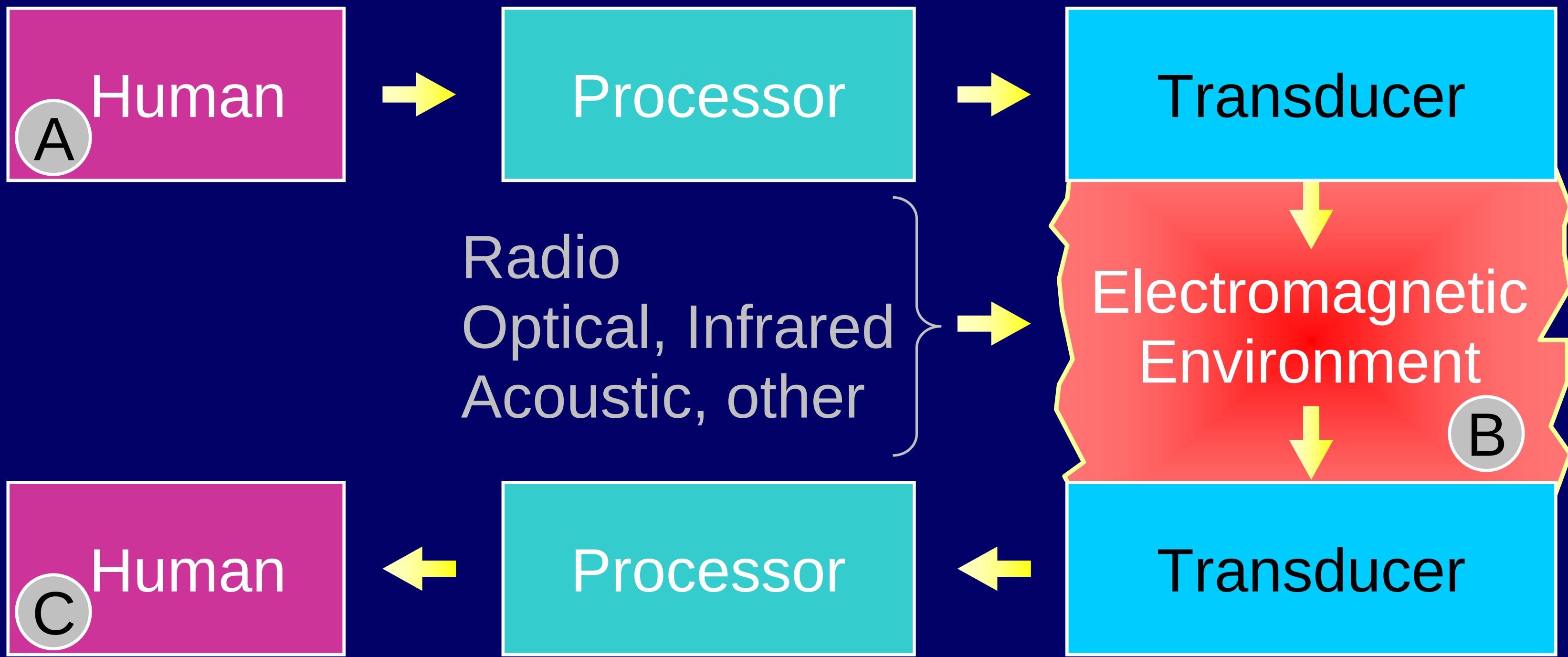
Receivers, Antennas, and Signals

Professor David H. Staelin

Subject Content



Communications: A → C (radio, optical)

Active Sensing: A → C (radar, lidar, sonar)

Passive Sensing: B → C (systems and devices: environmental, medical, industrial, consumer, and radio astronomy)

Subject Offers

- Physical concepts
- Mathematical methods, system analysis and design
- Applications examples
- Motivation and integration of prior learning

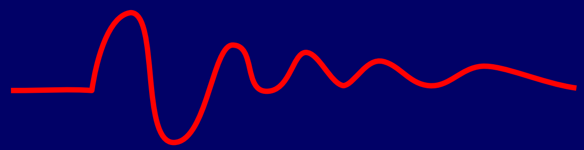
Subject Outline

- Review of signals and probability
- Noise in detectors and systems; physics of detectors
- Receivers and spectrometers; radio, optical, infrared
- Radiation, propagation, and antennas
- Signal modulation, coding, processing and detection
- Communications, radar, radio astronomy, and remote sensing
- Parameter estimation

