# 6.003 Homework 6

*Due at the beginning of recitation on **Wednesday, March 17, 2010**.*

## Problems

**1. Second-order systems**

The impulse response of a second-order CT system has the form

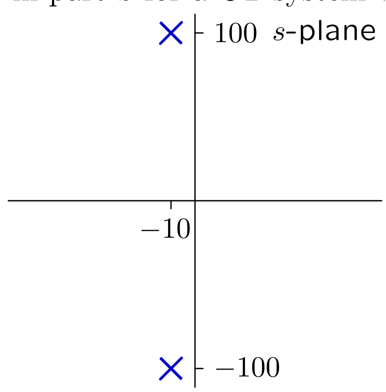
$$h(t) = e^{-\sigma t}\cos(\omega_d t + \phi)u(t)$$

where the parameters $\sigma$, $\omega_d$, and $\phi$ are related to the parameters of the characteristic polynomial for the system: $s^2 + Bs + C$.

**a.** Determine expressions for $\sigma$ and $\omega_d$ (not $\phi$) in terms of $B$ and $C$.

**b.** Determine
– the time required for the envelope $e^{-\sigma t}$ of $h(t)$ to diminish by a factor of $e$,
– the period of the oscillations in $h(t)$, and
– the number of periods of oscillation before $h(t)$ diminishes by a factor of $e$.
Express your results as functions of $B$ and $C$ only.

**c.** Estimate the parameters in part b for a CT system with the following poles:




The unit-sample response of a second-order DT system has the form

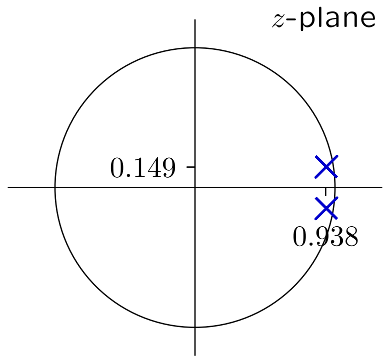
$$h[n] = r_0^n \cos(\Omega_0 n + \Phi)u[n]$$

where the parameters $r_0$, $\Omega_0$, and $\Phi$ are related to the parameters of the characteristic polynomial for the system: $z^2 + Dz + E$.

**d.** Determine expressions for $r_0$ and $\Omega_0$ (not $\Phi$) in terms of $D$ and $E$.

**e.** Determine
– the length of time required for the envelope $r_0^n$ of $h[n]$ to diminish by a factor of $e$.
– the period of the oscillations in $h[n]$, and
– the number of periods of oscillation in $h[n]$ before it diminishes by a factor of $e$.
Express your results as functions of $D$ and $E$ only.

**f.** Estimate the parameters in part e for a DT system with the following poles:




## 2. Maximum gain

For each of the following systems, find the frequency $\omega_m$ for which the magnitude of the gain is greatest.

**a.** $\dfrac{1}{1+s+s^2}$

**b.** $\dfrac{s}{1+s+s^2}$

**c.** $\dfrac{s^2}{1+s+s^2}$

Compare the $\omega_m$ for these systems and explain qualitatively any similarities or differences.

## 3. Phase

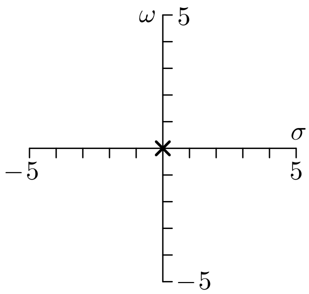
For a second-order system with poles at $-1$ and $-4$ (and no zeros), find the frequency at which the phase is $-90°$, using any method except for the vector method. Then illustrate and confirm that result using the vector method.

## 4. Matches

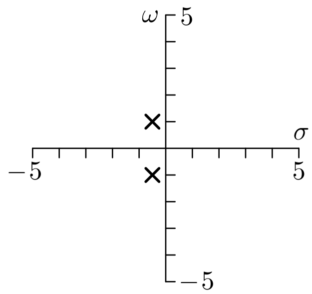
The following plots show pole-zero diagrams, impulse responses, Bode magnitude plots, and Bode angle plots for six causal CT LTI systems. Determine which corresponds to which and fill in the following table.

| | $h(t)$ | Magnitude | Angle |
|---|---|---|---|
| PZ diagram 1: | | | |
| PZ diagram 2: | | | |
| PZ diagram 3: | | | |
| PZ diagram 4: | | | |
| PZ diagram 5: | | | |
| PZ diagram 6: | | | |

