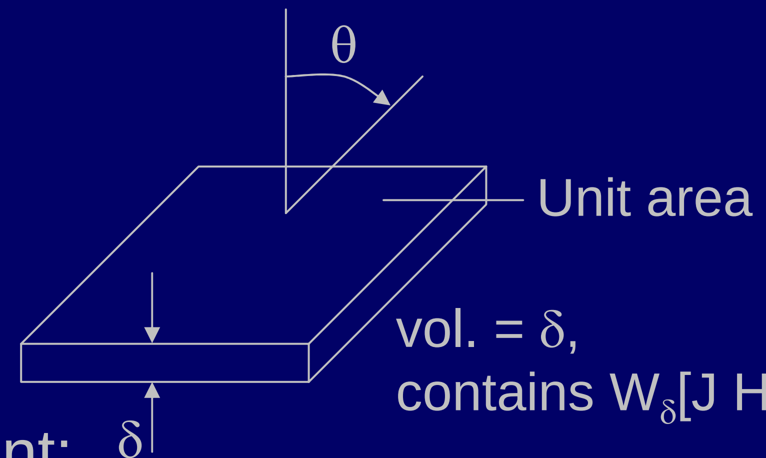
Find thermal radiation intensity:

Relate W(f) to I(θ,ϕ,f) = “radiation intensity”

i.e., $\int_{VOL}$ to $\int_{SURFACE}$

Consider slab of blackbody radiation:



Let slab radiate without replacement:

[Jm^{-3} Hz^{-1}] [Wm^{-2} ster^{-1} Hz^{-1}]

$$W_\delta = \int_V W(f)\,dV = \int I(f)\,dt\,dA\,d\Omega \quad [\text{J Hz}^{-1}]$$

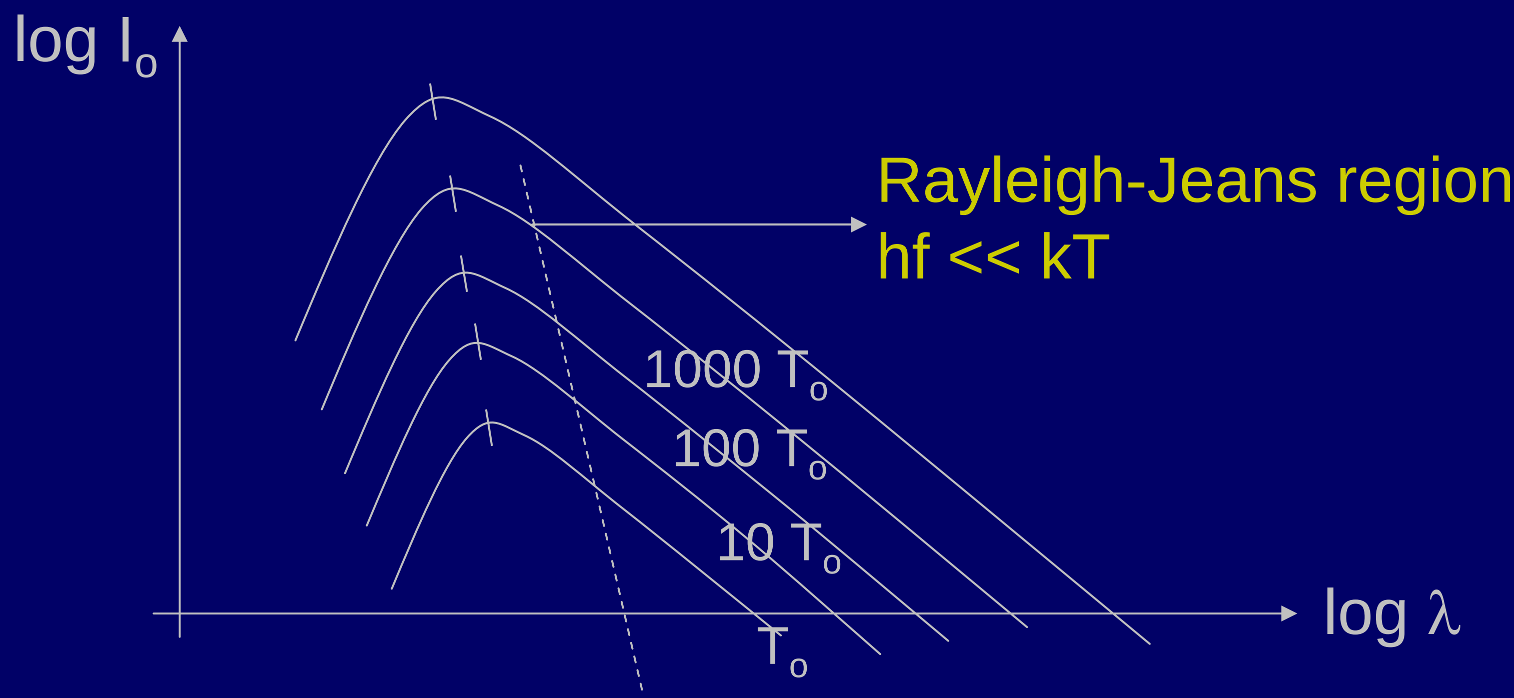
$$W_\delta = \delta \frac{8\pi}{c^3} hf^3 \Big/ \left(e^{hf/kT} - 1\right) = \int I_o \cos\theta \bullet \delta/(c \bullet \cos\theta) \bullet 1 \bullet d\Omega = \frac{4\pi I_o \delta}{c}$$