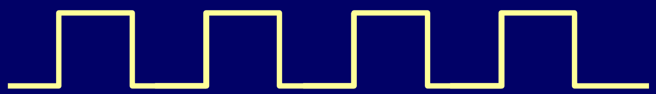
Review of Signals

Signal Types to be Reviewed:

- Pulses (finite energy)



- Periodic signals
  (finite energy per period)



- Random signals
  (finite power, infinite energy)

Pulses v(t)

Have Finite Energy : $\int_{-\infty}^{\infty} |v(t)|^2 \, dt < \infty$

Define Fourier Transform:

$$\underline{V}(f) \triangleq \int_{-\infty}^{\infty} \underline{v}(t) e^{-j\underbrace{2\pi f}_{\omega} t} \, dt \;\; [\text{volts/Hz} = \text{volt sec}]$$

$$\underline{v}(t) \triangleq \int_{-\infty}^{\infty} \underline{V}(f) e^{+j2\pi f t} \, df \;\; [\text{volts}]$$

Define notation "$\leftrightarrow$" for Fourier Transform :

e.g. $v(t) \leftrightarrow \underline{V}(f)$

Dimensions must only be self consistent;

e.g. v(t) can be dimensionless, volts, meters, newtons, etc.

Energy Spectral Density S(f)

$$\underline{V}(f) \triangleq \int_{-\infty}^{\infty} \underline{v}(t) e^{\overbrace{-j2\pi ft}^{\omega}}$$

$$S(f) \triangleq |\underline{V}(f)|^2$$

S(f) can have dimensions of :

sec^2	if t is time and v is dimensionless
m^2	if t is distance and v is dimensionless
$(volts/Hz)^2$	if t is time and v(t) is volts where $(v/Hz)^2 = \left(v^2/sec\right)(Hz) = (v/sec)^2$
Joules/Hz	if S(f) is dissipated in a 1-ohm resistor by v(t) volts, where Joules = $volts^2 \bullet sec/ohm$
Et cetera	

Autocorrelation Function

$$R(\tau) \underline{\underline{\Delta}} \int_{-\infty}^{\infty} v(t)v^*(t-\tau)dt \left[v^2 \sec\right] \text{or } [J], \text{ etc.}$$

Claim: $R(\tau) \leftrightarrow S(f)$

$$S(f) \underline{\underline{?}} \int_{-\infty}^{\infty} R(\tau)e^{-j2\pi f\tau}\, d\tau = \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} v(t)v^*\underbrace{(t-\tau)}_{\underline{\underline{\Delta}}\, t'} dt \right\} e^{-j2f \underbrace{\tau}_{t-t'}} \underbrace{d\tau}_{-dt'}$$

$$= \left\{ \int_{-\infty}^{\infty} v(t)e^{-j2\pi ft}\, dt \right\} \bullet \left\{ \int_{-\infty}^{\infty} v^*(t')e^{+j2\pi ft'}\, dt' \right\}$$

Reverses sign of - dt' for $t = \text{constant}$

$$S(f) = \underline{V}(f) \bullet \underline{V}^*(f) = \left|\underline{V}(f)\right|^2 \text{ Q.E.D.}$$

Therefore:

$$R(\tau) = \int_{-\infty}^{\infty} S(f)e^{+j2\pi f\tau}\, df$$

$$R(0) = \int_{-\infty}^{\infty} v^2(t)\, dt = \int_{-\infty}^{\infty} S(f)\, df \quad \text{Parseval's Theorem}$$

