2.019 Design of Ocean Systems

Lecture 5

Seakeeping (I)

February 18, 2011

Six-Degree-of-Freedom Motion of a Floating Body in Waves

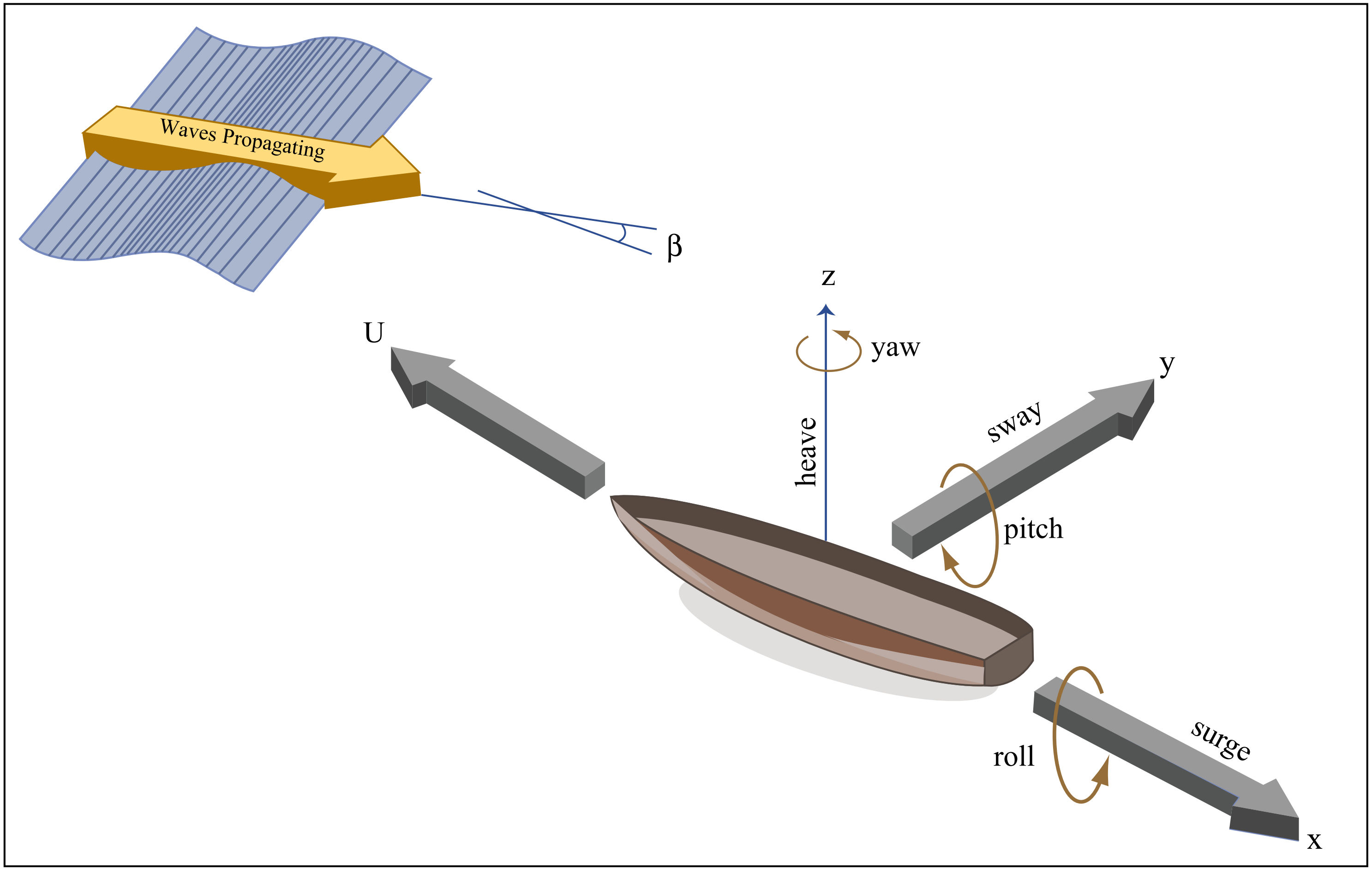
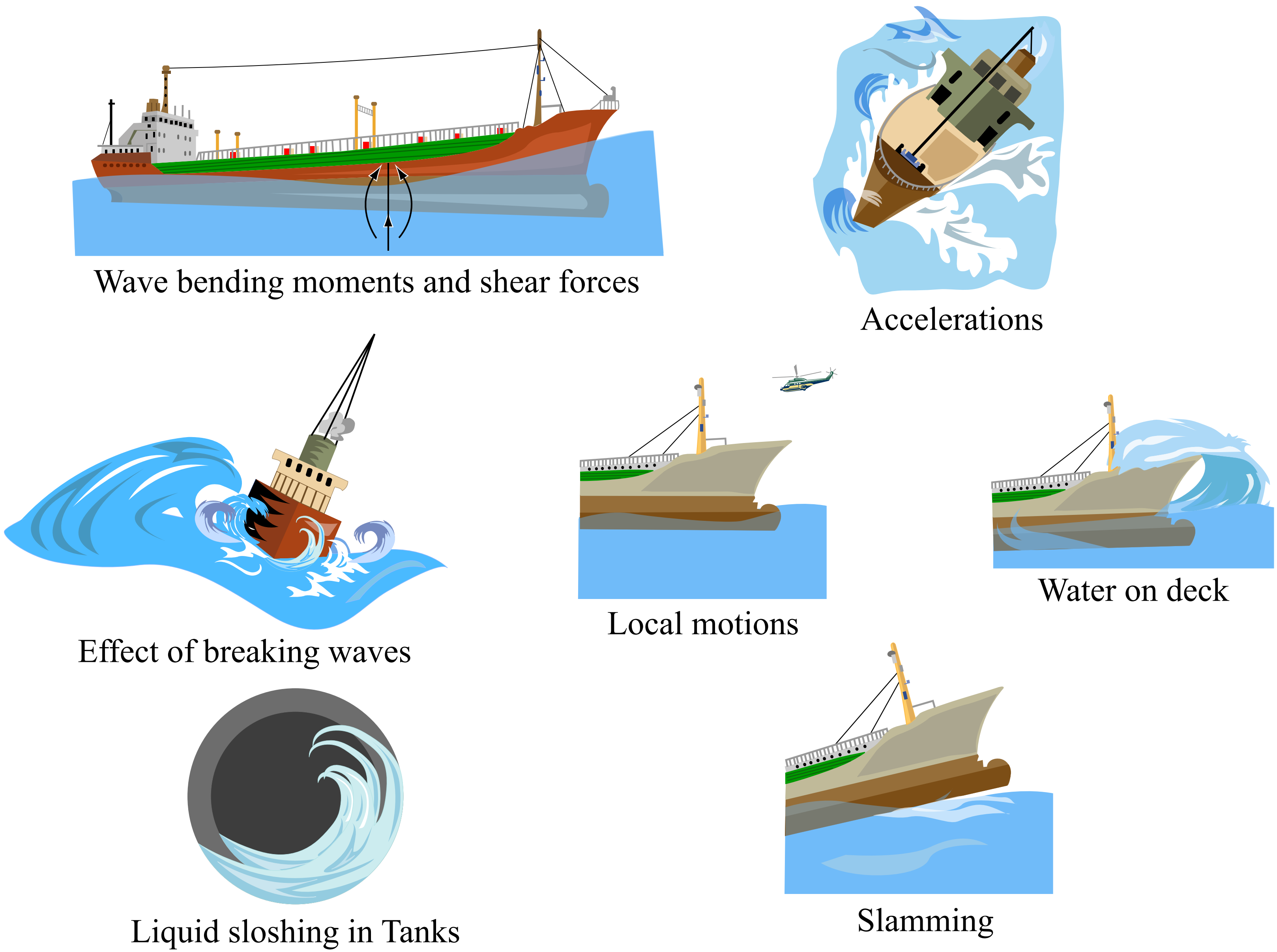