$$I_o(f,\theta,\phi) = 2hf^3 \Big/ c^2\left(e^{hf/kT} - 1\right) \quad \text{Wm}^{-2}\ \text{Hz}^{-1}\ \text{ster}^{-1}$$

Planck’s radiation law

$$I_o(f,\theta,\phi) \cong \frac{2kT}{\lambda^2} \text{ in the limit } hf \ll kT$$

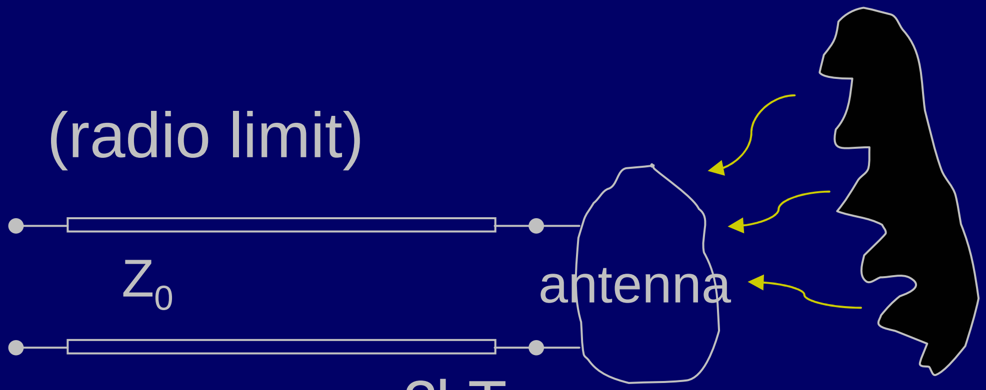
Rayleigh-Jeans Law



Paradox:

blackbody

(radio limit)



$$\frac{2kT}{\lambda^2}\ \mathrm{Wm^{-2}Hz^{-1}ster^{-1}}$$

KT $\mathrm{WHz^{-1}}$

$\underbrace{\neq f(\lambda)}_{\text{single mode}}$

$\longleftrightarrow$ how?

$$\frac{2kT}{\lambda^2} = f(\lambda)$$

Answer: kT $\mathrm{WHz^{-1}}$ $\mathrm{mode^{-1}}$

Two polarizations $\longrightarrow$ 2kT

Number of modes/Hz•m^2•ster $\longrightarrow$ λ^2

$$\frac{2kT}{\lambda^2}$$

Note: In the radio limit: hf_j...

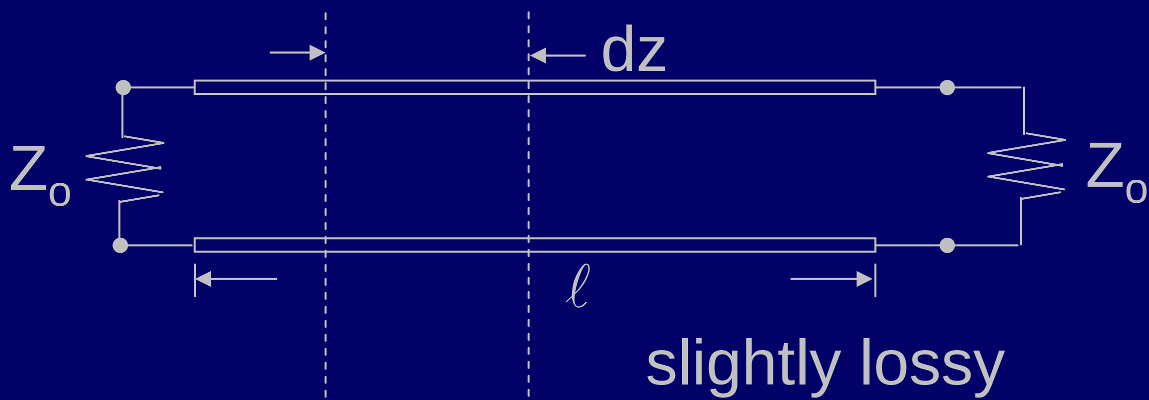
$$hf_j \bullet \bar{n}_j = hf \frac{1}{e^{hf/kT} - 1} \cong kT \left[J\,\text{mode}^{-1} \right] \text{[TEM line cavity]}$$