Compact Transform Notation

$$v(t) \leftrightarrow \underline{V}(f)$$

$$\downarrow \qquad \downarrow$$

$$R(\tau) \leftrightarrow |\underline{V}(f)|^2 \triangleq S(f)$$

$$[v] \leftrightarrow [v/Hz]$$

$$\downarrow \qquad \downarrow$$

$$[v^2 \sec] \leftrightarrow [v/Hz]^2$$

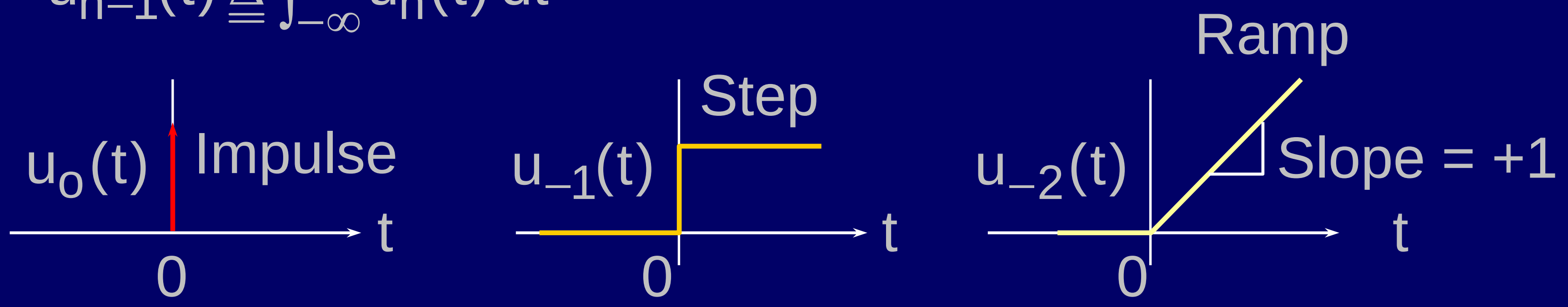
$$[Joules] \leftrightarrow [J/Hz]$$

(If power is dissipated in a 1-ohm resistor)

Define "Unit Impulse"

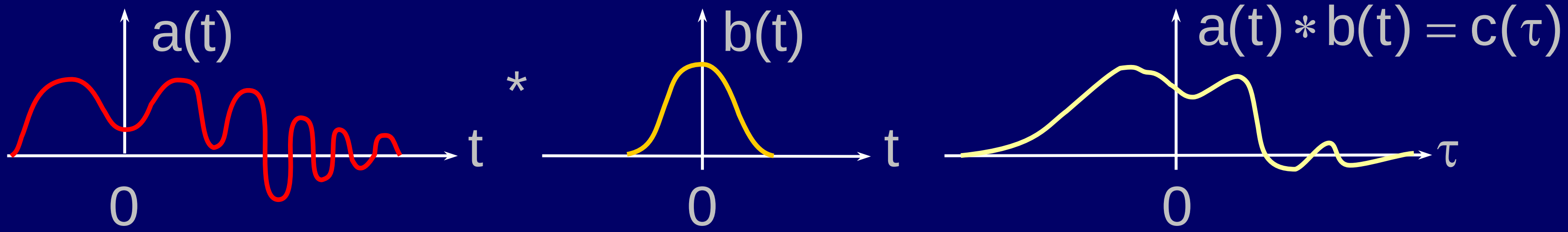
$$u_o(t) \triangleq \delta(t) \text{ where } \lim_{\varepsilon \to 0} \int_{-\varepsilon}^{\varepsilon} u_o(t)\,dt = 1,\; u_o(t) = 0 \text{ for } |t| > 0$$

$$u_{n-1}(t) \triangleq \int_{-\infty}^{t} u_n(t)\,dt$$



Define "Convolution"

$$a(t) * b(t) \triangleq \int_{-\infty}^{\infty} a(t)\, b(t-\tau)\, dt = c(\tau)$$



Useful Transformation Pairs for Pulses

$$\underline{A}(f) \triangleq \int_{-\infty}^{\infty} a(t)\, e^{-j2\pi ft} dt \qquad \omega \triangleq 2\pi f$$

$a(t) \leftrightarrow \underline{A}(f)$

$u_o(t) = \delta(t) \leftrightarrow 1$

$1 \leftrightarrow u_o(f)$

Have ∞ energy (treated as special pulses)

