Key Concepts for this section

1: Lorentz force law, Field, Maxwell's equation

2: Ion Transport, Nernst-Planck equation

3: (Quasi)electrostatics, potential function,

4: Laplace's equation, Uniqueness

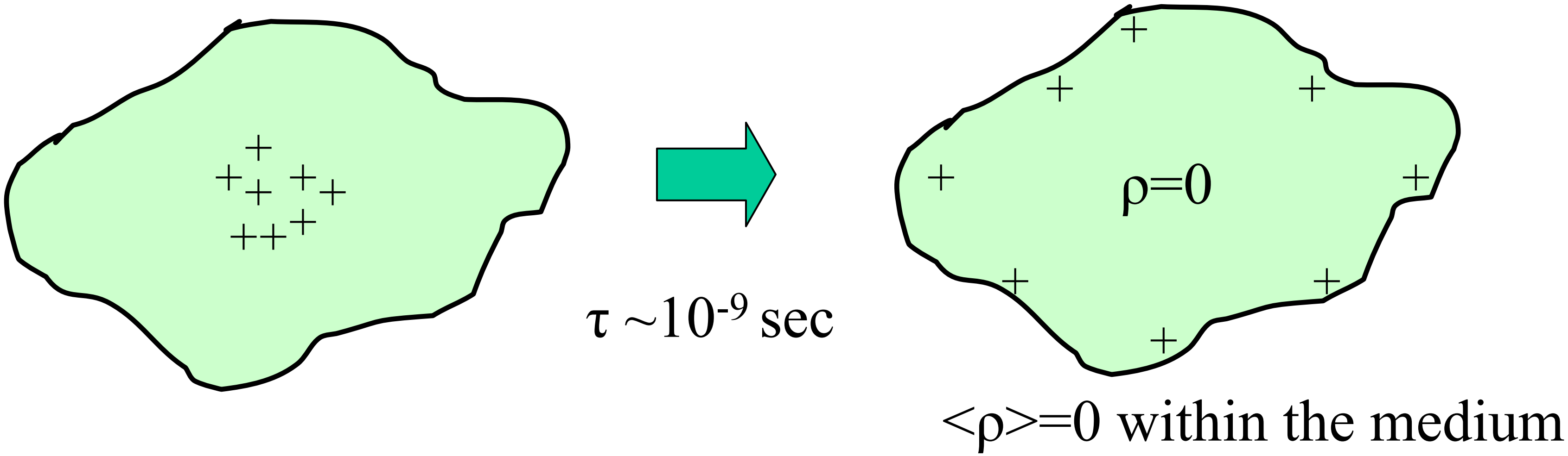
5: Debye layer, electroneutrality

Goals of Part II:

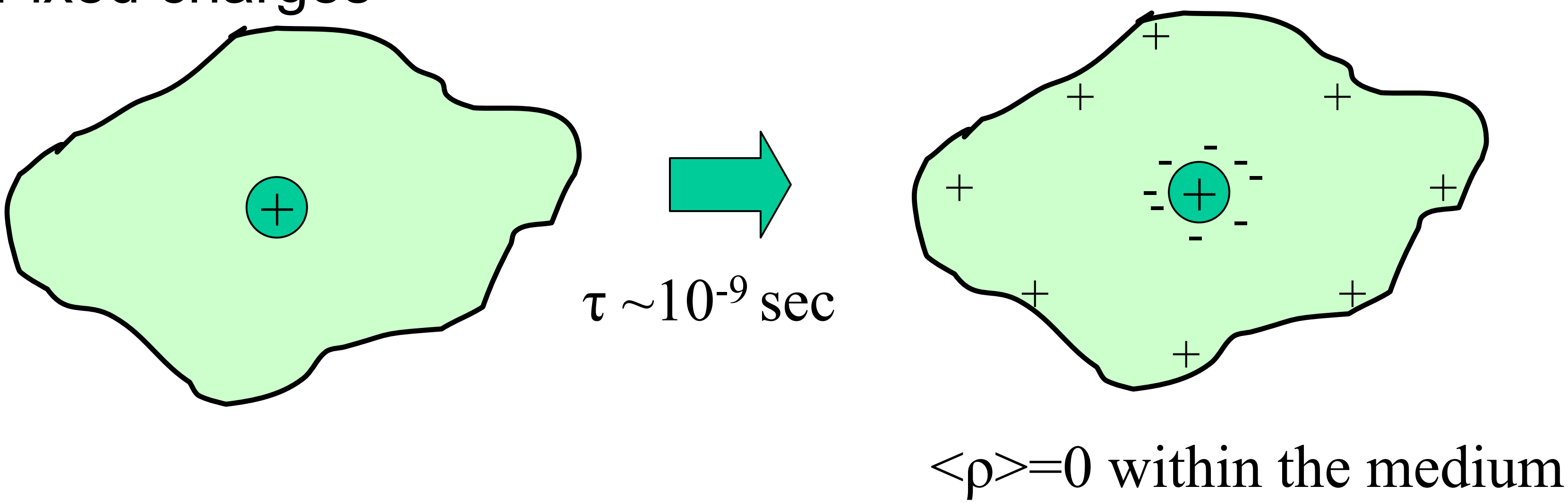
(1) Understand when and why electromagnetic (E and B) interaction is relevant (or not relevant) in biological systems.

(2) Be able to analyze quasistatic electric fields in 2D and 3D.