Image by MIT OpenCourseWare.

Translation in x: surge $\zeta_1(t)$
Translation in y: sway $\zeta_2(t)$;
Translation in z: heave $\zeta_3(t)$;
Rotation with x: roll $\zeta_4(t)$;
Rotation with y: pitch $\zeta_5(t)$;
Rotation with z: yaw $\zeta_6(t)$;

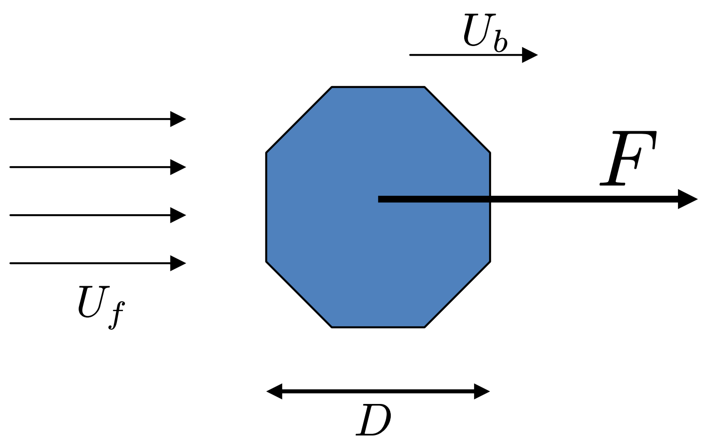
Examples of Seakeeping and Wave Load Problems for Ships and Offshore Structure



Concerns of Seakeeping in FPSO Design

- Increasing maximum loads (due to dynamic pressure)
- Affecting operation
  - Production by Risers
  - Gas-oil, oil/water separation
  - Normally, heave amplitude < 4m, pitch amplitude < 5 degrees, roll amplitude < 10 degrees, excursion < (5~8)% water depth
- Vibration of superstructures
- Fatigue life of hull structures, risers, etc.
- Survival in extreme seas
- Local extreme structure damage (bottom slamming, breaking wave impact, green water on deck etc.)
- Human safety

Hydrodynamic Forces on a Body in Unbounded Fluid



$$(1)\ U_f=0,\ U_b(t)\neq 0$$

$$F(t) = -m_a \frac{\mathrm{d}U_b(t)}{\mathrm{d}t}$$

m_a : Added mass
Depending on body geometry, motion direction, fluid density

$$(2)\ U_f(t)\neq 0,\ U_b=0$$

Morrison's formula:

$$F(t) = \rho\nabla \frac{\mathrm{d}U_f(t)}{\mathrm{d}t} + m_a \frac{\mathrm{d}U_f(t)}{\mathrm{d}t}$$

Froude-Krylov force　　　Added mass effect

ρ: Fluid density
∇: Body volume

$$(3)\ U_f(t)\neq 0,\ U_b(t)\neq 0$$

$$F(t) = \rho\nabla \frac{\mathrm{d}U_f(t)}{\mathrm{d}t} + m_a \left\{ \frac{\mathrm{d}U_f(t)}{\mathrm{d}t} - \frac{\mathrm{d}U_b(t)}{\mathrm{d}t} \right\}$$

Potential Flow

- In typical marine engineering applications such as ships, offshore platforms,

$$R_e = \frac{UL}{\nu} = 10^{6\sim 10}$$

  Thus, viscous effect can be neglected in general.

- Flow can be considered as a irrotational flow $(\text{i.e. vorticity } \nabla \times \vec{v} = 0)$ except under some special conditions where flow separation occurs.

- Fluid motion in the ocean is normally assumed as a potential flow:

