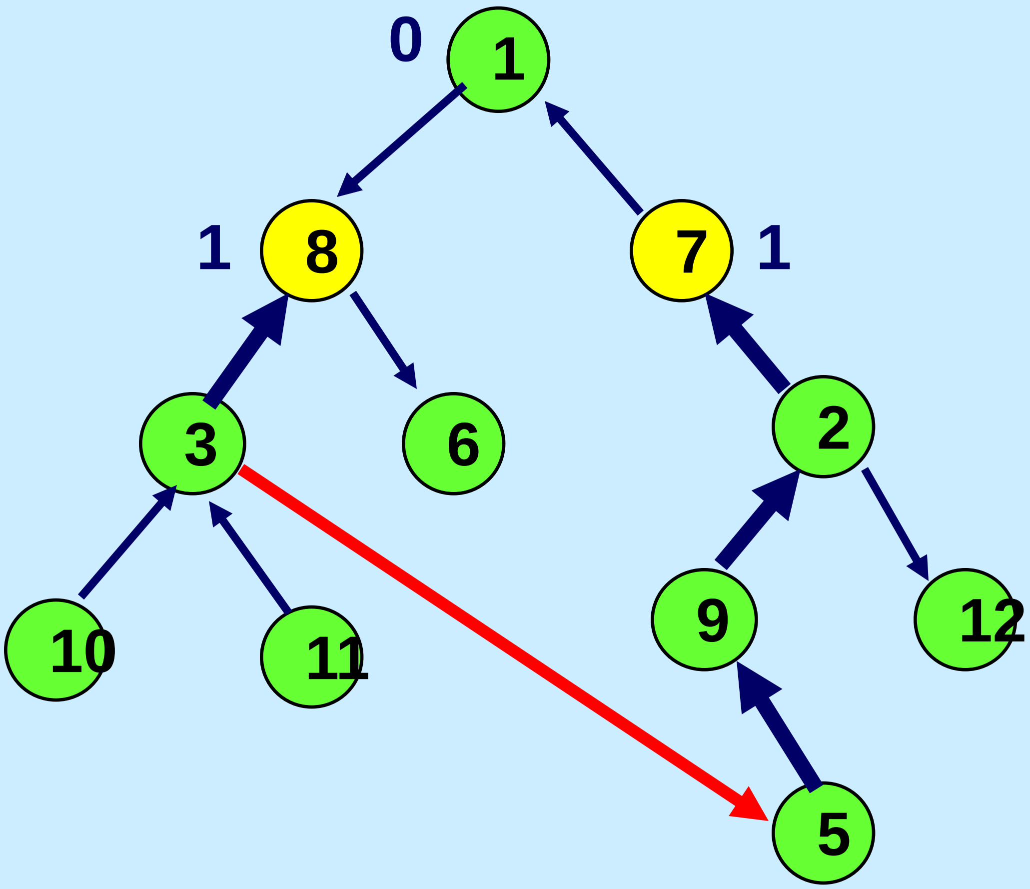

depth(2) = 2;

depth(3) = 2;

replace node 2 by pred(2);

replace node 3 by pred(3)