$a'(t) \leftrightarrow j\omega\underline{A}(f)$

$$a(t) = \int_{-\infty}^{\infty} \underline{A}(f) e^{j2\pi ft} df$$

$u_n(t) \leftrightarrow (j\omega)^n$

$a(t)\, e^{j\omega_o t} \leftrightarrow \underline{A}(f - f_o)$

$\omega_o \equiv 2\pi f_o$

$a(t - t_o) \leftrightarrow \underline{A}(f)\, e^{-j\omega t_o}$

$u_{-1}(t)\, e^{-\alpha t} \leftrightarrow 1/(j\omega + \alpha)$

Transforms: Even/Odd Functions

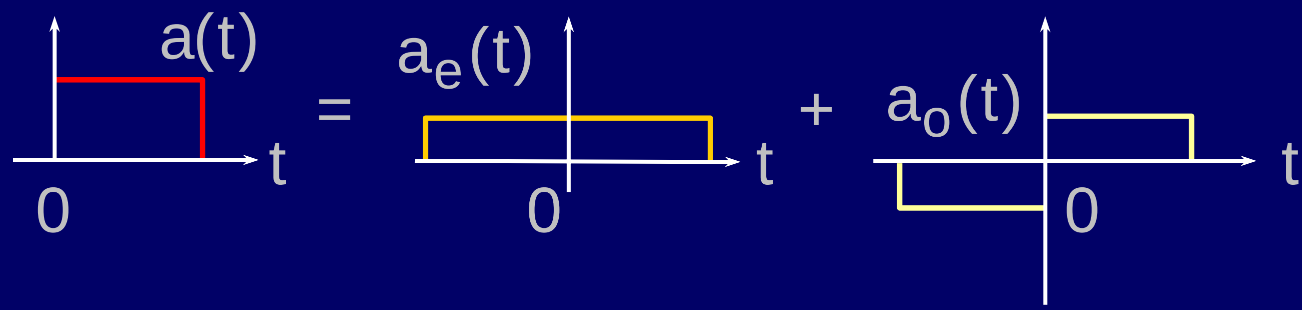
$$a_e(t) \leftrightarrow R_e\{\underline{A}(f)\}$$

where: $a_e(t) \triangleq [a(t) + a(-t)]/2 = a_e(-t)$ EVEN

$a_o(t) \triangleq [a(t) - a(-t)]/2 = a_o(-t)$ ODD

so: $a(t) = a_e(t) + a_o(t)$

e.g.

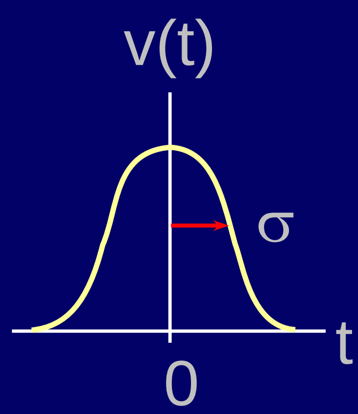


$$a_o(t) \leftrightarrow j\,\mathrm{Im}\{\underline{A}(f)\}$$

Transforms: Operators and Gaussians

$$a_1(t) \bullet a_2(t) \;\leftrightarrow\; \underline{A}_1(f) * \underline{A}_2(f)$$

$$a_1(t) * a_2(t) \;\leftrightarrow\; \underline{A}_1(f) \bullet \underline{A}_2(f)$$



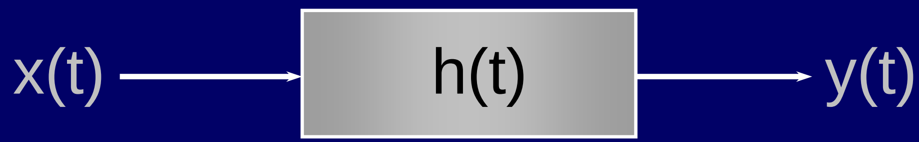
$$v(t) \triangleq \frac{A}{\sigma\sqrt{2\pi}} e^{-(t/\sigma)^2/2} \;\leftrightarrow\; A e^{-(\sigma\omega)^2/2}$$

$$\downarrow \qquad\qquad \downarrow$$

$$\frac{A^2}{2\sigma\sqrt{\pi}} e^{-(\tau/\sigma\sqrt{2})^2/2} \;\leftrightarrow\; A^2 e^{-(\sigma\omega)^2}$$

} All Gaussians

Linear Systems



Characterized by:

$h(t) = $ "Impulse Response," where

$$y(t) \triangleq x(t) * h(t) \triangleq \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau \leftarrow \text{"superposition integral"}$$