$$\text{Velocity: } \vec{v}(x,y,z,t) = \nabla\phi(x,y,z,t)$$

$$\text{Continuity equation: } \nabla^2\phi = 0$$

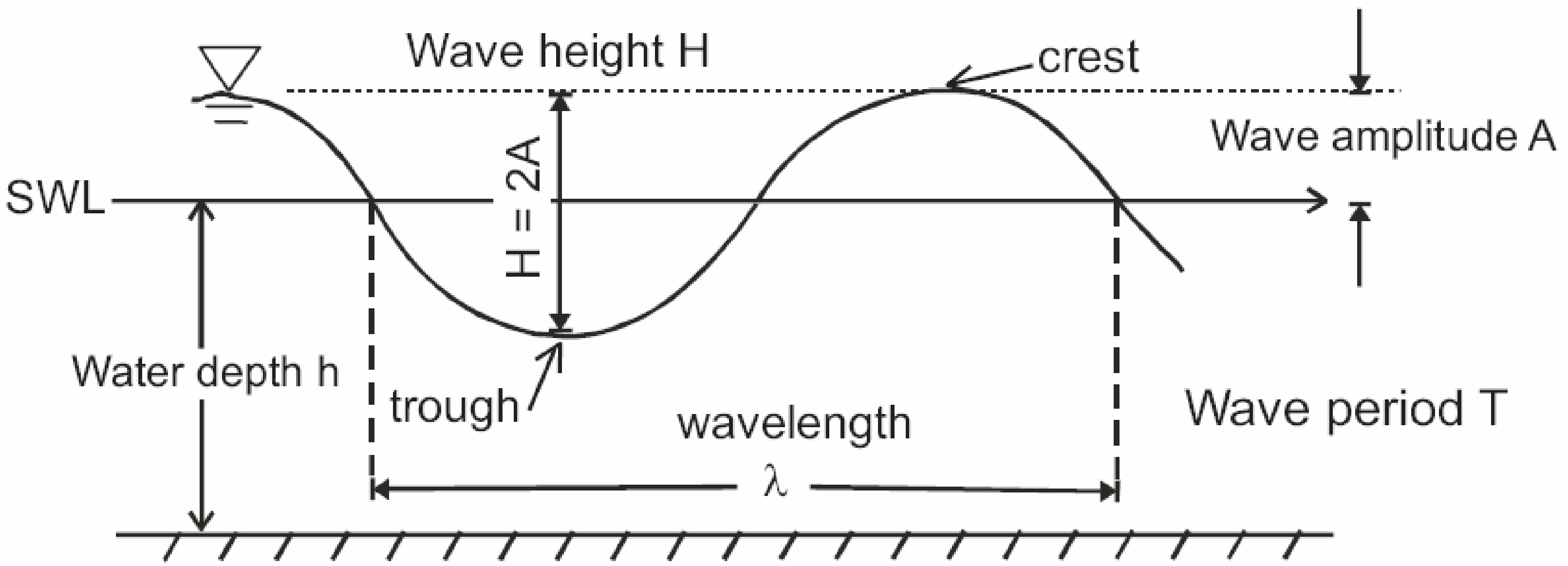
$$\text{Momentum eqaution: } \frac{p(x,y,z,t)}{\rho} = -\frac{\partial\phi}{\partial t} - \frac{1}{2}|\nabla\phi|^2 - gz$$

- The key is to solve the Laplace equation with certain boundary conditions for the velocity potential $\phi(x,y,z,t)$

Linearized (Airy) Wave Theory

Assume small wave amplitude compared to wavelength, i.e., small free surface slope

$$\frac{A}{\lambda} << 1$$



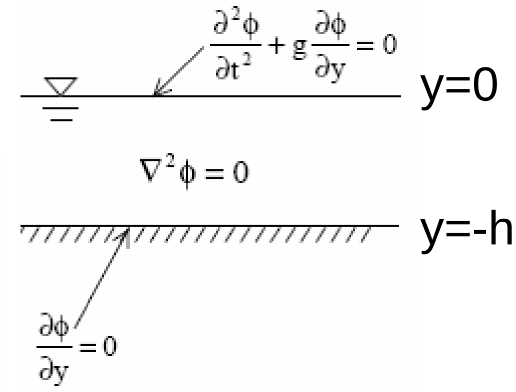
Consequently

$$\frac{\phi}{\lambda^2/T}, \frac{\eta}{\lambda} << 1$$

We keep only linear terms in ϕ, η.