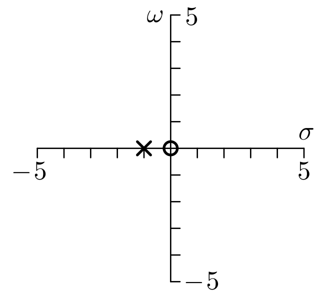
Pole-zero diagram 1



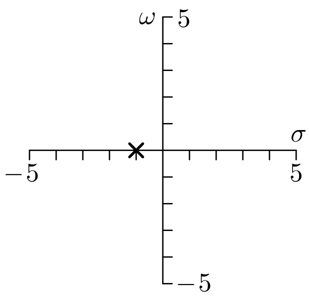

Pole-zero diagram 2



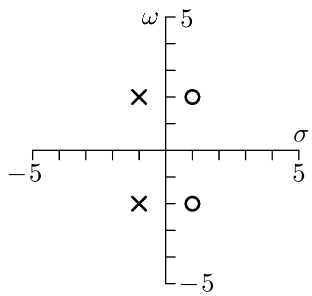

Pole-zero diagram 3



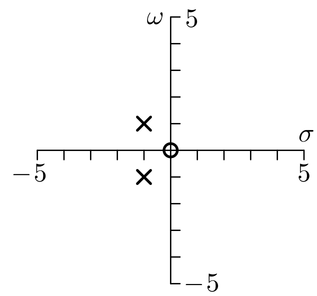

Pole-zero diagram 4



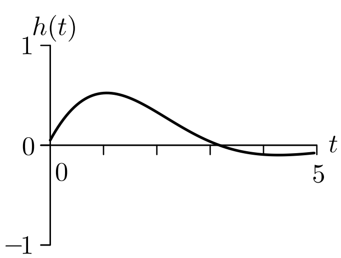

Pole-zero diagram 5



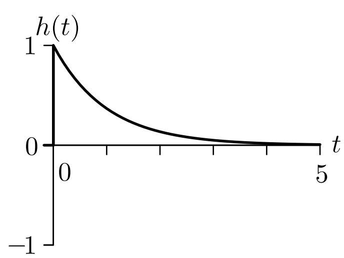

Pole-zero diagram 6



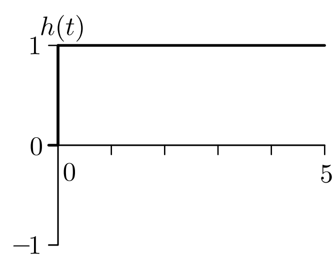

Impulse response 1



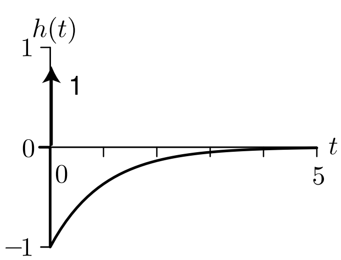

Impulse response 2



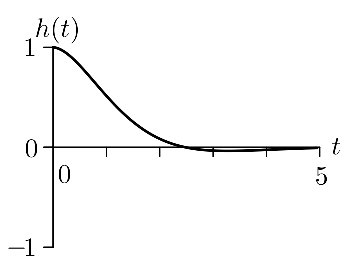

Impulse response 3



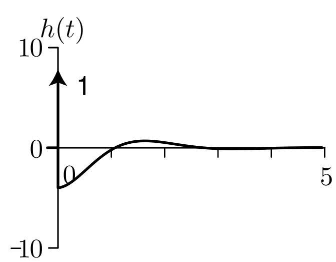

Impulse response 4



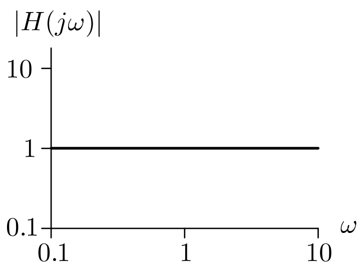

Impulse response 5



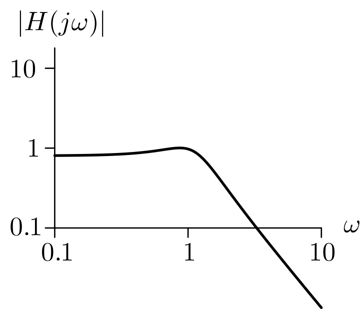

Impulse response 6



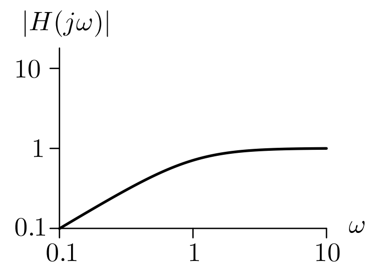
Bode Magnitude 1



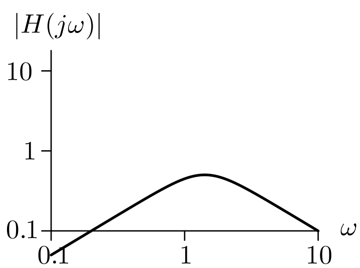

Bode Magnitude 2



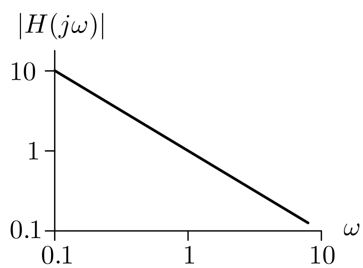

Bode Magnitude 3



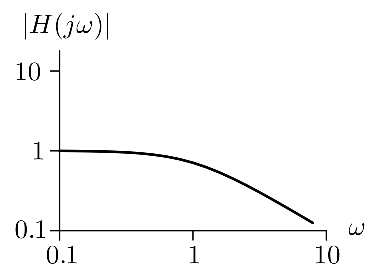

Bode Magnitude 4



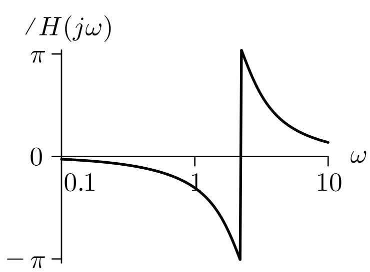

Bode Magnitude 5



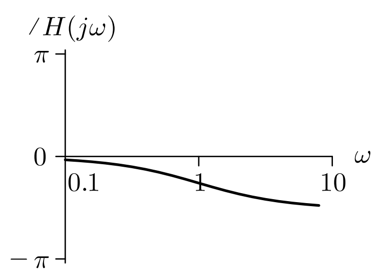