Recall: kT/2 J/degree of freedom • m mechanical systems

$$\left.\begin{array}{l} \sin \omega t \\ \cos \omega t \end{array}\right\} \text{orthogonal degrees of freedom, 2 per mode}$$

Therefore we also obtain kT/2 J/degree of freedom
for thermal radiation

Thermal radiation from lossy lines:



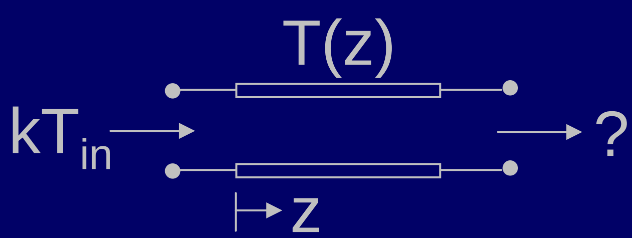
$P_+ = kT \rightarrow \quad \rightarrow kT\, e^{-\alpha dz}$ + emission = kT in equilibrium

$$\therefore \text{ Emission} = kT\left(1 - e^{-\alpha dz}\right) \cong kT\left(1 - [1 - \alpha dz]\right) = kT\alpha\, dz$$

What is thermal emission when not in equilibrium?

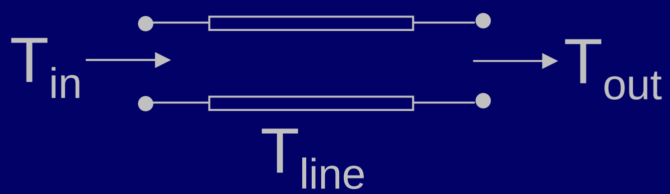
Answer: same, recall linearity of Maxwell's equations

$$kT_{out} = kT_{in}\, e^{-\int_o^L \alpha\, dz} + k\int_o^L T(z)\alpha(z)\, e^{-\int_z^L \alpha\, dz}\, dz$$



$$T_{out} = T_{in} e^{\underbrace{-\int_o^L \alpha dz}_{\tau_{MAX}}} + \int_0^{\tau_{max}} T(\tau) e^{-\tau} d\tau$$

Equation of radiative transfer



$$T_{out} = T_{in} e^{-\tau} + T_{line}\left(1 - e^{-\tau}\right)$$

Definition of a decibel:

$$\text{dB gain} \triangleq 10\log_{10} P_2/P_1$$

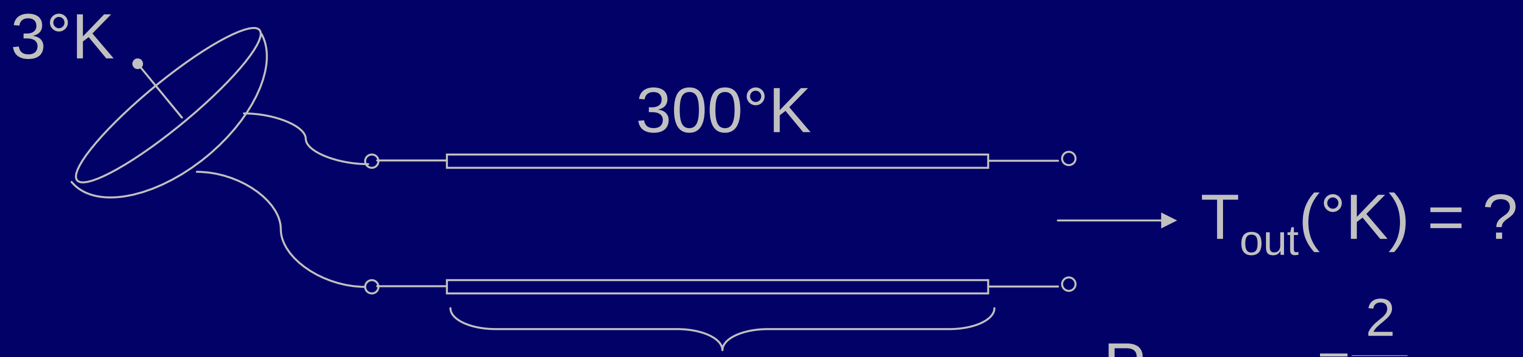
e.g.:

0 dB if $P_2 = P_1$

10 dB if $P_2 = 10P_1$

20 dB if $P_2 = 100P_1$

Example of thermal noise from lossy line:



