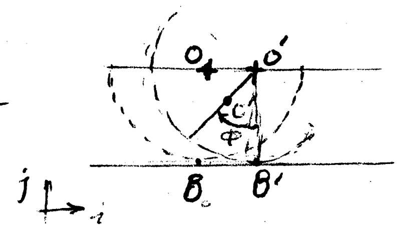
φ: Generalized Coordinates
(Complete & independent set of coordinates)



Start with F.B.D.

rolling means there is no relative motion between two surfaces involved

Use angular momentum principle Start at B'

$$\dot{H}_B + \underline{V}_B \times \underline{P} = M_B$$

$$\underline{H}_B = \underline{r}_{BC} \times \underline{P} = \underline{r}_{BC} \times m\underline{v}_C$$

$$\underline{r}_C = (R\varphi - a\sin\varphi)\,\underline{i} + (R - a\cos\varphi)\,\underline{j}$$

$$\underline{V}_C = \dot{\underline{r}}_C = (R - a\cos\varphi)\dot{\varphi}\,\hat{i} + a\sin\varphi\,\dot{\varphi}\,\underline{j}$$

$$\underline{r}_{BC} = -a\sin\varphi\,\underline{i} + (R - a\cos\varphi)\,\underline{j}$$

$$\Rightarrow \underline{H}_B = m\left[-a^2 - R^2 + 2aR\cos\varphi\right]\dot{\varphi}\,\underline{k}$$

$$\dot{\underline{H}}_B = m\left[(2aR\cos\varphi - a^2 - R^2)\ddot{\varphi} - 2aR\sin\varphi\,\dot{\varphi}^2\right]\underline{k}$$

$$\underline{V}_B = \frac{d}{dt}(R\varphi)\,\hat{i} = R\dot{\varphi}\,\hat{i}$$

$$\underline{P} = m\underline{v}_C \qquad \underline{V}_B \times \underline{P} = a R\sin\varphi\, m\dot{\varphi}^2\,\underline{k} = maR\sin\varphi\,\dot{\varphi}^2\,\underline{k}$$

$$\overline{M}_B = mga\sin\varphi\,\underline{k}$$

we get

$$(R^2 + a^2 - 2aR\cos\varphi)\ddot{\varphi} + aR\sin\varphi\,\dot{\varphi}^2 + ag\sin\varphi = 0$$

eq of motion

- 2nd order ODE, nonlinear
- 2nd order ODE
- initial conditions $\varphi(t_0) = \varphi_0$, $\dot{\varphi}(t_0) = \dot{\varphi}_0$

For numerical solutions, let $x_1 = \varphi$, $x_2 = \dot{\varphi}$. Eq of motion becomes

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{aR\sin x_1\, x_2^2 + ag\sin x_1}{R^2 + a^2 - 2aR\cos x_1}$$

$$x_1(0) = \varphi_0$$

$$x_2(0) = \dot{\varphi}_0$$

1st order system of ODE's

To obtain the reaction forces use linear momentum $\dot{\underline{P}} = \underline{F}$

(x) equation $m\ddot{x}_c = S$ $\qquad x_c = R\varphi - a\sin\varphi$

$$\Rightarrow S = m\left[a\sin\varphi\,\dot{\varphi}^2 + (R - a\cos\varphi)\ddot{\varphi}\right]$$

use eq of motion $\Rightarrow S = m\left[a\sin\varphi\,\dot{\varphi}^2 - \dfrac{(R-a\cos\varphi)(aR\sin\varphi\,\dot{\varphi}^2 + ag\sin\varphi)}{R^2+a^2-2aR\cos\varphi}\right]$

(y) equation $m\ddot{y}_c = N - mg \Rightarrow N = m(\ddot{y}_c + g)$

$y_c = R - a\cos\varphi$

S and N are non-potential forces because their expersion is also dependent on $\dot{\varphi}$. if it was just dependent on φ it was.

S, N act on the chair at point $\tilde{B}$ a point at the disc instantaneously at B

$\underline{V}_{\tilde{B}} = \underline{V}^{rel}_{B/O'} + \underline{V}_{O'}$ (to be justified)

$= -R\dot{\varphi}\hat{i} + R\dot{\varphi}\hat{i}$

$= 0$

$$\Rightarrow \int_{t_1}^{t_2} (\underline{N} + \underline{S})\cdot\underline{V}_{\tilde{B}(t)}\,dt \Rightarrow W_{12} = 0 \Rightarrow \text{System is Conservative}$$

$$\Rightarrow T + V = \frac{1}{2}m(\dot{x}_c^2 + \dot{y}_c^2) + mgy_c = \text{Const}$$

$\frac{d}{dt}(T+V) = 0$ Substituting of x_c and y_c gives the same eq. of motion

this procedure works if the system has one degree of freedom (1 generalized Coordin[ate]) the system is Conservative

Frequency of Small Oscillations

linearize eq. of motion about $\varphi = 0$, $\dot{\varphi} = 0$ i.e. taylor expand in two variables φ and $\dot{\varphi}$ and keep linear terms Only

$$[R^2 + a^2 - 2aR(1 + \cdots)] + aR(\cancel{0 + \cdots})\dot{\varphi}^2 + ag(\varphi + \cdots) = 0$$

$$\underbrace{(R-a)^2}_{\text{mass Coef.}}\ddot{\varphi} + \underbrace{ag}_{\text{Spring Coef.}}\varphi = 0 \Rightarrow \boxed{\omega = \sqrt{\frac{ag}{(R-a)^2}}}$$