Bode Magnitude 6



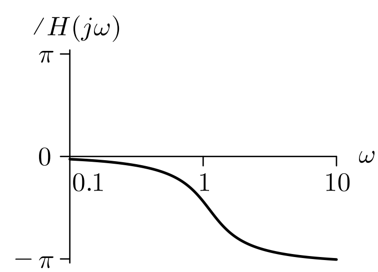

Bode Angle 1



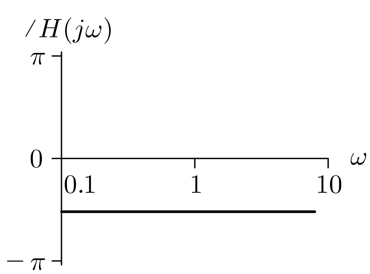

Bode Angle 2



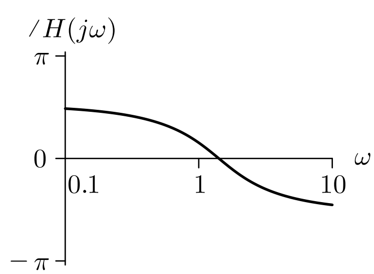

Bode Angle 3



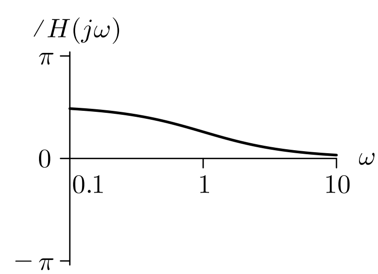

Bode Angle 4




Bode Angle 5



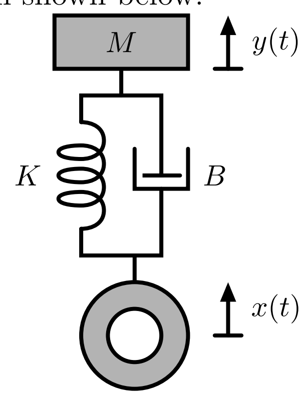

Bode Angle 6



# Engineering Design Problems

**5. Automotive suspension**

Wheels are attached to an automobile through a suspension system that is designed to minimize the vibrations of the passenger compartment that result when traveling over bumpy terrain. The suspension system consists of a spring and shock absorber that are both compressed when the wheel passes over a bump, so that the sudden motion of the wheel is not directly transmitted to the passenger compartment. The spring generates a force to hold the passenger compartment at a desired distance above the surface of the road, and the shock absorber adds frictional damping. In this problem, you will determine how much damping is desireable by analyzing a simple model of an automobile's suspension system shown below.