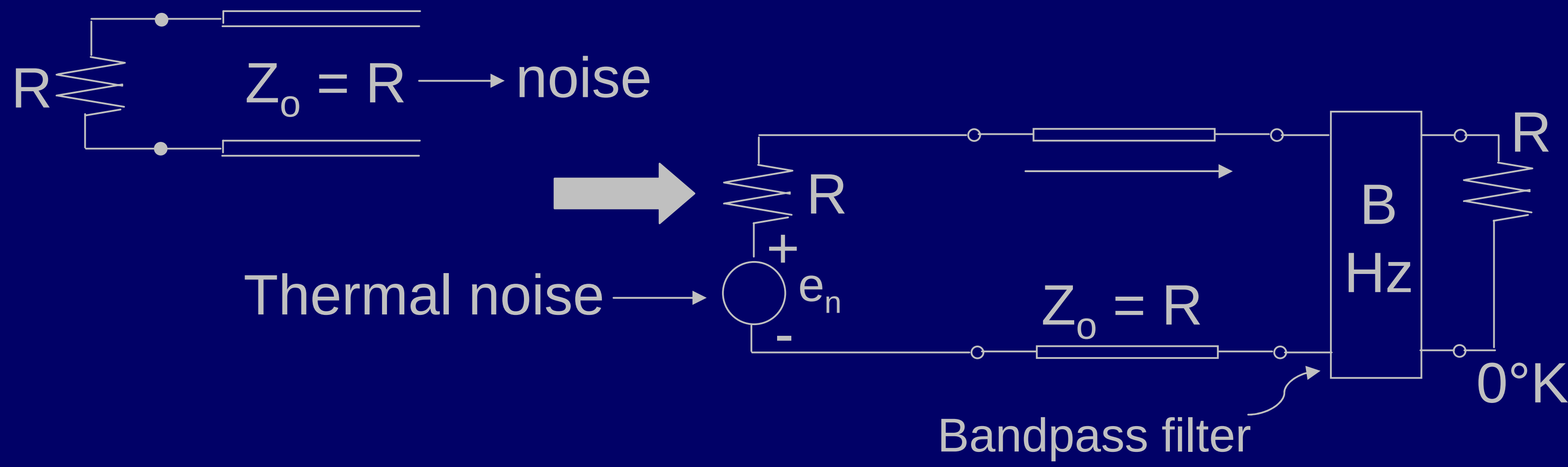
2 dB loss $\left(e^{-\tau} = \frac{P_2}{P_1} = 10^{-\frac{2}{10}} = 0.63\right)$

Case 1: $\tau = 0 \Rightarrow T_{out} = 3K$

Case 2: 2 dB loss $\Rightarrow T_{out} = 3 \times 0.63 + 300(1 - 0.63) \cong 113K$

Case 3: $\tau = \infty \Rightarrow T_{out} = 300K$

Thermal noise voltage (Johnson noise) in circuits



Watts dissipated in load R: $kTB = (e_{rms}/2)^2/R$

$$e_{rms}(\text{thermal noise}) = \sqrt{4kTBR} \text{ volts (in B Hz)}$$

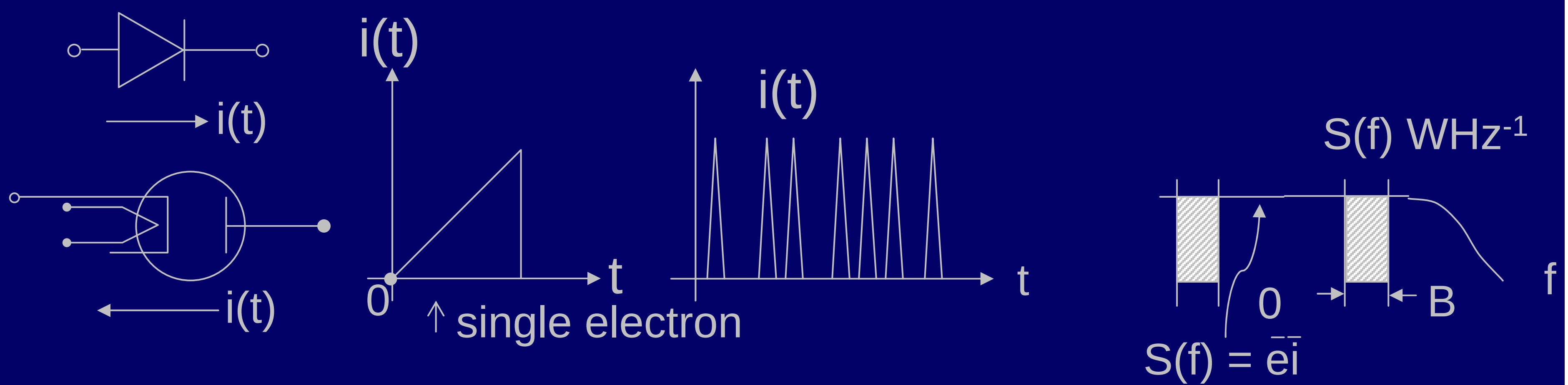
Example: Amplifier, 50Ω input, B = 100 MHz, T = 300K

$$e_{rms} = \sqrt{4 \bullet 1.38 \times 10^{-23} \times 300 \times 10^{8} \times 50} = 9.1\mu v$$

(9.1 mv if R = 50MΩ)

Shot Noise

For example, occurs in diodes:



If each charge moves independently, arrivals are poisson distributed:

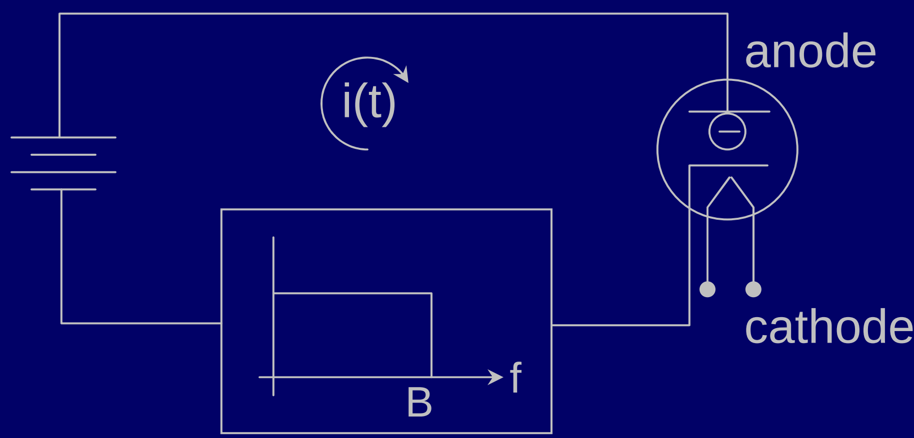
For example, let $\bar{i}$ = 1 ma, B = 10^8 Hz

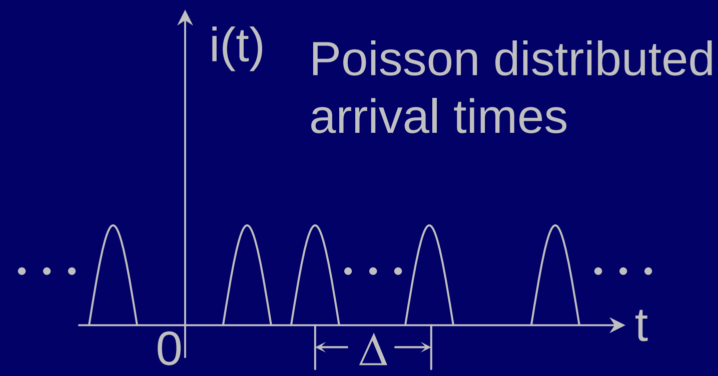
We later show $\overline{i_{AC}^2} = 2e\bar{i}B = 2 \times 1.6 \bullet 10^{-19} \times 10^{-3} \times 10^8$

Then $i_{AC_{rms}} \times 50\Omega \cong 9\mu V$ shot noise

Approximate derivation of shot noise

Sounds like falling shot



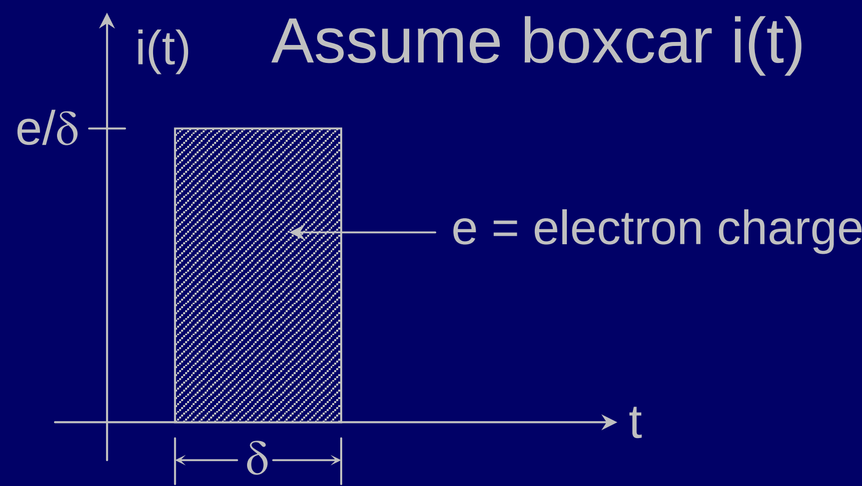


$\bar{i} = \bar{n}e$

$\bar{n} \triangleq$ avg. # electrons/sec

$\bar{n} >> B$, by assumption

Assume boxcar i(t)



