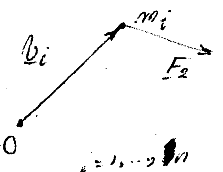
Session 15

$$2\,[x - V(t - t_s)]\,(dx - V\,dt) + 2y\,dy = 0$$

(dx, dy, dt) true infinitesimal displacement

(4) Virtual Work

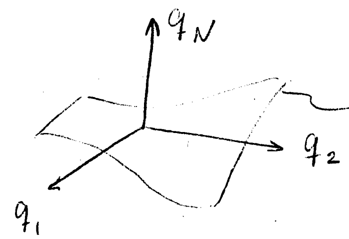


$$\delta W_{F_i} = \underline{F}_i \cdot \delta \underline{r}_i$$

Definition: A Constraint is Called ideal if The associated ideal force does zero virtual work

Consider specifically holonomic Constraints

$$f_j(q, t) = 0 \qquad q = \begin{Bmatrix} q_1 \\ \vdots \\ q_N \end{Bmatrix}$$ Space of q_i's are Called Configuration Space



$f_j(q, t) = 0$ Surface of dimension $N-1$ (instantaneous position of this surface)

NOTE: the Constraint force $\underline{K}_j$ (Corresponding to $f_j(q,t) = 0$ must be orthogonal to the above surface)

$$\Rightarrow \underline{K}_j = \lambda \nabla f_j \qquad (\nabla \equiv \nabla_q) \qquad (1)$$

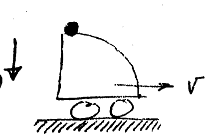
On the other hand, by definition, $\nabla f_j \cdot \delta q = 0$ (2)

$$(1), (2) \longrightarrow \frac{1}{\lambda} \boxed{\underline{K}_j \cdot \delta q = 0}$$

$\hookrightarrow \delta W_{\underline{K}} = 0$

Any holonomic Constraint is ideal

Example:



$dW_N \neq 0$ because $f_j(q, t) = 0$

$$\underbrace{\nabla f_j \cdot dq}_{\frac{1}{\lambda} dW_N \neq 0} + \frac{\partial f}{\partial t} dt = 0$$

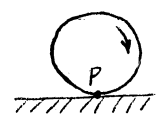
(because $\partial f / \partial t \neq 0$)

work done by the Constraint is non-zero but virtual work is zero (ideal Constraint)

General Result

true infinitesimal of the Constraint is only equal to the virtual work for scleronomic Constraint

(2) Rolling in 2D



$v_P = 0$ integrate

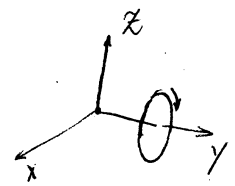
$$x_P - R\varphi_P - (x_P^0 - R\varphi_P^0) = 0$$

$$f(q, t) = 0$$

ideal Constraint

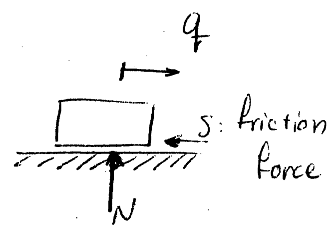
(3) 3D rolling

truly non holonomic



General argument
given for holonomic Systems does not apply.
But One Can Still Show that this Constraint is ideal.

(4) Sliding over rough Surface



~~[illegible]~~ Scleronomic System

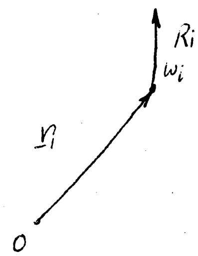
$$dW_S = \delta W_S \neq 0$$

$$\Downarrow$$

Constraint is not ideal

Trick: Consider S as an active force that is not part of the Constraint
$\Rightarrow$ Sliding becomes an ideal Constraint, and
$$S = S(\mu, N; q, \dot{q}, \ldots)$$
is just another active force

(5) Generalized Forces $\approx$ forces ~~expressed~~ acting in the direction of generalized ~~[illegible]~~ Coordinate



$\underline{R}_i = \underline{F}_i + \underline{K}_i$

Virtual work on ith particle

$\therefore \delta W^i = \underline{P}_i \cdot \delta r_i = (F_i + K_i)\,\delta r_i$

Total virtual work does on System $\delta W = \sum_{i=1}^{n} \underline{F}_i \cdot \delta \underline{r}_i + \underbrace{\sum_{i=1}^{n} \underline{K}_i\, \delta \underline{r}_i}_{=0}$

$= \sum_i F_i \cdot \sum_{j=1}^{N} \frac{\partial r_i}{\partial q_j} \delta q_j$ if all Constraints are ideal

$= \sum_{j=1}^{N} \Big(\underbrace{\sum_{i=1}^{n} F_i \cdot \frac{\partial r_i}{\partial q_j}}_{Q_j} \Big) \delta q_j$

Q_j the generalized force associated with q_j

Special Case: generalized forces in the Case when F_i's are all potential forces.

$\delta W = \sum_i \underline{F}_i\, \delta \underline{r}_i = \sum_i \left(-\frac{\partial V}{\partial \underline{r}_i}\right) \cdot \delta r_j$

$= -\delta V = \sum_{j=1}^{N} -\frac{\partial V}{\partial q_j} \delta q_j$

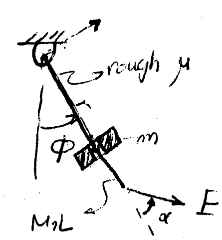
$\Rightarrow$ for potential forces by definition $Q_j = -\frac{\partial V}{\partial q_j}$

Example: Sliding Collar on a rough beam (pendulum)
Subjected to a follower force

#DOF: $3+3-2-2=2$ (holonomic)

$\Rightarrow$ 2 generalized Coordinates

$g \downarrow$



Question: Generalized forces Q_r & Q_φ

$\delta W = \delta W^{potential} + \delta W^{nonpotential}$

$\delta W^{potential} = -\delta V = -\delta\left(-Mg\frac{L}{2}\cos\varphi - mgr\cos\varphi\right)$

$= Mg\frac{L}{2}(-\sin\varphi)\,\delta\varphi + mgr(-\sin\varphi)\,\delta\varphi + mg\cos\varphi\,\delta r$

$Q_\varphi^{potential} = -\left(Mg\frac{L}{2} + mgr\right)\sin\varphi$

$Q_r^{potential} = mg\cos\varphi$