For example: $()|_{y=\eta} = \underbrace{()_{y=0}}_{\text{keep}} + \underbrace{\eta \frac{\partial}{\partial y} ()|_{y=0}}_{\text{discard}} + \ldots$ Taylor series

• Boundary-Value Problem (BVP) for linearized (Airy) wave:



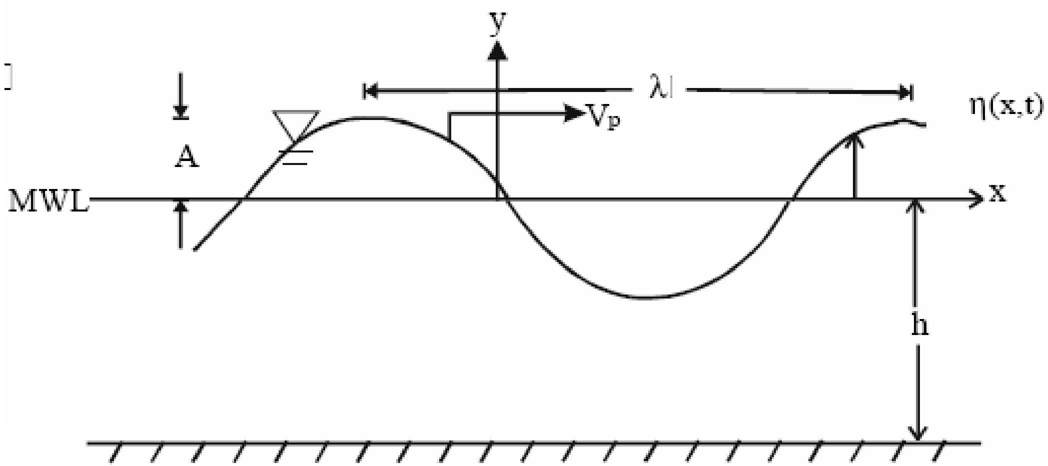
	Finite depth $h = const$	Infinite depth
GE:	$\nabla^2 \phi = 0, \quad -h < y < 0$	$\nabla^2 \phi = 0, \quad y < 0$
BKBC:	$\frac{\partial \phi}{\partial y} = 0, \quad y = -h$	$\nabla \phi \to 0, \quad y \to -\infty$
FSKBC:	$\frac{\partial \phi}{\partial y} = \frac{\partial \eta}{\partial t}, \quad y = 0$	$\to \frac{\partial^2 \phi}{\partial t^2} + g \frac{\partial \phi}{\partial y} = 0$
FSDBK:	$\frac{\partial \phi}{\partial t} + g\eta = 0, \quad y = 0$	

• Given velocity potential ϕ, find free-surface elevation η and pressure p:

$$\eta(x,t) = -\frac{1}{g} \left. \frac{\partial \phi}{\partial t} \right|_{y=0}$$

$$p - p_a = \underbrace{-\rho \frac{\partial \phi}{\partial t}}_{\text{dynamic}} - \underbrace{\rho g y}_{\text{hydrostatic}}$$

Solution of 2D Periodic Progressive (Airy) Waves



Potential: $$\phi = \frac{gA}{\omega}\sin\left(kx - \omega t\right)\frac{\cosh k\left(y + h\right)}{\cosh kh}$$

Free-surface elevation: $$\eta = A\cos(kx - \omega t)$$

A = H/2: wave amplitude; k= 2π/λ: wavenumber; ω =2π/T : frequency

Dispersion relation: $$\omega^2 = gk\tanh kh$$

Phase velocity: $$V_p \equiv \frac{\lambda}{T} = \frac{\omega}{k} = \sqrt{\frac{g}{k}\tanh kh}$$

Characteristics of a Linear Plane Progressive Wave

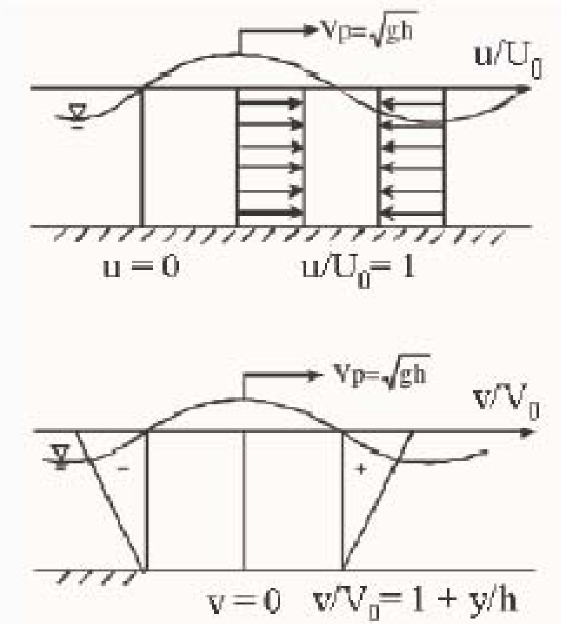
- Velocity Field:

Velocity on free surface $\vec{v}(x, y=0, t)$	
$u(x,0,t) \equiv U_o = A\omega \frac{1}{\tanh kh} \cos(kx - \omega t)$	$v(x,0,t) \equiv V_o = A\omega \sin(kx - \omega t) = \frac{\partial \eta}{\partial t}$
Velocity field $\vec{v}(x, y, t)$	
$u = \frac{\partial \phi}{\partial x} = \frac{Agk}{\omega} \frac{\cosh k(y+h)}{\cosh kh} \cos(kx - \omega t)$ $= \underbrace{A\omega}_{U} \frac{\cosh k(y+h)}{\sinh kh} \cos(kx - \omega t) \Rightarrow$ $\frac{u}{U_o} = \frac{\cosh k(y+h)}{\cosh kh} \begin{cases} \sim e^{ky} & \text{deep water} \\ \sim 1 & \text{shallow water} \end{cases}$	$v = \frac{\partial \phi}{\partial y} = \frac{Agk}{\omega} \frac{\sinh k(y+h)}{\cosh kh} \sin(kx - \omega t)$ $= \underbrace{A\omega}_{U} \frac{\sinh k(y+h)}{\sinh kh} \sin(kx - \omega t) \Rightarrow$ $\frac{v}{V_o} = \frac{\sinh k(y+h)}{\sinh kh} \begin{cases} \sim e^{ky} & \text{deep water} \\ \sim 1 + \frac{y}{h} & \text{shallow water} \end{cases}$
• u is in phase with η	• v is out of phase with η