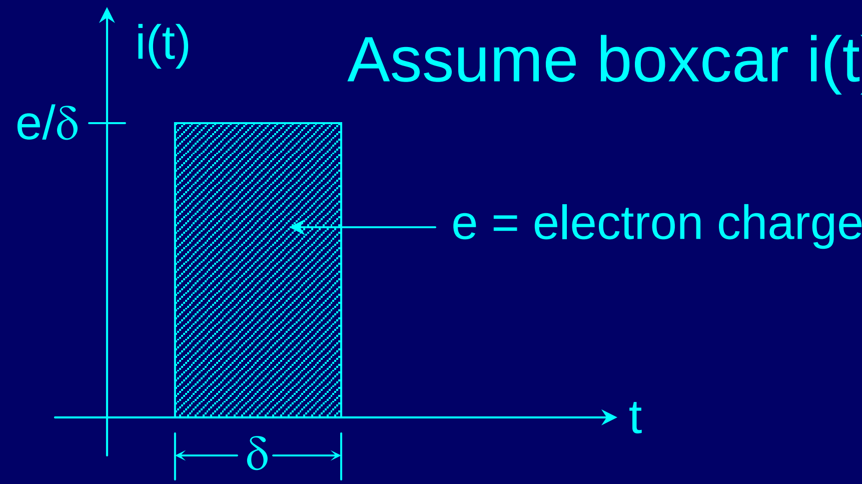
Approximate derivation of shot noise

$\bar{i} = \bar{n}e$

$\bar{n} \triangleq$ avg. # electrons/sec

$\bar{n} \gg B$, by assumption

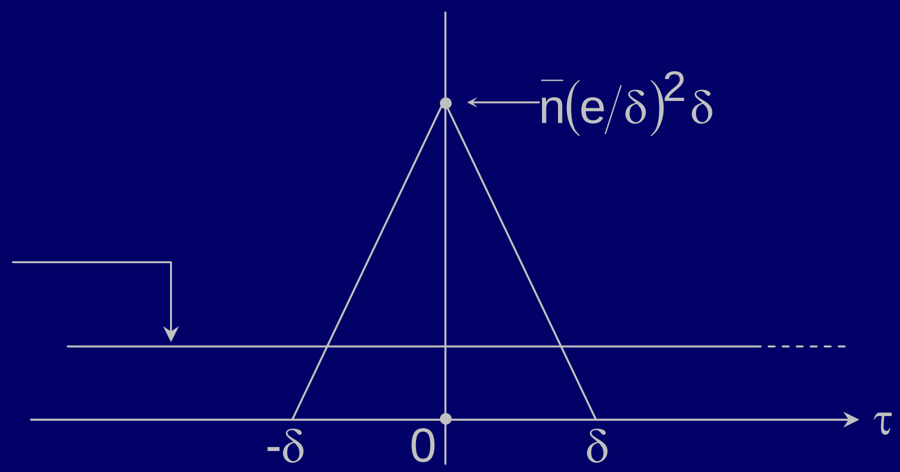
Assume boxcar i(t)



$$i(t) \;\downarrow\; \phi_i(\tau) \leftrightarrow \Phi_i(f)$$

$$\phi_i(\tau) \triangleq E\left[i(t)\,i(t-\tau)\right]$$

$$\left(\frac{e}{\delta} \bullet \frac{\delta}{\Delta}\right)^2 = \bar{i}^2 = \left(\bar{n}e\right)^2$$



