Recitation 2: Solutions
February 14, 2006

1. Problem 1.12, page 55 of text. See online solutions.

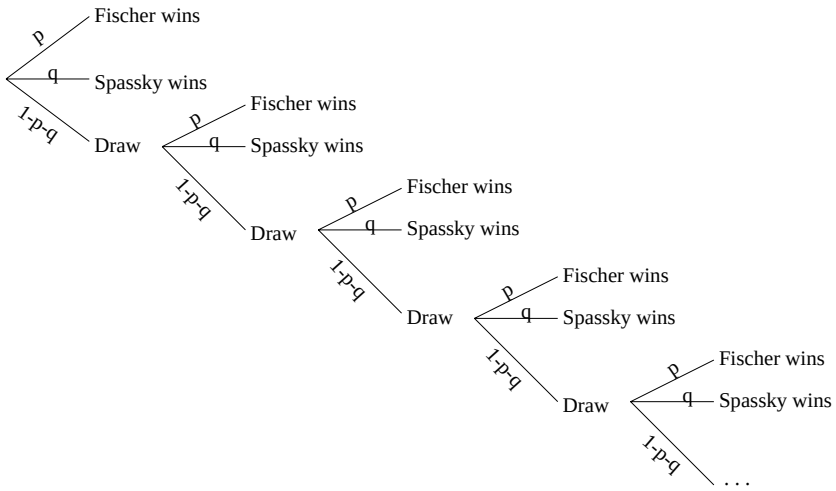
2. Problem 1.13, page 55 of text. See online solutions.

3. (a)

$$\begin{aligned}\mathbf{P}(\text{Fischer wins}) &= p + p(1-p-q) + p(1-p-q)^2 + \cdots \\ &= \frac{p}{1-(1-p-q)} \\ &= \boxed{\frac{p}{p+q}}\end{aligned}$$

We may also find the solution through a simpler method:

$$\begin{aligned}\mathbf{P}(\text{Fischer wins} \mid \text{Someone wins}) &= \frac{\mathbf{P}(\text{Fischer wins})}{\mathbf{P}(\text{Someone wins})} \\ &= \boxed{\frac{p}{p+q}}\end{aligned}$$



(b) $\mathbf{P}$(the match lasted no more than 5 games)
$= (p+q) + (p+q)(1-p-q) + (p+q)(1-p-q)^2 + (p+q)(1-p-q)^3 + (p+q)(1-p-q)^4$
$= \frac{(p+q)[1-(1-p-q)^5]}{1-(1-p-q)}$
$= 1-(1-p-q)^5$

$\mathbf{P}$(Fischer wins in the first game $\cap$ the match lasted no more than 5 games)
$= p$

Therefore, $\mathbf{P}$(Fischer wins | the match lasted no more than 5 games)
$= \frac{\mathbf{P}(\text{Fischer wins} \cap \text{the match lasted no more than 5 games})}{\mathbf{P}(\text{the match lasted no more than 5 games})}$
$= \boxed{\frac{p}{1-(1-p-q)^5}}$

(c) $\mathbf{P}$(the match lasted no more than 5 games)
$= 1 - (1-p-q)^5$

$\mathbf{P}$(Fischer wins $\cap$ the match lasted no more than 5 games)
$= p + p(1-p-q) + p(1-p-q)^2 + p(1-p-q)^3 + p(1-p-q)^4$
$= \frac{p[1-(1-p-q)^5]}{1-(1-p-q)}$
$= \frac{p[1-(1-p-q)^5]}{p+q}$

Therefore, $\mathbf{P}$(Fischer wins | the match lasted no more than 5 games)
$= \frac{\mathbf{P}(\text{Fischer wins} \cap \text{the match lasted no more than 5 games})}{\mathbf{P}(\text{the match lasted no more than 5 games})}$
$= \boxed{\frac{p}{p+q}}$

(d) $\mathbf{P}$(Fischer wins at or before the 5th game | Fischer wins)
$= \frac{\mathbf{P}(\text{Fischer wins at or before the 5th game} \cap \text{Fischer wins})}{\mathbf{P}(\text{Fischer wins})}$
$= \left(\frac{p[1-(1-p-q)^5]}{p+q}\right) / \left(\frac{p}{p+q}\right)$
$= \boxed{1-(1-p-q)^5}$

This part may be solved by observing that the events {Fischer wins} and {the match lasted no more than 5 games} are independent (we know this from parts (a) and (c)):
$\mathbf{P}$(the match lasted no more than 5 games | Fischer wins)
$= \mathbf{P}$(the match lasted no more than 5 games)
$= \boxed{1-(1-p-q)^5}$