Test: If $x(t) = \delta(t)$, then $y(t) = h(t)$

If $h(t) = \delta(t)$, then $y(t) = x(t)$

Note:

$A * (B + C) = (A * B) + (A * C)$ "Distributive"

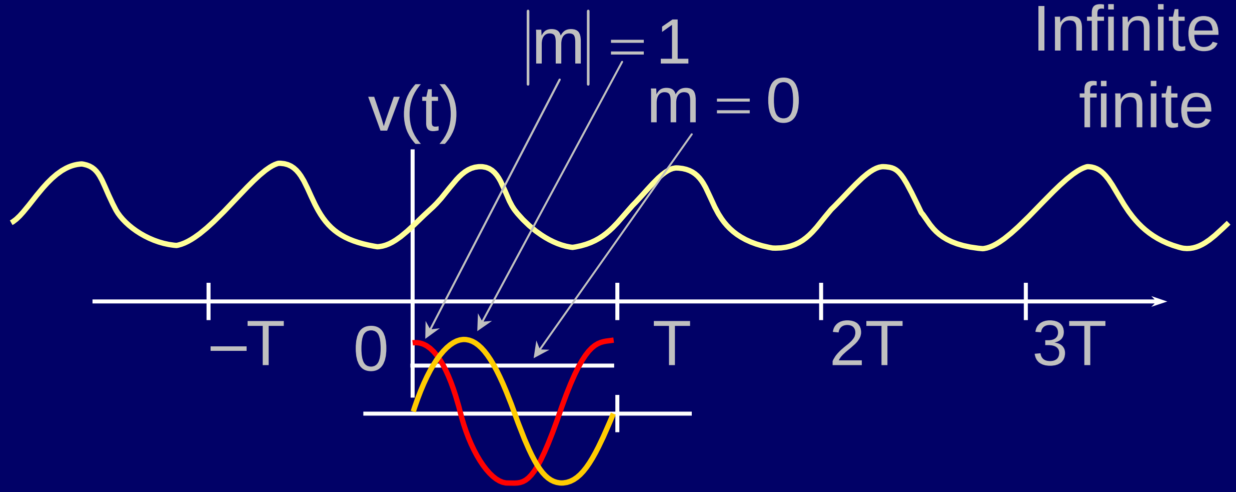
$A * B = B * A$ "Commutative"

$A * (B * C) = (A * B) * C$ "Associative"

Periodic Signals

Although $\int_{-\infty}^{\infty} v^2(t)dt = \infty$, $\int_{o}^{T} v^2(t)dt < \infty$ where Period $\triangleq$ T

Infinite energy, finite power



Fourier Series:

$$\underline{V}_m \triangleq \frac{1}{T}\int_{-T/2}^{T/2} v(t)e^{-jm\overbrace{(2\pi/T)}^{}t}\,dt \qquad \text{where } m = 0, \pm 1, \pm 2, ...$$

$$\omega_o = 2\pi f_o$$

$$v(t) = \sum_{m=-\infty}^{\infty} \underline{V}_m\, e^{jm\,2\pi f_o\, t} \quad \left(f_o \triangleq 1/T\right)$$

Autocorrelation, Power Spectrum

Autocorrelation Function:

$$R(\tau) \triangleq \frac{1}{T}\int_{-T/2}^{T/2} v(t)v^*(t-\tau)dt = \sum_{m=-\infty}^{\infty} \left|\underline{V}_m\right|^2 e^{jm2\pi f_o \tau}$$

Power Spectrum:

$$\Phi_m \triangleq \left|\underline{V}_m\right|^2 = \frac{1}{T}\int_{-T/2}^{T/2} R(\tau)e^{-jm\,2\pi f_o \tau}\,dt$$

Compact Notation

$$v(t) \leftrightarrow \underline{V}_m$$

$$\downarrow \qquad\qquad \downarrow$$

$$R(\tau) \leftrightarrow \left|\underline{V}_m\right|^2 \triangleq \Phi_m \leftrightarrow \Phi(f)$$