Approximate derivation of shot noise

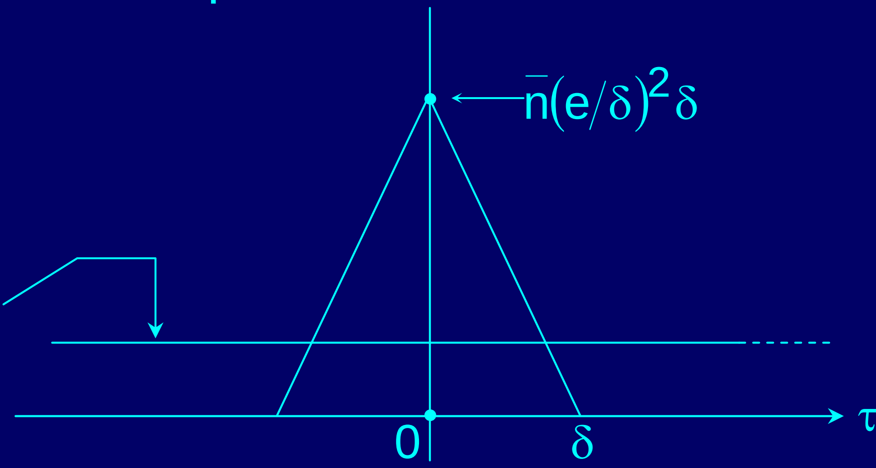
$i(t)$

$\downarrow$

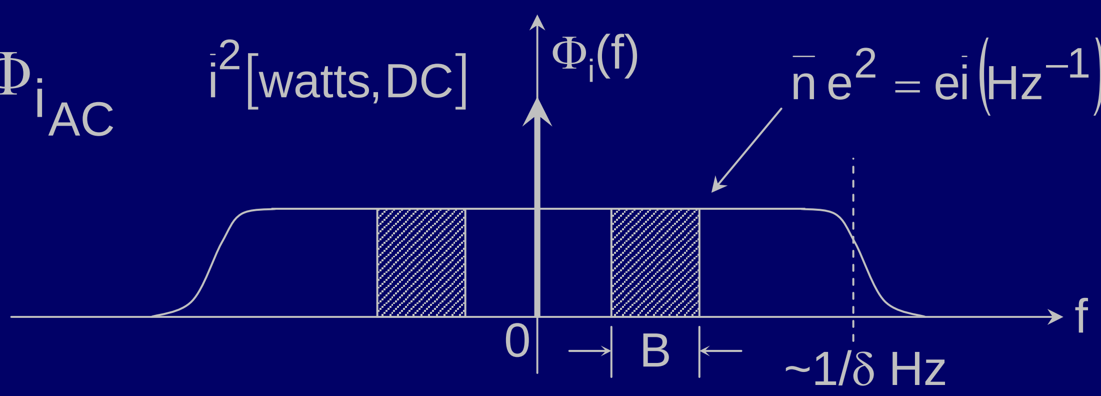
$\phi_i(\tau) \leftrightarrow \Phi_i(f)$

$$\phi_i(\tau) \triangleq E[i(t)i(t-\tau)]$$

$$\left(\frac{e}{\delta}\bullet\frac{\delta}{\Delta}\right)^2 = \bar{i}^2 = (\bar{n}e)^2$$



$$\Phi_i(f) = \Phi_{i_{DC}} + \Phi_{i_{AC}}$$



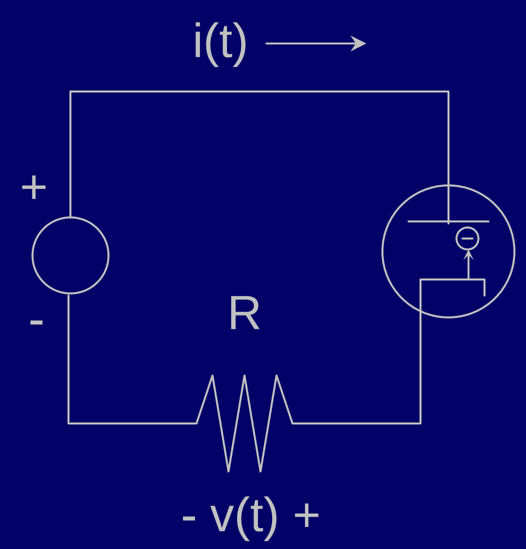
$$\therefore \text{in B Hz}: \ \sigma_{i_{AC}}^2 = 2Be\bar{i}\left[\text{Amp}^2\right]$$

Shot noise example

$R = 5K\Omega$

$B = 10^6 Hz$

$\bar{i} = 1ma$



$\bar{v} = \bar{i}R = 10^{-3} \bullet 5K = 5 \text{ volts}$

$\sigma_{i\ AC}^2 = 2Be\bar{i}$

$v_{rms}(shot) = \sqrt{\sigma_{i\ AC}^2} R = \sqrt{2 \bullet 10^6 \bullet 1.6 \times 10^{-19} \times 10^{-3}}\, 5000 \cong 0.1mv$

$$\left[\begin{array}{l} v_{rms}(thermal) = \sqrt{4kTBR} \\ \cong \sqrt{4 \times 1.38 \times 10^{-23} \times 300 \times 10^6 \times 5000} \cong 0.01mv \end{array} \right]$$

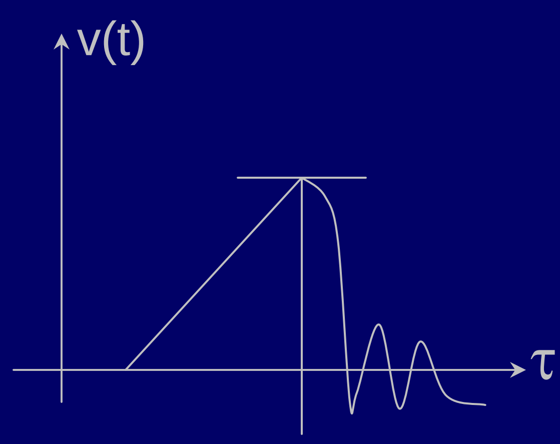
Receiver Architecture

Professor David H. Staelin

Graphics: Scott Bressler

Uses of receivers

I. Power measurement
II. Finite set of transmitted signals is possible; which is it?
III. Infinite set possible; estimate one or more parameters, e.g. arrival time, amplitude, Doppler, etc., e.g.

