

# 6.003 (Spring 2010)

## Quiz #2

*April 7, 2010*

**Name:**

**Kerberos Username:**

**Please circle your section number:**

<i>Section</i>	<i>Instructor</i>	<i>Time</i>
1	Peter Hagelstein	10 am
2	Peter Hagelstein	11 am
3	Rahul Sarpeshkar	1 pm
4	Rahul Sarpeshkar	2 pm

**Grades will be determined by the correctness of your answers (explanations are not required).**

**Partial credit will be given for ANSWERS that demonstrate some but not all of the important conceptual issues.**

You have **two hours**.

Please put your initials on all subsequent sheets.

Enter your answers in the boxes.

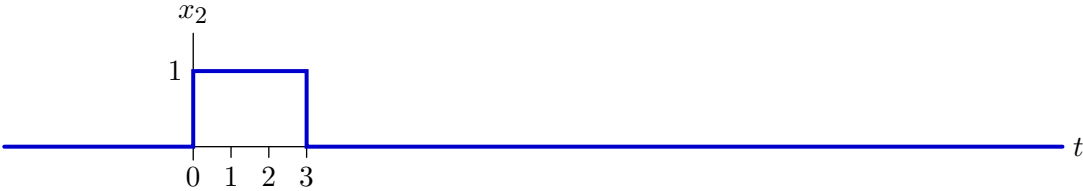
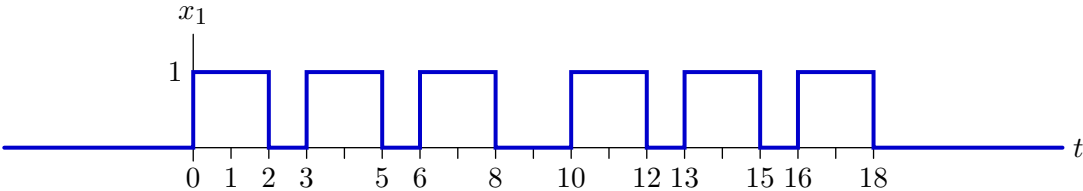
This quiz is closed book, but you may use two  $8.5 \times 11$  sheets of paper (four sides total).

No calculators, computers, cell phones, music players, or other aids.

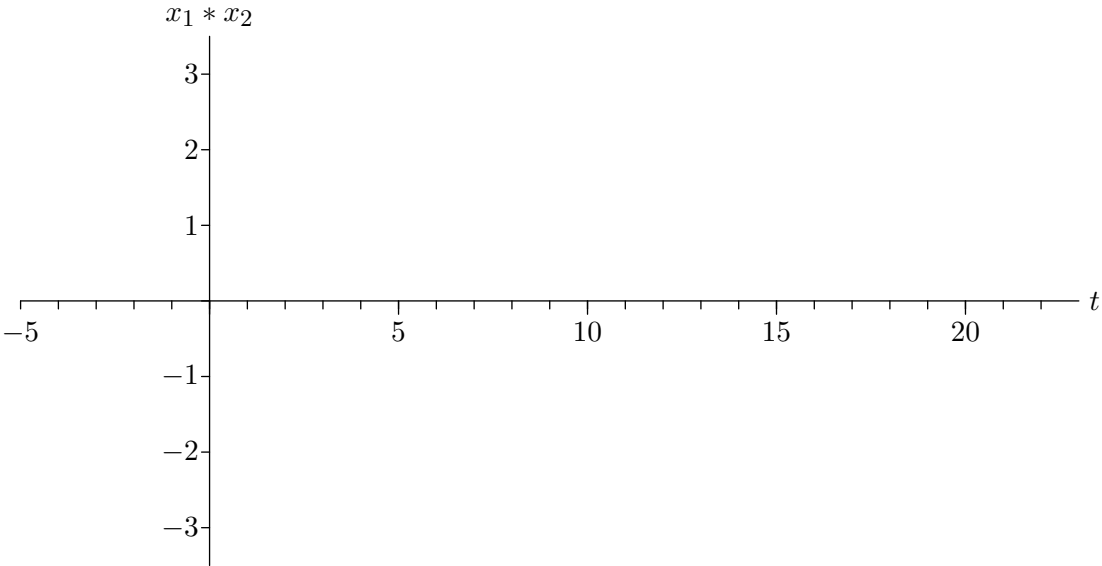
1	/20
2	/30
3	/20
4	/30
Total	/100

1. Convolution [20 points]

Signals  $x_1(t)$  and  $x_2(t)$  are shown in the plots below, and are zero outside the indicated intervals.

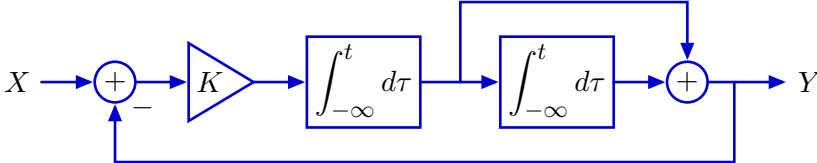


Plot the result of convolving  $x_1(t)$  with  $x_2(t)$ . Make sure that the important break-points are clear.