Typical dimensions:

$$[\text{volts}] \leftrightarrow [\text{volts}]$$

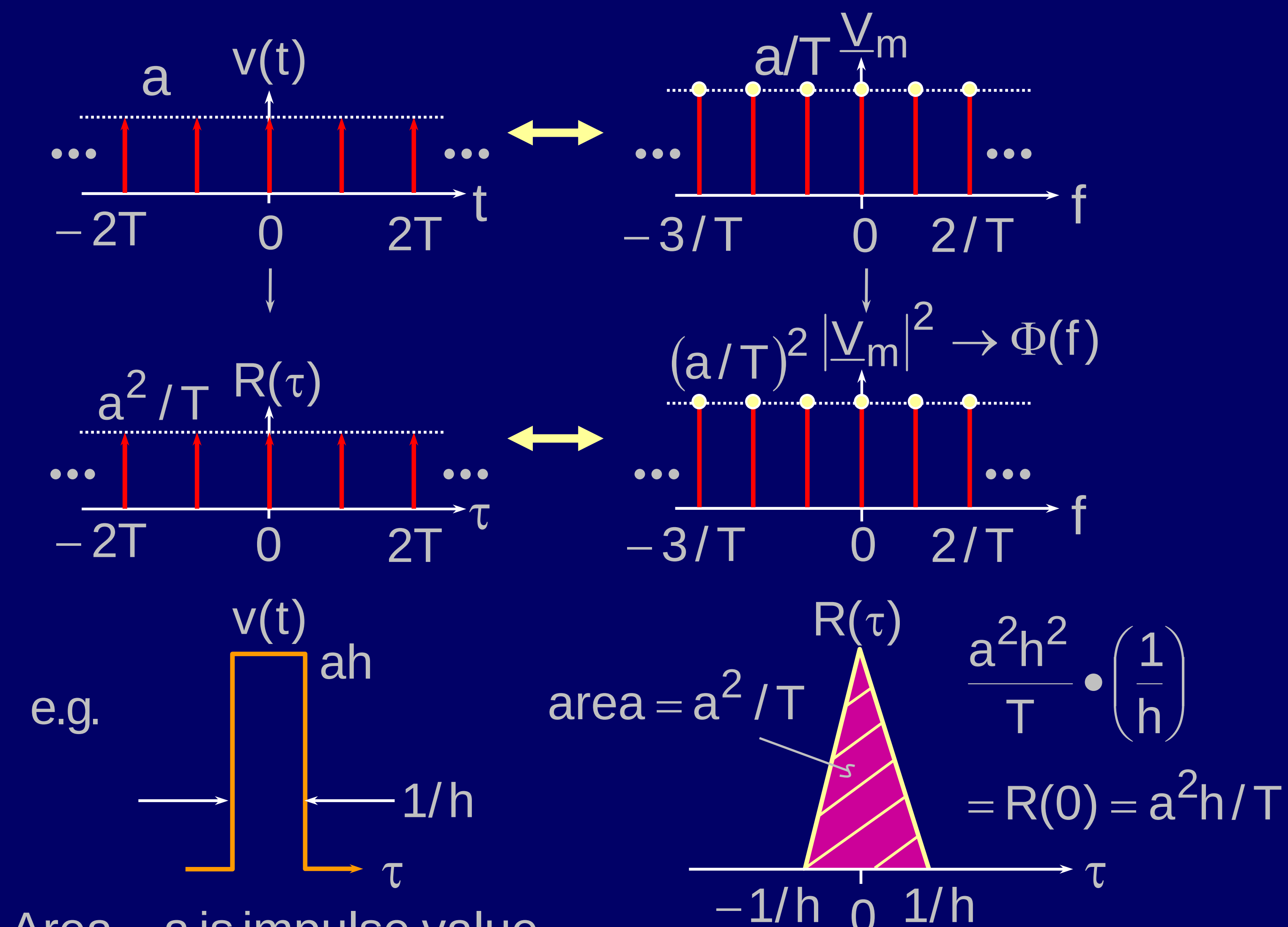
$$\downarrow \qquad\qquad \downarrow$$

$$\left[\text{volts}^2\right] \leftrightarrow \left[\text{volts}^2\right]$$

In 1-ohm resistor:

$$[\text{watts}] \leftrightarrow [W]$$

Transforms of Impulse Trains



Area = a is impulse value

Let $h \rightarrow \infty$ so area a^2/T becomes impulse value

Receiver Noise Processes

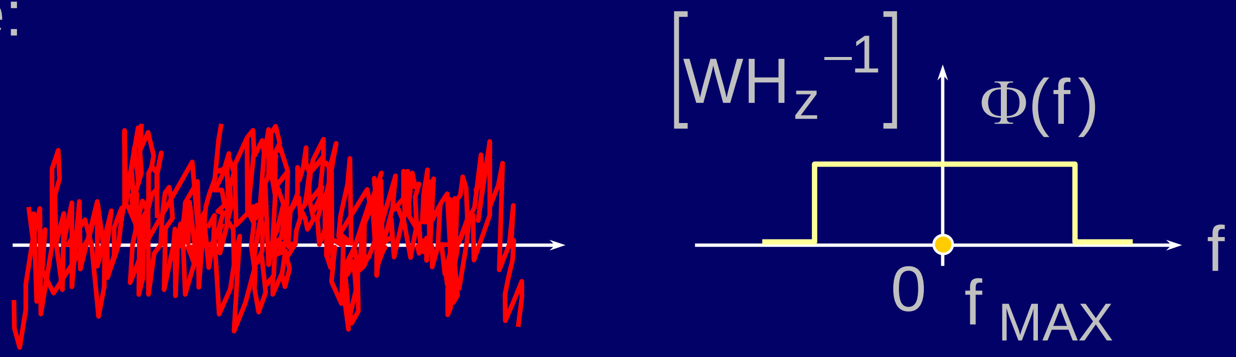
Receivers, Antennas, and Signals

Professor David H. Staelin

Random Signals

Random signals generally have finite power, infinite energy, and are unpredictable

Example:



Since we have infinite information for infinite time and a finite frequency band, then "V(f)" is not an analytic function and our approach must be different.

New definitions are required.

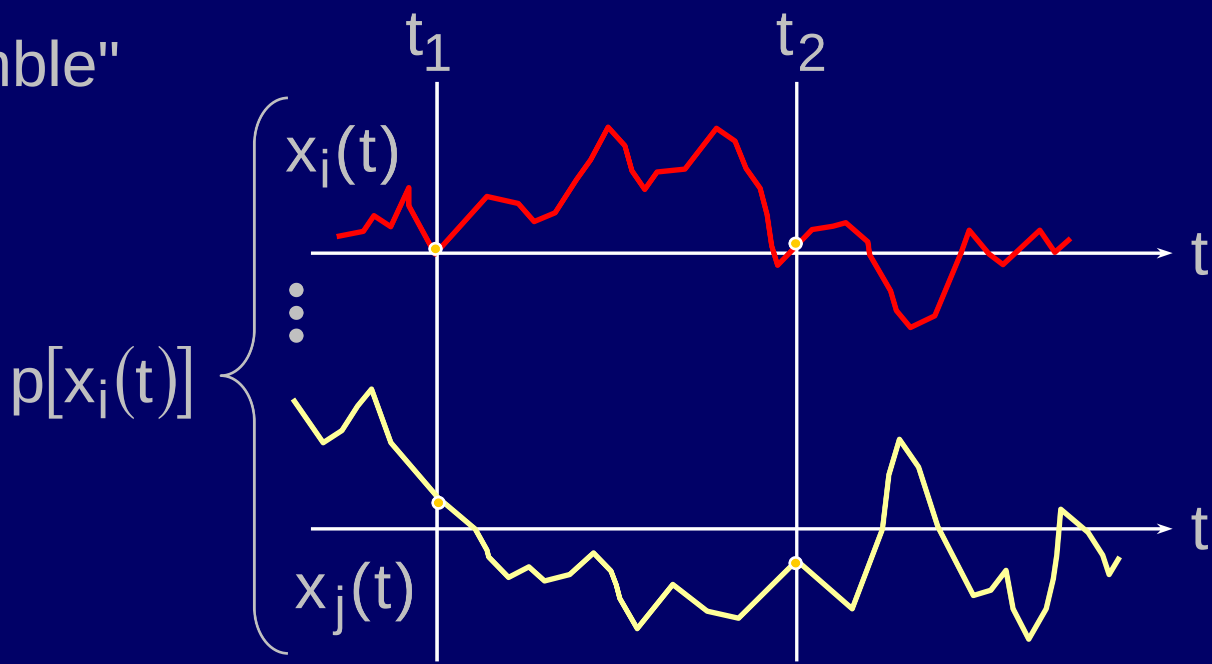
Expected Value of x

Finite or infinite ensemble of $x_i(t)$

$$E[x(t)] \triangleq \sum_i x_i(t)\, p\{x_i(t)\} \rightarrow \int_{-\infty}^{\infty} x\, p(x)\, dx$$