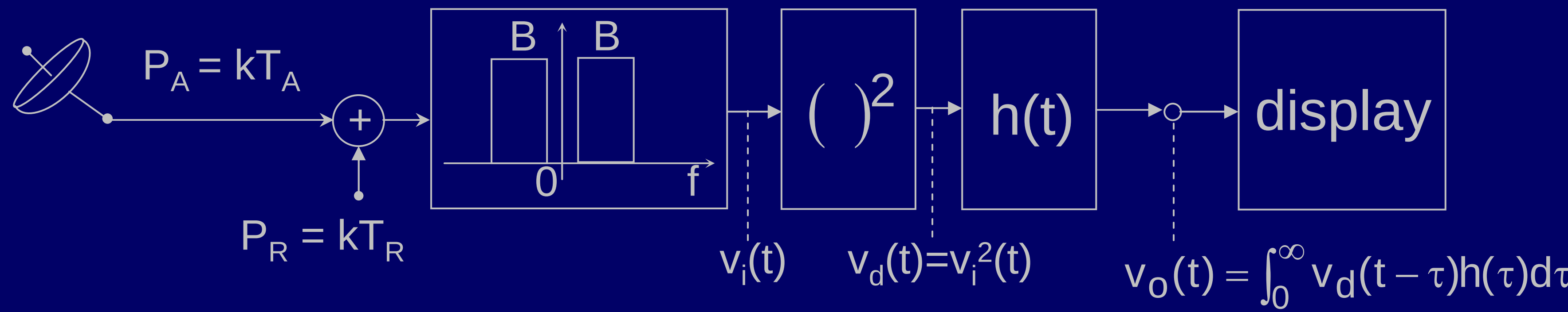


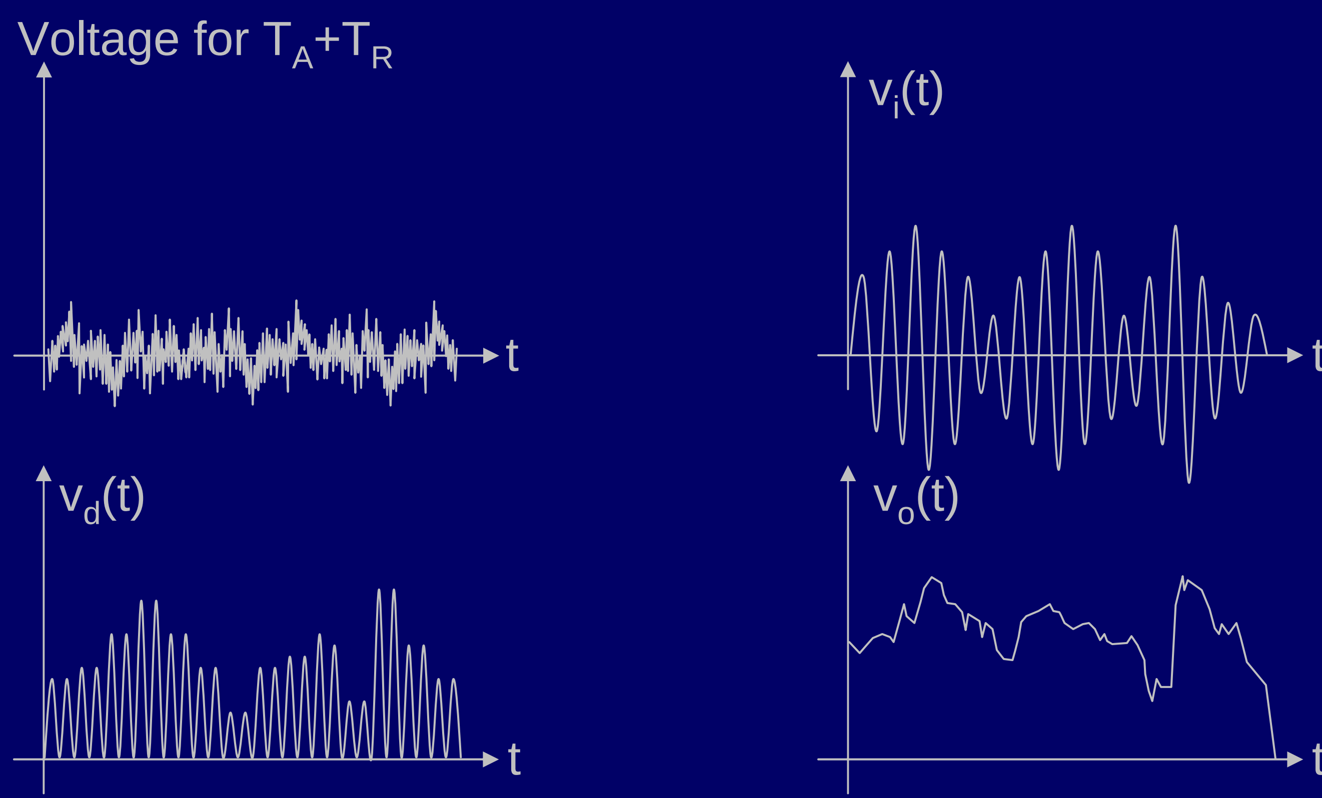
Design of waveform sets is part of our problem

Measurement of noise power in B Hz

Simply compute average output power over τ sec: $\propto \langle v^2(t) \rangle$

Total power radiometer





Total power radiometer