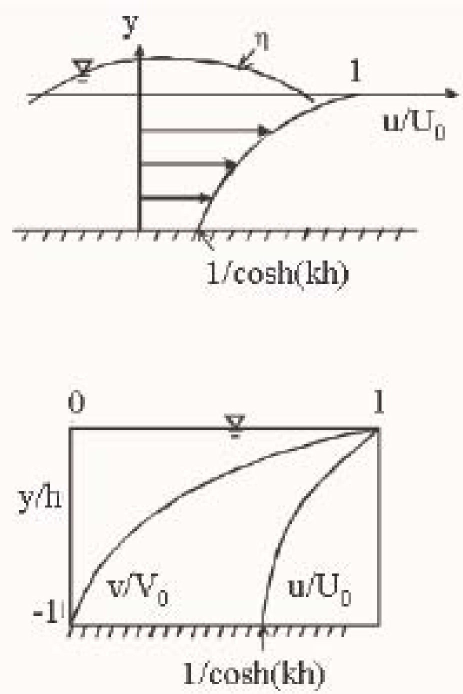
Velocity field $\vec{v}(x,y)$

Shallow water	Intermediate water	Deep water

Shallow water



Intermediate water

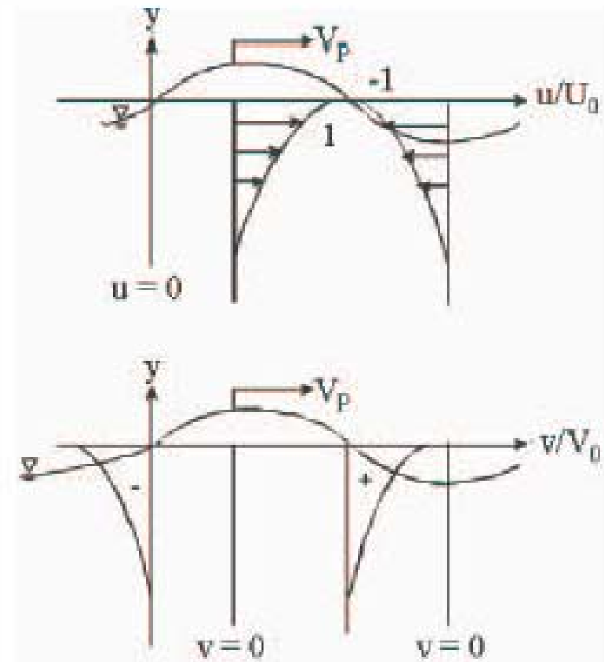


Shallow water / Long waves: kh << 1

$$u = \frac{A\omega}{kh}\cos(kx - \omega t) = \eta\sqrt{\frac{g}{h}}$$

$$v = A\omega\left(1 + \frac{y}{h}\right)\sin(kx - \omega t)$$

Deep water



Rule of thumb $\frac{u}{u_o} = \frac{v}{v_o} \approx 4\%$ at $y = -\frac{\lambda}{2}$

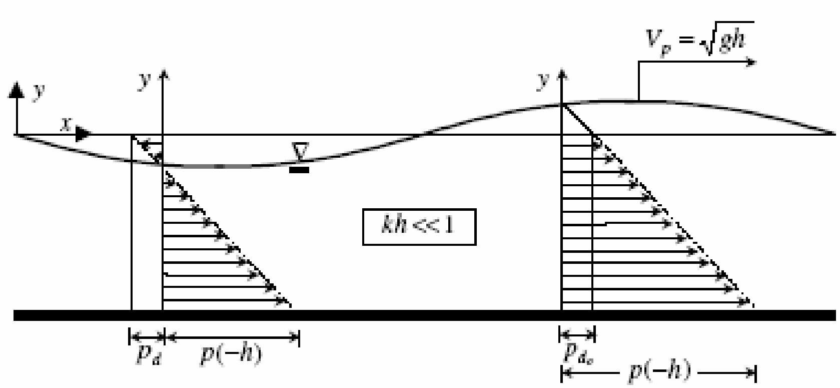
$(\cosh\ kh - 1, \sinh\ kh - kh)$

- **Pressure Field:**

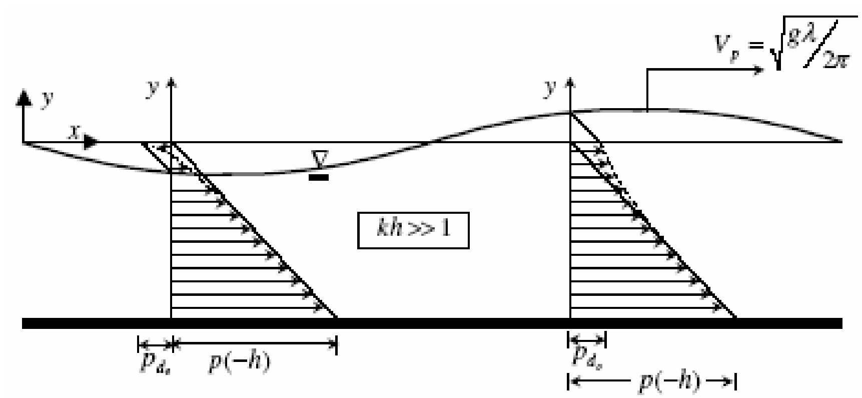
Total pressure $p = p_d - \rho g y$.

Dynamic pressure $p_d = -\rho \frac{\partial \phi}{\partial t}$.

Pressure field		
Shallow water	Intermediate water	Deep water
$p_d = \rho g \eta$	$p_d = \rho g A \frac{\cosh k(y+h)}{\cosh kh} \cos(kx - \omega t)$ $= \rho g \frac{\cosh k(y+h)}{\cosh kh} \eta$	$p_d = \rho g e^{ky} \eta$
$\frac{p_d}{p_{d_o}}$ same picture as $\frac{u}{U_o}$		
$\frac{p_d(-h)}{p_{d_o}} = 1$ (no decay)	$\frac{p_d(-h)}{p_{d_o}} = \frac{1}{\cosh kh}$	$\frac{p_d(-h)}{p_{d_o}} = e^{-ky}$
$p = \underbrace{\rho g(\eta - y)}_{\text{"hydrostatic" approximation}}$		$p = \rho g \left(\eta e^{ky} - y\right)$