$x_i \; \varepsilon$ "ensemble"

$$\sum_i p(x_i) \triangleq 1$$



A “random signal” is drawn from some ensemble

Autocorrelation Function : $\phi_v(t_1, t_2) \triangleq E\left[v(t_1)v^*(t_2)\right]$

Stationarity

v(t) is "wide-sense stationary" if:

$$\phi_v(t_1, t_2) = \phi_v(t_1 + \Delta, t_2 + \Delta) = \phi(\tau)$$

where $\tau \triangleq t_2 - t_1$ for all t_1, t_2, Δ

v(t) is "strict-sense stationary" if:

$$E[g\{v(t_1), v(t_2), \ldots, v(t_n)\}] = E[g\{v(t_1 + \Delta), v(t_2 + \Delta), \ldots, v(t_n + \Delta)\}]$$

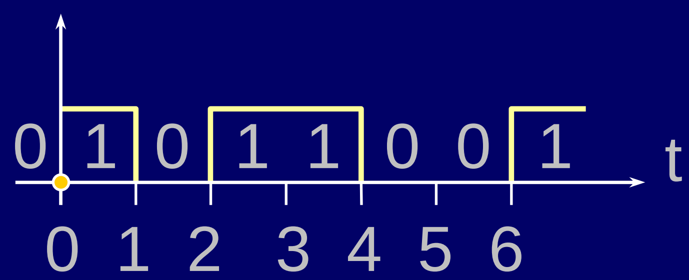
for any function g

v(t) is "Ergodic" if: v(t) is wide-sense stationary and

$$\phi_v(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} v(t)\, v^*(t - \tau)\, dt,$$

i.e., ensemble average = time average

Otherwise v(t) is "Non-stationary" – e.g.:
(time-origin sensitive)



transitions occur only at clock ticks

Transform Diagram: Random Signals

$$v(t) \leftrightarrow (?)$$

$$\downarrow \qquad\qquad \downarrow$$

$$\phi_v(\tau) \leftrightarrow \Phi(f)$$

Typical Sets of Units

[V]	$\leftrightarrow$	(?)	[V]	$\leftrightarrow$	(?)
$\downarrow$		$\downarrow$	$\downarrow$		$\downarrow$
$[V^2]$	$\leftrightarrow$	$[V^2/Hz]$	[W]	$\leftrightarrow$	[W/Hz]
			Power to 1-ohm resistor		Power spectral density

Power Spectral Density

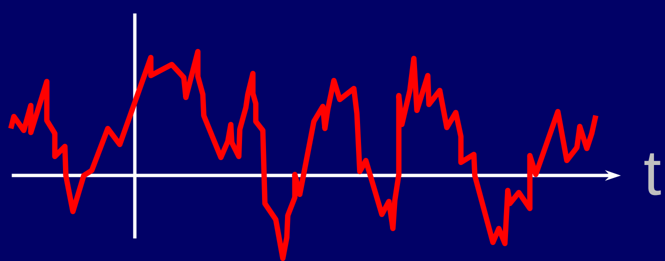
$$\Phi(f) = \lim_{T\to\infty} E\left[\frac{1}{2T}\left|\int_{-T}^{T} v(t)e^{-j2\pi ft}\,dt\right|^2\right]$$

Why use E[ ] if v(t) is ergodic?

Because $\lim \sigma_T^2(f) \neq 0!$ where $\sigma_T^2(f) \triangleq E[\Phi_T(f) - \Phi(f)]^2$

Spectral resolution increases with T, becoming infinite as $T \to \infty$

e.g.



Infinite information in finite bandwidth unless ensemble is averaged

Power Spectral Density Computation:

For a single ergodic waveform, take ensemble average over successive intervals of width 2T. Use T adequate to yield desired or meaningful spectral resolution.