# Use Depth and Predecessor



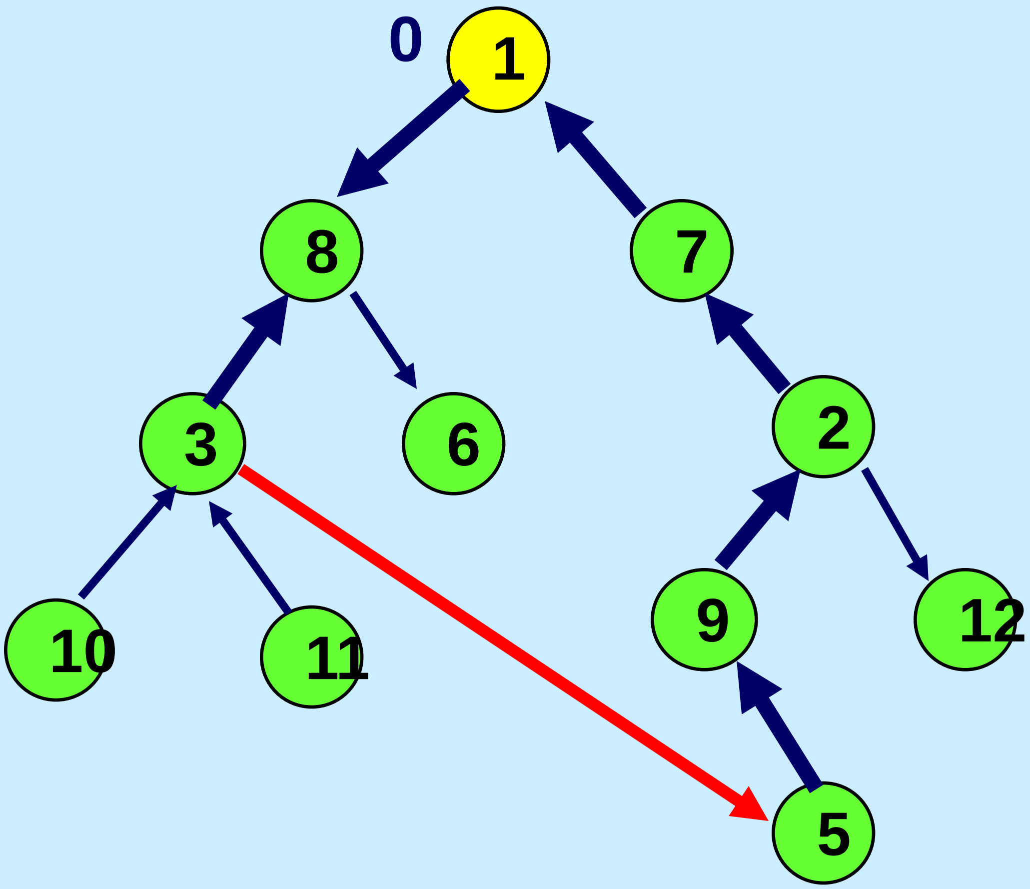

depth(8) = 1;

depth(7) = 1;

replace node 8 by pred(8);

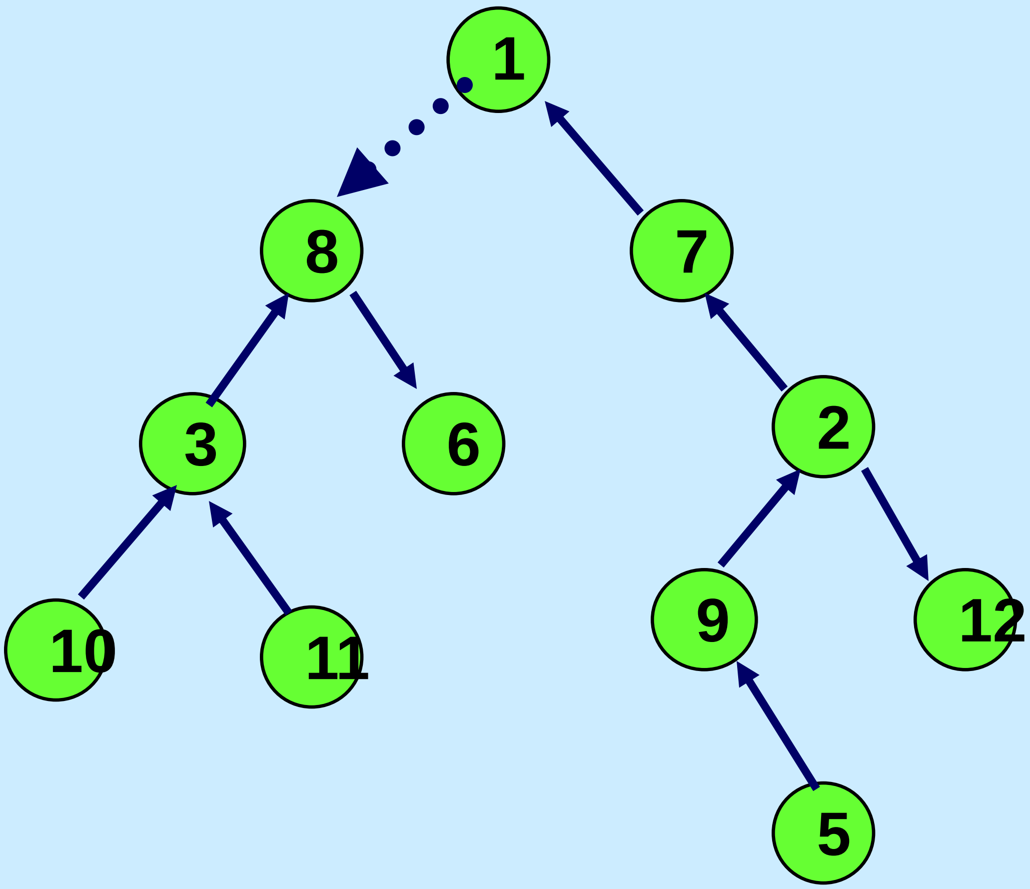
replace node 7 by pred(1)

