

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2006)

Tutorial 05 Answer
March 16-17, 2006

1. (a)

$$f_Y(y) = 2y, \text{ for } 0 \leq y \leq 1.$$

(b)

$$f_Y(y) = e^{-y}, \text{ for } 0 \leq y < \infty.$$

2. (a)

$$f_Y(y) = \frac{1}{\pi\sqrt{1-y^2}}, \text{ for } -1 < y < 1.$$

(b)

$$f_Y(y) = \frac{1}{\pi} \cdot \frac{1}{1+y^2}, \text{ for } -\infty < y < \infty.$$

A random variable with the above density is called a Cauchy random variable

3. Optional

(a)

$$\begin{aligned} f_Y(y) &= \frac{1}{2\sqrt{y}} \cdot f_X(\sqrt{y}) + \frac{1}{2\sqrt{y}} \cdot f_X(-\sqrt{y}) \\ &= \frac{1}{\sqrt{y}} \cdot f_X(\sqrt{y}), \text{ for } 0 \leq y < \infty. \end{aligned}$$

(b)

$$f_Y(y) = \frac{1}{y} f_X(\ln y), \text{ for } 0 \leq y < \infty.$$

Note that f_X is the standard normal density in both (a) and (b).

4. (a)

$$f_{V,W}(v,w) = \frac{\log(1/v)}{2\sqrt{w}}, \quad 0 \leq v, w \leq 1$$

(b)

$$\begin{aligned} P(XY \leq Z^2) &= P(V \leq W) = \int_0^1 \int_0^w \frac{\log(1/v)}{2\sqrt{w}} dv dw = \int_0^1 \frac{v(1-\log v)}{2\sqrt{w}} \Big|_{v=0}^w dw \\ &= \int_0^1 \frac{\sqrt{w}(1-\log w)}{2} dw = \left[\frac{w^{3/2}}{3} \left(\frac{5}{3} - \log w \right) \right]_{w=0}^1 = \frac{5}{9} \end{aligned}$$