**2. Impulse response** [30 points]

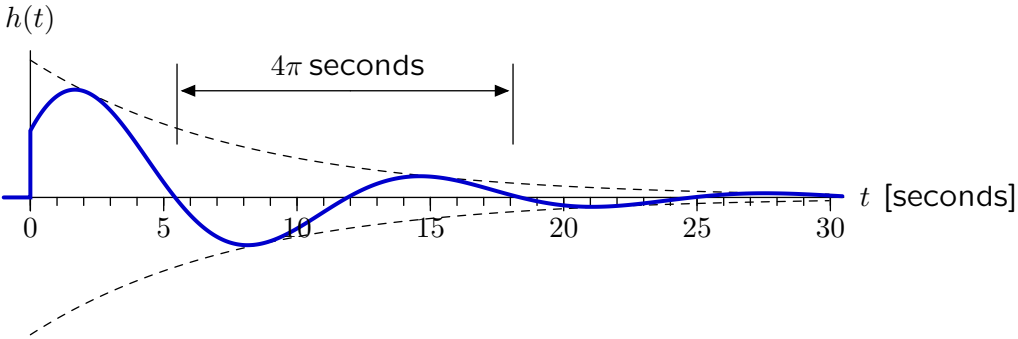
Consider the following control system where the gain  $K$  is a real-valued constant.



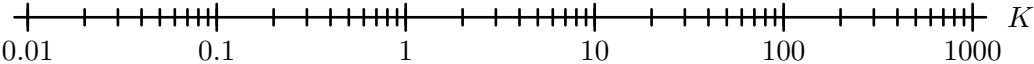
**Part a.** [10 points] Determine the system function  $H(s) = \frac{Y(s)}{X(s)}$  as a function of  $K$ .

$H(s) =$

**Part b.** [20 points] The following plot shows the impulse response  $h(t)$  of the closed-loop system for a particular value of  $K$ . [The dashed curves are exponential functions of time, shown for reference.]

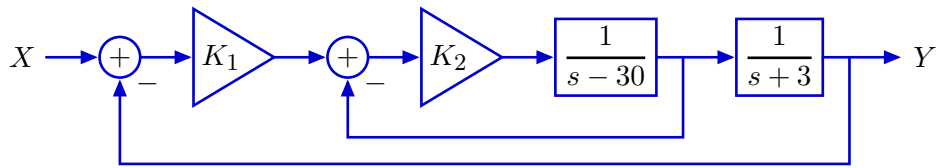


Determine  $K$  and indicate its value by placing an **X** on the following scale.

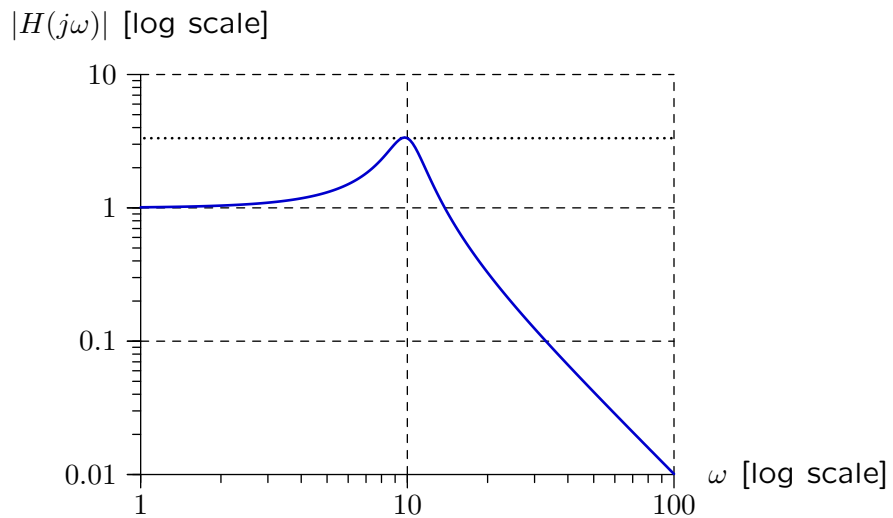


**3. Frequency Response** [20 points]

Let  $H(s) = Y(s)/X(s)$  for the following system.



Find  $K_1$  and  $K_2$  so that  $|H(j\omega)|$  matches the plot below.



Enter numbers (or numerical expressions) for  $K_1$  and  $K_2$  in the boxes.

$K_1 =$

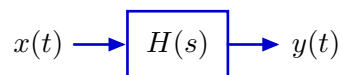
$K_2 =$

**4. Fourier series** [30 points]

Let  $x(t)$  represent a periodic signal (period  $T = 8$  seconds) whose Fourier series coefficients are

$$a_k = \begin{cases} \frac{1}{j\pi k} & \text{for integers } k \neq 0 \\ 0 & k = 0. \end{cases}$$

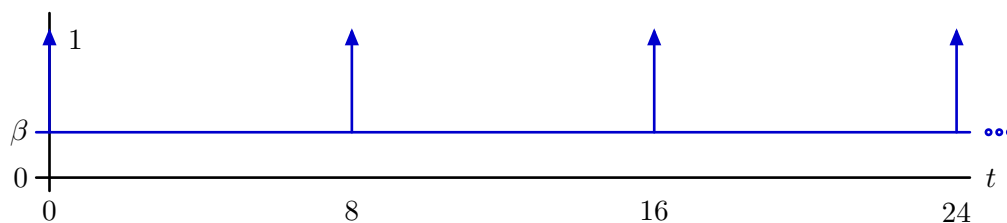
When  $x(t)$  is the input to a linear, time-invariant system with system function  $H(s)$



the output  $y(t)$  is the sum of a constant  $\beta$  plus a periodic train of impulses with area 1,

$$y(t) = \beta + \sum_{k=-\infty}^{\infty} \delta(t - 8k)$$

as shown below.



**Part a.** [15 points] Determine  $\beta$ .

$$\beta = \boxed{\phantom{0}}$$

**Part b.** [15 points] Consider the output of the same system if the period of the input signal is changed to  $T = 4$  seconds while keeping the Fourier series coefficients  $a_k$  unchanged.

Is it possible to determine the new output signal from the information provided?

possible? (Yes or No)

If **Yes**, sketch and fully label the new output signal on the axes below.



If **No**, briefly explain why not.

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6.003 Signals and Systems  
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