

Massachusetts Institute of Technology

Biological Engineering Division
Department of Mechanical Engineering
Department of Electrical Engineering and Computer Science
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Fields, Forces and Flows in Biological Systems, Spring 2007

Problem Set #4.

Issued: March 9th (Friday)
Due date: March 16th (Friday)

Problem 1: Electromagnetic Wave

Starting from the differential form of Maxwell's equation in free space (no charge and current), show that \vec{E} and \vec{B} fields in free space satisfy the following wave equation.

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) E_i = \frac{1}{v^2} \frac{\partial^2 E_i}{\partial t^2} \quad (i = x, y, \text{ or } z)$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) B_i = \frac{1}{v^2} \frac{\partial^2 B_i}{\partial t^2} \quad (i = x, y, \text{ or } z)$$

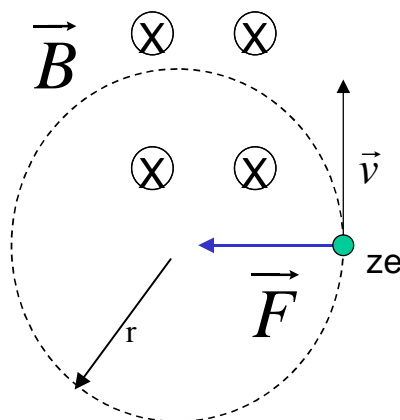
Use the following vector identity.

$$\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - (\nabla \cdot \nabla) \vec{A}$$

What is the propagation speed v of this "electromagnetic wave"? Calculate the numerical value.

Problem 2: FT-ICR MS

Fourier Transform Ion-Cyclotron Resonance Mass Spectrometer (FT-ICR-MS) is the current state-of-the-art mass spectrometer for analyzing biomolecules. It has a very high mass resolving power ($\Delta M/M \sim 10^{-5}$), which is high enough to detect the mass shift by one mass unit (one proton). Therefore, it is a viable tool for analyzing small changes (post-translation modification of



proteins, for example) in biomolecules. In FT-ICR-MS, biomolecules are electro-sprayed into a vacuum chamber, and accelerated to a velocity v by the accelerating potential V_{ac} . Then, biomolecules are introduced into the area where a magnetic field perpendicular to the direction of motion exists, essentially trapping the charged biomolecules into a circular orbit (radius r), as shown in the figure.

(a) Get the cyclotron angular velocity $\omega_c = v/r$ as a function of B , z (charge number), e , and m (mass of the molecule).

(b) One of the issue of this type of mass spectrometer is the strength of magnetic field to trap heavy biomolecules within a reasonable distance, say $r \sim 10\text{mm}$. (Large r requires larger vacuum chambers and larger vacuum pumps, which is very costly.) Using the following typical values,

$$V_{ac} \sim 1000\text{V}$$

$$m \sim 10\text{kD (small proteins)}$$

$$z \sim 1$$

calculate the required magnetic field to trap the biomolecule within the $\sim 20\text{mm}$ size vacuum chamber ($r \sim 10\text{mm}$).

Problem 3: Quasistatic Approximation

For each of the following experimental situations, determine if the quasistatic (QS) approximation is appropriate or not. Explain your reasoning. Use the typical, approximate size / time scales when necessary.

(a) Wireless telephone in our home uses 2.4GHz frequency for its communication between the base station and the phone. When we use the phone, electric fields could affect brain tissue, perhaps inducing currents.

(b) In recent work by Prof. Hamad-Schifferli (Biological Engineering, MIT), metallic (gold) nanoparticles (with diameter of $\sim 3\text{nm}$) are excited by the 1GHz oscillating magnetic field. (The end result of this is the heating of the particle, which could (locally) denature DNA molecules that are attached to the particle. See the K. Hamad-Schifferli, J.J. Schwartz, A.T. Santos, S. Zhang, J.M. Jacobson, "Remote electronic control of DNA hybridization through inductive coupling to an attached metal nanocrystal antenna," *Nature*, 2002, **415**, 152-155.)

Problem 4: Isoelectric Focusing

In isoelectric focusing (IEF) a pH-gradient is established along the microchannel or a capillary column by special buffer called carrier ampholytes, as shown in the figure below. When the protein is in the environment where its pH is above (below) the isoelectric point of the protein (pI), they have net negative (positive) charges. When an external field is applied, proteins will get focused around the point $x=x_{ip}$, where the mobility (and net charge) of the molecule becomes zero. Approximately, one can say that the electrical mobility near the pI of the molecule is linear, as in