# Use Depth and Predecessor



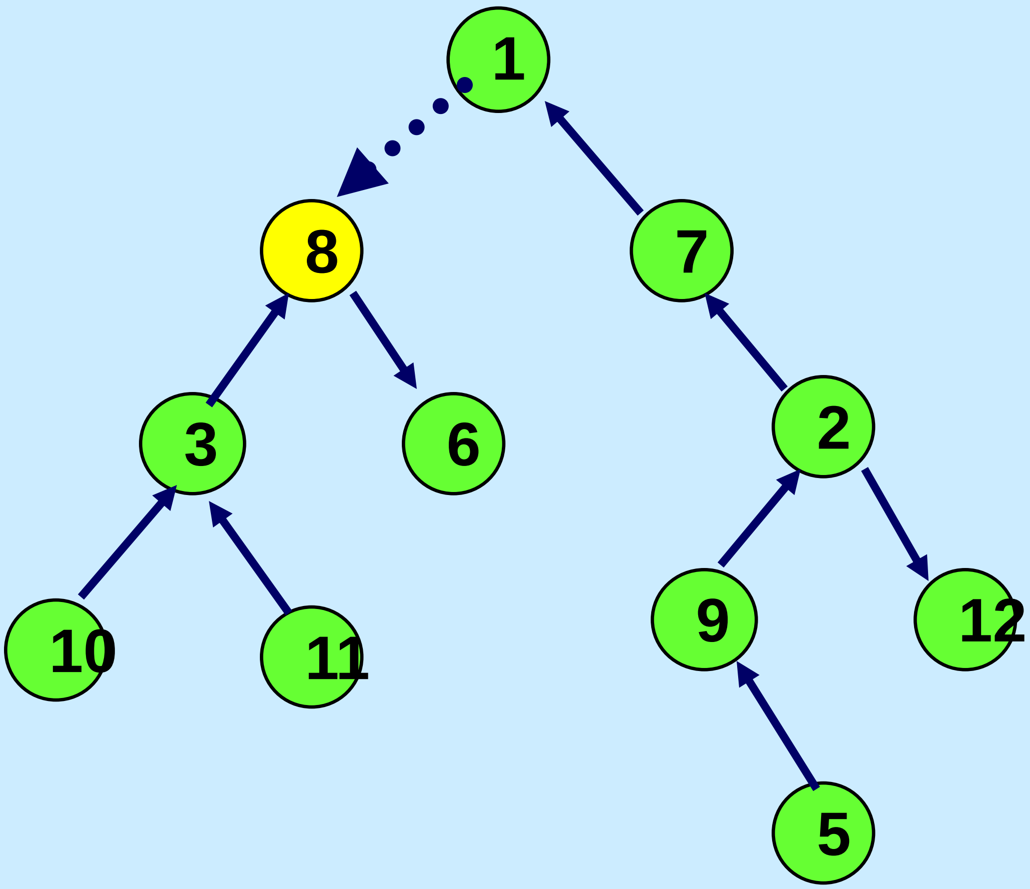

The least common ancestor of nodes 3 and 5 has been found.