Pressure field in shallow water



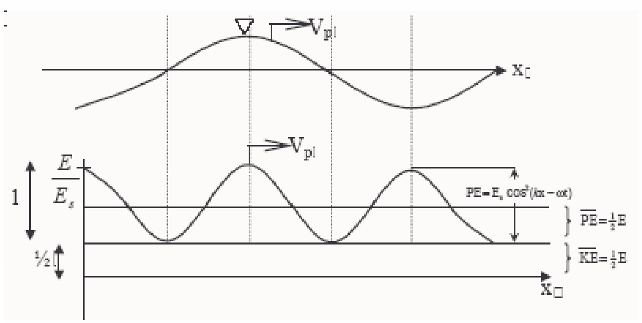
Pressure field in deep water

Wave Energy

For a single plane progressive wave:

Energy per unit surface area of wave	
• Potential energy PE	• Kinetic energy KE
PE without wave $= \int_{-h}^{0} \rho g y dy = -\frac{1}{2}\rho g h^2$	$\text{KE}_{wave} = \int_{-h}^{\eta} dy \frac{1}{2}\rho\left(u^2+v^2\right)$
PE with wave $\int_{-h}^{\eta} \rho g y dy = \frac{1}{2}\rho g\left(\eta^2 - h^2\right)$	Deep water $= \cdots = \underbrace{\frac{1}{4}\rho g A^2}_{\text{KE const in x,t}}$ to leading order
$\text{PE}_{wave} = \frac{1}{2}\rho g \eta^2 = \frac{1}{2}\rho g A^2 \cos^2(kx-\omega t)$	Finite depth $= \cdots$
Average energy over one period or one wavelength	
$\overline{\text{PE}}_{\text{wave}} = \frac{1}{4}\rho g A^2$	$\overline{\text{KE}}_{\text{wave}} = \frac{1}{4}\rho g A^2$ at **any** h

- Total wave energy in **deep** water:
  $\text{E} = \text{PE} + \text{KE} = \frac{1}{2}\rho g A^2\left[\cos^2(kx-\omega t) + \frac{1}{2}\right]$
- Average wave energy E (over 1 period or 1 wavelength) for **any** water depth:
  $\overline{\text{E}} = \frac{1}{2}\rho g A^2[\underset{\underset{PE}{\uparrow}}{\frac{1}{2}} + \underset{\underset{KE}{\uparrow}}{\frac{1}{2}}] = \frac{1}{2}\rho g A^2 = E_s,$
  $E_s \equiv$ Specific Energy: total average wave energy per unit surface area.



Recall: $\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$

- Linear waves: $\overline{\text{PE}} = \overline{\text{KE}} = \frac{1}{2}E_s$ (equipartition).
- Nonlinear waves: $\overline{\text{KE}} > \overline{\text{PE}}$.
- Wave energy propagation speed: group velocity: $V_g = \frac{\mathrm{d}\omega}{\mathrm{d}k}$

Example: Wave Loads on Vertical Wall

A vertical wall is located at *x*=0 in in a water of depth h:

$$\eta(x,t) = A\cos(kx-\omega t) + A\cos(kx+\omega t)$$

$$\phi(x,y,t) = \frac{gA}{\omega}\frac{\cosh k(y+h)}{\cosh kh}\left[\sin(kx-\omega t) - \sin(kx+\omega t)\right]$$

$$p(x,y,t) = -\rho\frac{\partial\phi}{\partial t} - \rho g y = \rho g A\frac{\cosh k(y+h)}{\cosh kh}\left[\cos(kx-\omega t) + \cos(kx+\omega t)\right] - \rho g y$$

$$p(x=0,y,t) = 2\rho g A\cos\omega t\frac{\cosh k(y+h)}{\cosh kh} - \rho g y$$

$$F_x = \int_{-h}^{0} p(x=0,y,t)\mathrm{d}y$$

$$= \frac{2\rho g A\cos\omega t}{\cosh kh}\int_{-h}^{0}\cosh k(y+h)\mathrm{d}y - \int_{-h}^{0}\rho g y\mathrm{d}y$$

$$= \frac{2\rho g A}{k}\tanh kh\cos\omega t + \frac{\rho g h^2}{2}$$

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