$$u = -p(x - x_{ip}) \quad (p: \text{positive constant})$$

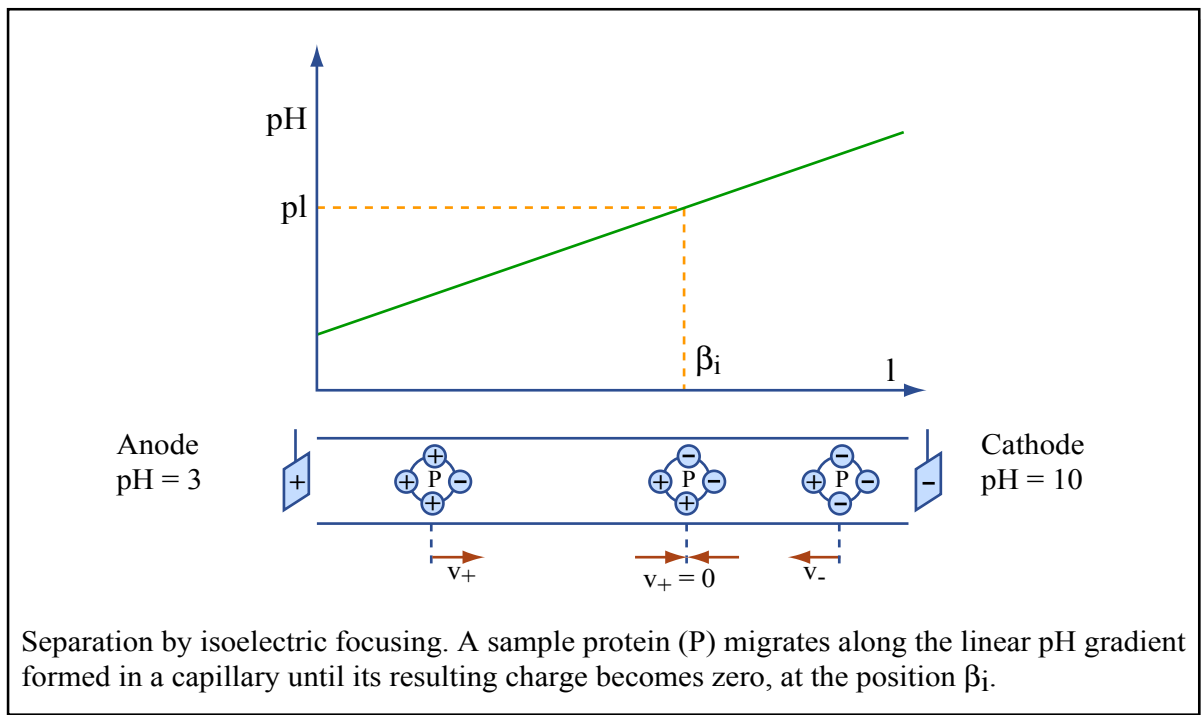
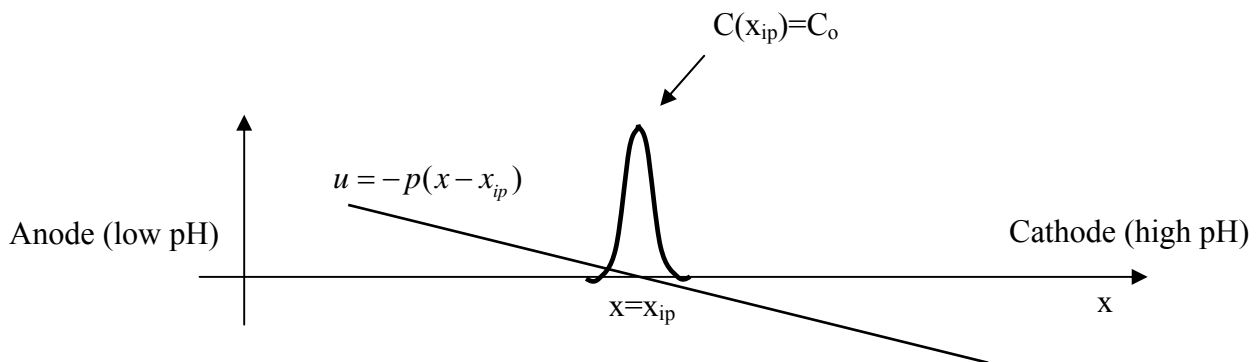


Figure by MIT OCW.



After a certain time, all the proteins will be focused around the $x=x_{ip}$, reaching a steady state peak concentration $C(x_{ip})=C_0$. However, the resulting peak will have a finite peak width, due to the diffusional transport. The diffusion constant of the protein is given as D , and the electric field in the microchannel/capillary is uniform ($\vec{E} = E_o \hat{x}$). One can ignore convection in this case.

- At steady state, derive the expression for $C(x)$, the concentration of a protein near the isoelectric point.
- Estimate the approximate width of the focused protein peak as a function of other parameters.