2.003/1.053 Dynamics and Controls I
Spring 2007
Problem Set 7

Issued on Monday, April 9^{th}
Due in lecture on Monday, April 16^{th}

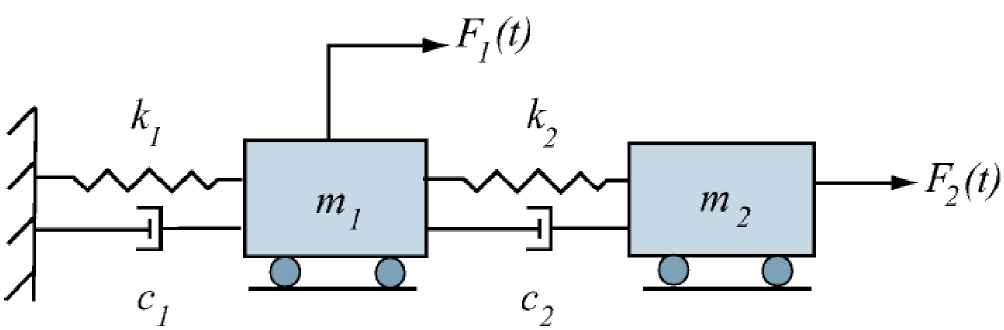
1 Two carts

The two carts shown below roll on massless wheels on a horizontal surface. Derive the equations of motion for this system using Lagrange's equations. Use x_1 and x_2 as generalized coordinates describing the position of masses m_1 and m_2 respectively, measured from the static equilibrium configuration in the absence of forces F_1 and F_2.

Put your equations of motion in the following matrix form:

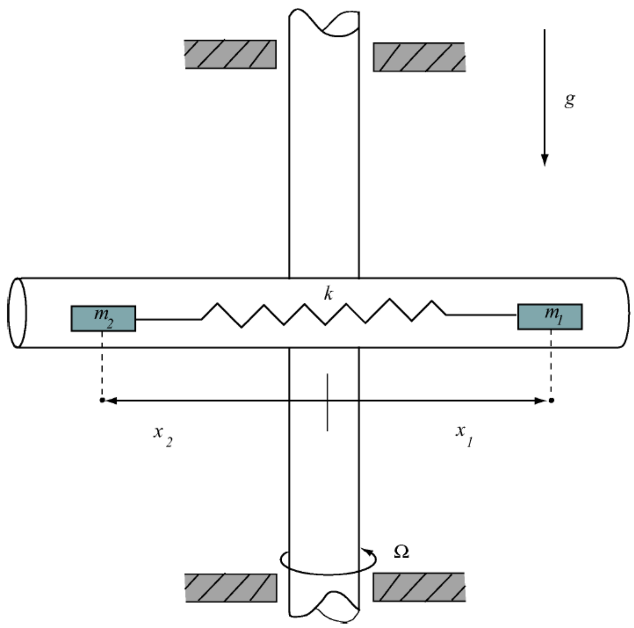
$$\mathrm{M}\begin{Bmatrix}\ddot{x}_1\\ \ddot{x}_2\end{Bmatrix}+\mathrm{C}\begin{Bmatrix}\dot{x}_1\\ \dot{x}_2\end{Bmatrix}+\mathrm{K}\begin{Bmatrix}x_1\\ x_2\end{Bmatrix}=\mathbf{F}$$

where M, C and K are 2×2 matrices and $\mathbf{F}$ is a 2×1 vector.



2 Centrifuge

A horizontal frictionless tube rotates about a vertical axis at a constant angular velocity Ω. Two particles, m_1 and m_2, (connected by a spring), are placed inside the tube as shown. Derive equations of motion for this system. The unstretched length of the spring is l_0.



3 Rigid body pendulum

A pendulum consists of a rod of length L, mass m, and centroidal moment of inertia $\frac{1}{12}mL^2$ with a frictionless pivot at one end. The pendulum is suspended from a flywheel of radius R and mass M which can rotate about the fixed point O, as shown below.

i. Select a complete and independent set of generalized coordinates.

ii. Derive the Lagrangian equations of motion without making any approximations (small angles, etc.).