# Updating the multipliers: use the thread and depth



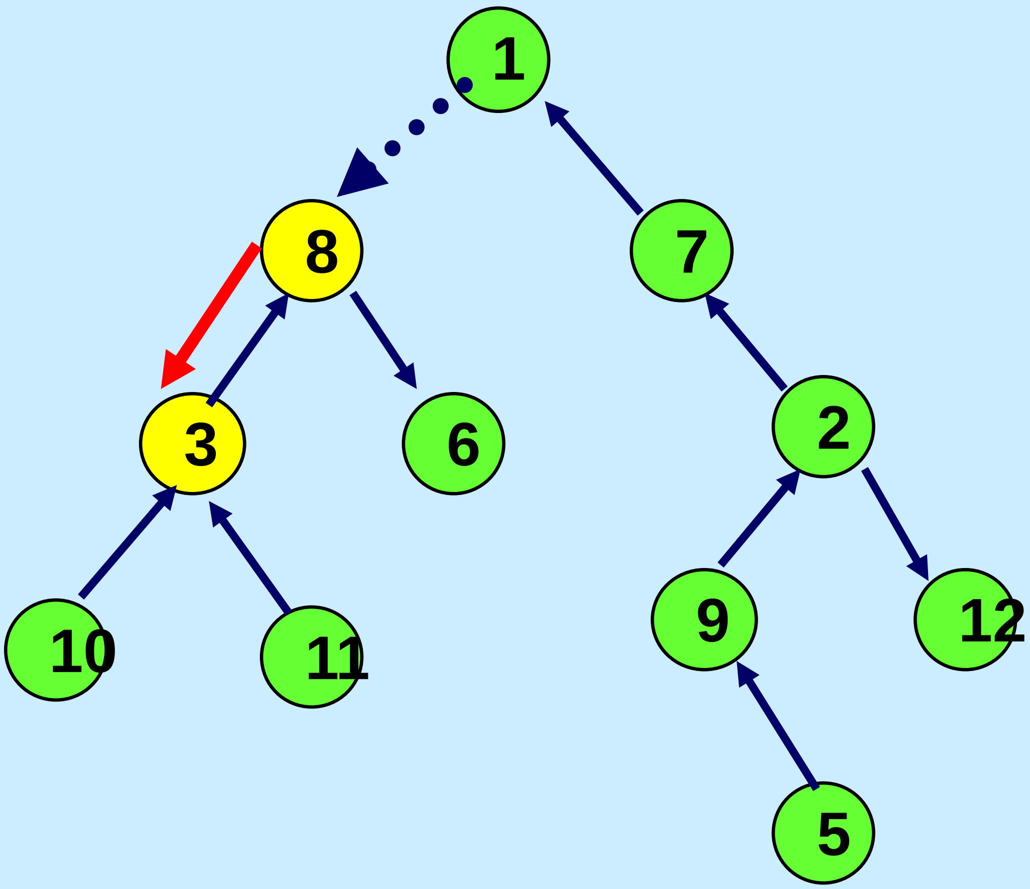

**Suppose that arc (1,8) will drop out of the tree. What is the subtree rooted at node 8?**

# Follow the thread starting with node 8



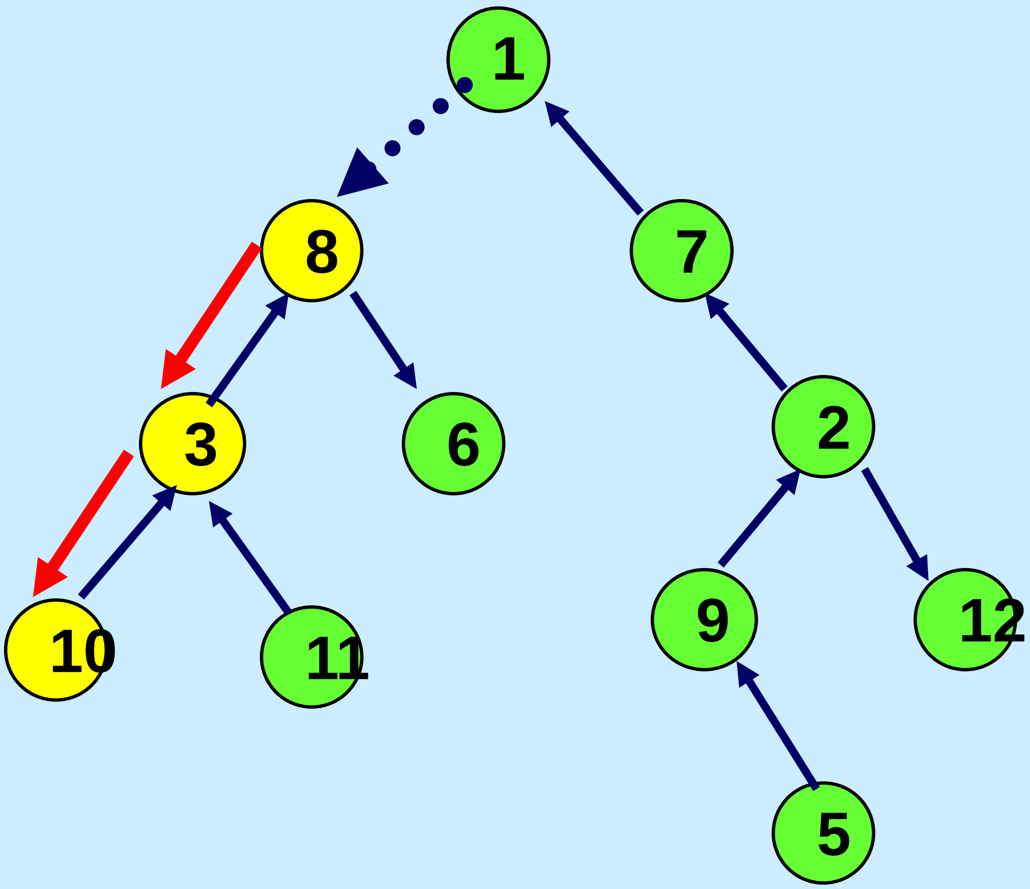

