MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2006)

Recitation 19
May 4, 2006

1. (Example 6.3) A machine can be either working or broken down on a given day. If it is working, it will break down in the next day with probability b, and will continue working with probability $1 - b$. If it breaks down on a given day, it will be repaired and be working in the next day with probability r, and will continue to be broken down with probability $1 - r$. Suppose that whenever the machine remains broken for a given number of l days, despite the repair efforts, it is replaced by a new working machine. Model this machine as a Markov chain.

2. (Problem 6.3) A spider and a fly move along a straight line in unit increments. The spider always moves towards the fly by one unit. The fly moves towards the spider by one unit with probability 0.3, moves away from the spider by one unit with probability 0.3, and stays in place with probability 0.4. The initial distance between the spider and the fly is integer. When the spider and the fly land in the same position, the spider captures the fly.

   (a) Construct a Markov chain that describes the relative location of the spider and fly.

   (b) Identify the transient and recurrent states.

   (c) Assume that the initial distance between the spider and the fly is 2 units. Provide a recursive formula to evaluate the n-step transition probabilities $r_{ij}(n)$, $i, j \in \{0, 1, 2\}$. Compute $r_{2i}(3)$, $i \in \{0, 1, 2\}$. Can you infer the limiting behavior of the n-step transition probabilities ?

3. (Problem 6.4) **Existence of a recurrent state.** Show that in a Markov chain at least one recurrent state must be accessible from any given state, i.e., for any i, there is at least one recurrent j in the set $A(i)$ of accessible states from i.