The model consists of a mass $M$ that represents the mass of the car, which is connected through a spring and dashpot to the wheel. The vertical displacement of the wheel from it's equilibrium position is taken as the input $x(t)$. The vertical displacement of the mass from it's equilibrium position is taken as the output $y(t)$. The spring is assumed to obey Hooke's law, so that the force it generates is a constant $K$ times the amount that the spring is compressed relative to it's equilibrium compression. The shock absorber is assumed to generate a force that is a constant $B$ times the velocity with which the shock absorber is compressed. Notice that by referring $x(t)$ and $y(t)$ to their equilibrium positions, the force due to gravity can be ignored. Assume that $M = 1$ and $K = 1$.

**a.** Determine the differential equation that relates the input $x(t)$ and output $y(t)$.

**b.** Determine and plot the impulse response of the system when $B = 0$. Based on this result, give a physical explanation of the problem that would result if there were no shock absorber in the system.

**c.** Determine an expression for the smallest positive damping constant $B$ for which the poles of the system have real values. Sketch the impulse response of the system for this value of $B$. Based on this result, give a physical explanation of how the shock absorber improves performance of the suspension system.

**d.** Consider what would happen if $B$ were very large. Sketch the impulse response for the system if $B = 100$. Describe how this response might be less desireable than that in part c. Provide a physical explanation for how a stiff shock absorber can degrade system performance.

## 6. Dial tones

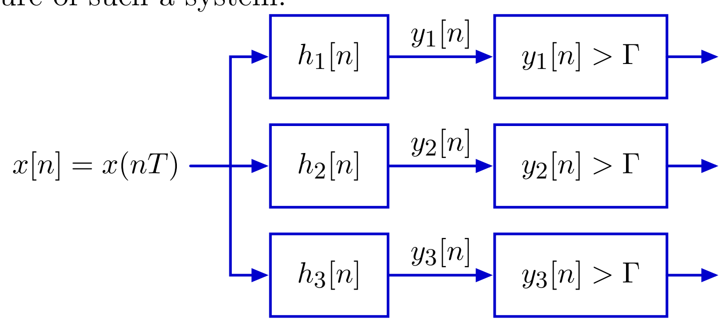
Pressing the buttons on a touch-tone phone generates tones that are used for dialing. Each button produces a pair of tones of the form

$$x(t) = \cos(2\pi f_1 t) + \cos(2\pi f_2 t)$$

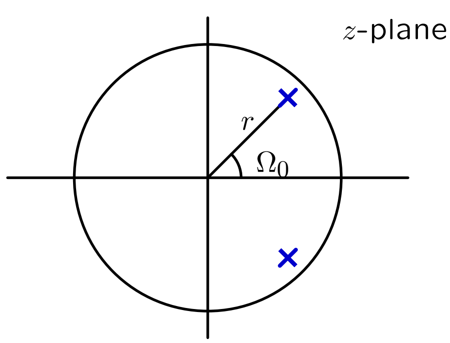
where $f_1$ and $f_2$ code the row and column of the button as shown in the following table.

| | $f_2$[Hz] | | |
|---|---|---|---|
| $f_1$[Hz] | 1209 | 1336 | 1477 |
| 697 | 1 | 2 | 3 |
| 770 | 4 | 5 | 6 |
| 852 | 7 | 8 | 9 |
| 941 | * | 0 | # |

This problem concerns the design of a system to detect the row and column numbers that were pressed by analyzing the signal $x(t)$. The following block diagram illustrates the basic structure of such a system.




The input $x(t)$ is first sampled with $T = 10^{-4}$ seconds. The samples are then passed through LTI systems that generate intermediate signals so that $y_1[n]$ is large when a button in column 1 is pressed, $y_2[n]$ is large when a button in column 2 is pressed, and $y_3[n]$ is large when a button in column 3 is pressed. These intermediate signals are then passed through detectors that determine when the signals are bigger than a threshold value $\Gamma$. Your task is to design the LTI systems. Each should consist of a system with 2 poles of the form shown in the following pole-zero diagram.




Such systems can be simulated by finding the difference equation that corresponds to the system and then iteratively solving that difference equation.

**a.** Determine values of $r$ and $\Omega_0$ so that the $h_1[n]$ system generates a large response when the "1" key is pressed and a small response when the "2" or "'3" keys are pressed. Your solution should work not only when the input consists of a single key press but also when it consists of sequences of key presses (as when dialing a phone number). Submit hardcopies of your code to generate $y_1[n]$ along with a plot of $y_1[n]$.

**b.** Describe how the choice of $\Omega_0$ affects the output signal $y_1[n]$.

**c.** Describe how the choice of $r$ affects the output signal $y_1[n]$. In particular, what limits the maximum acceptable value of $r$? Also, what limits the minimum acceptable value of $r$?