**What is thread(8)?**

# Follow the thread starting with node 8



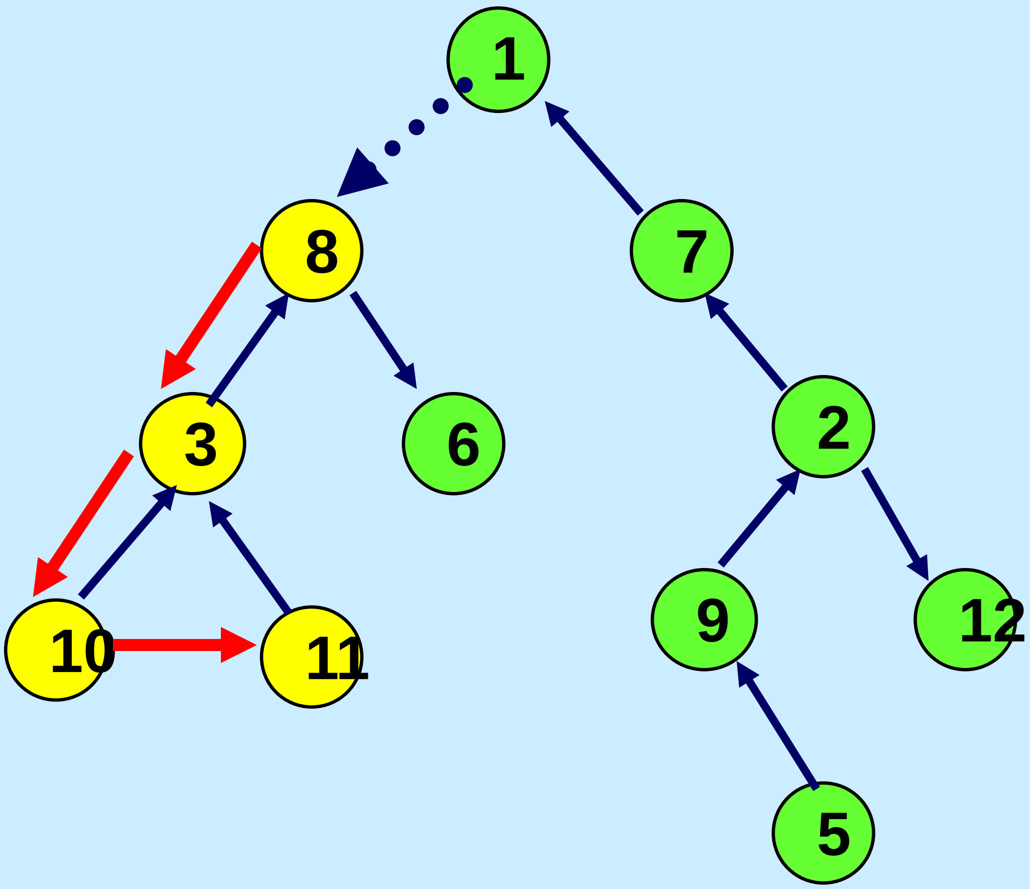

**What is thread(3)?**

# Follow the thread starting with node 8



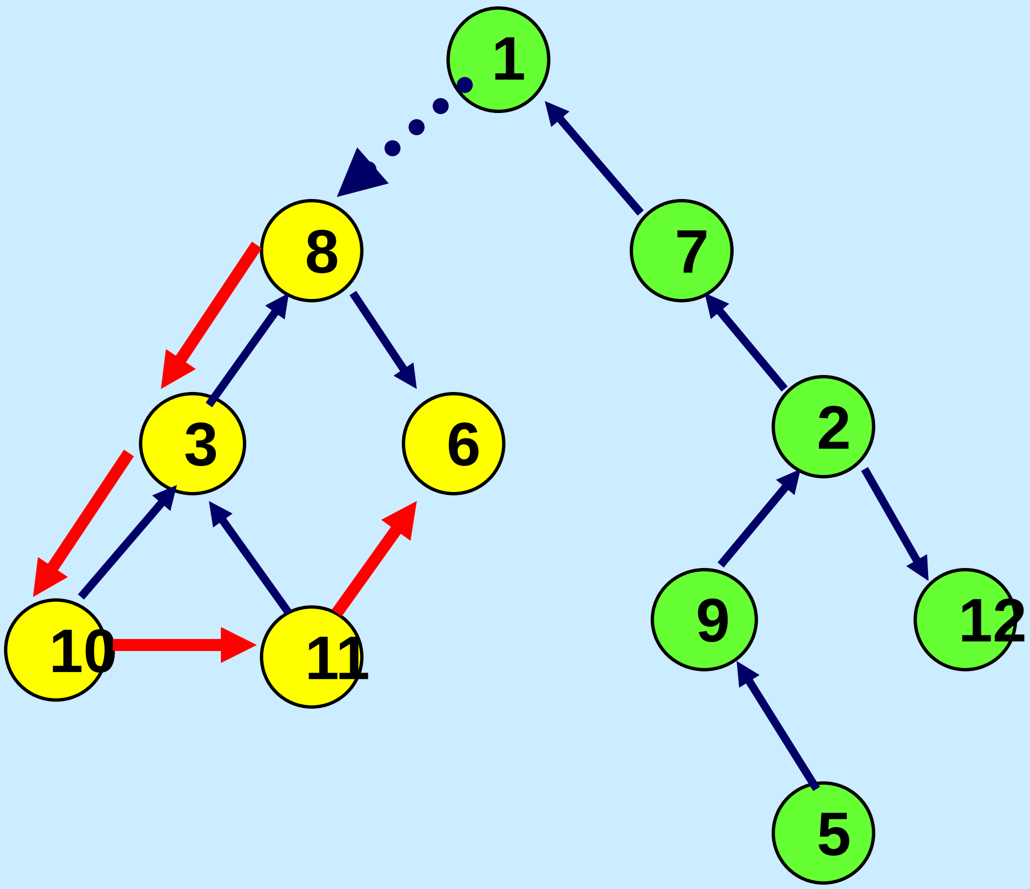

**What is thread(10)?**

# Follow the thread starting with node 8




**What is thread(11)?**

# Follow the thread starting with node 8




**What is thread(6)?**

# The stopping rule

**Stopping rule: stop when depth(current node) ≤ depth(8)**