Charge Relaxation in electrolyte

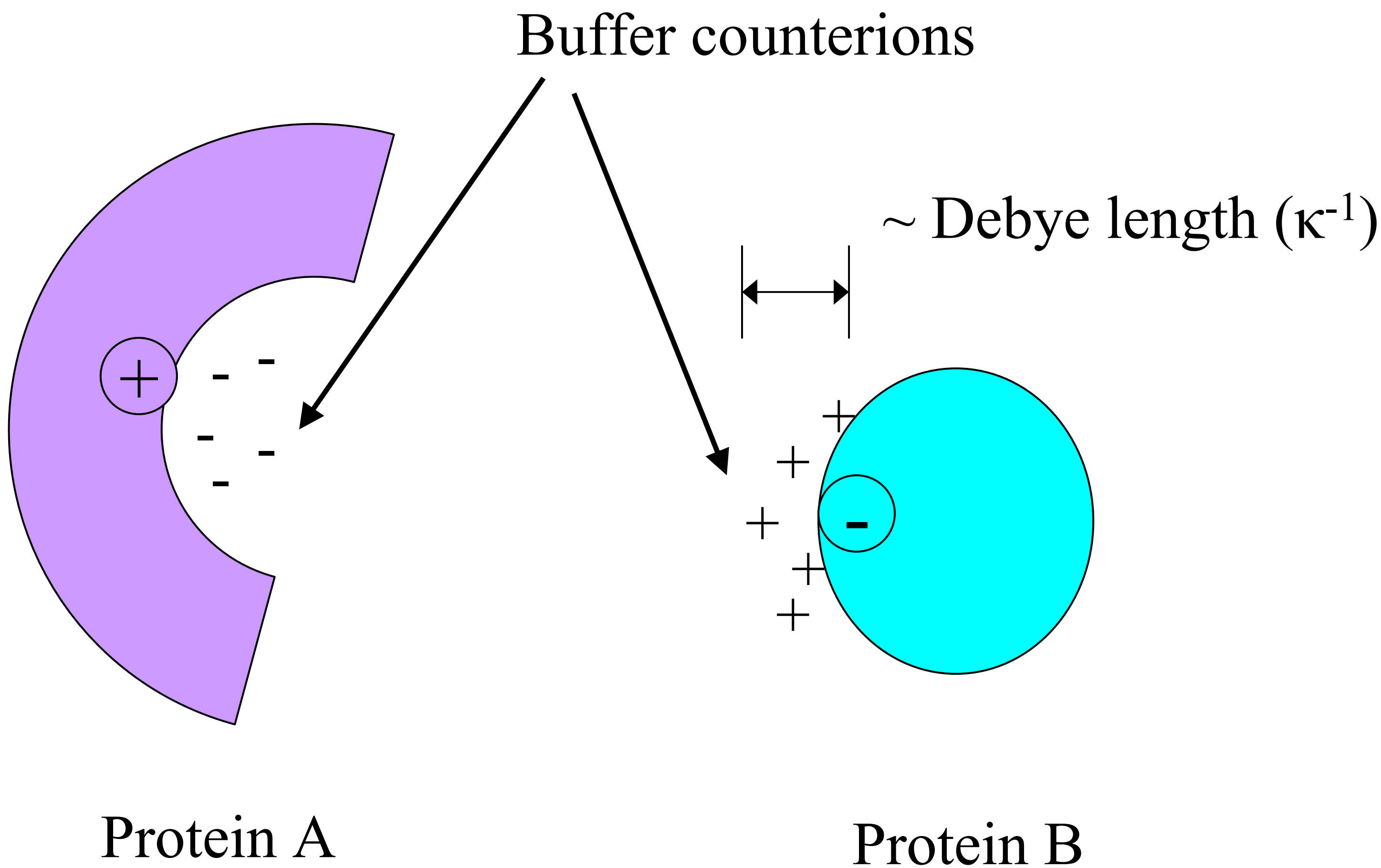


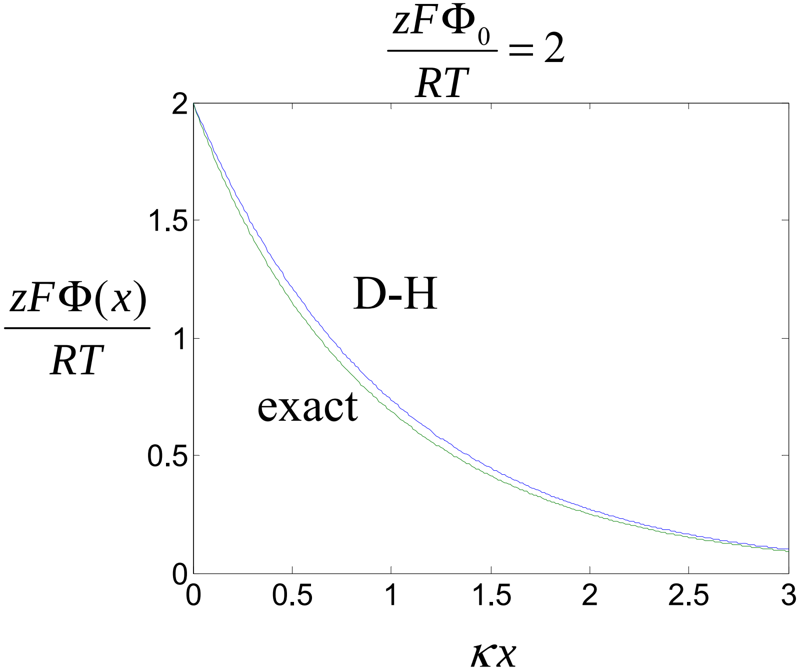
Fixed charges

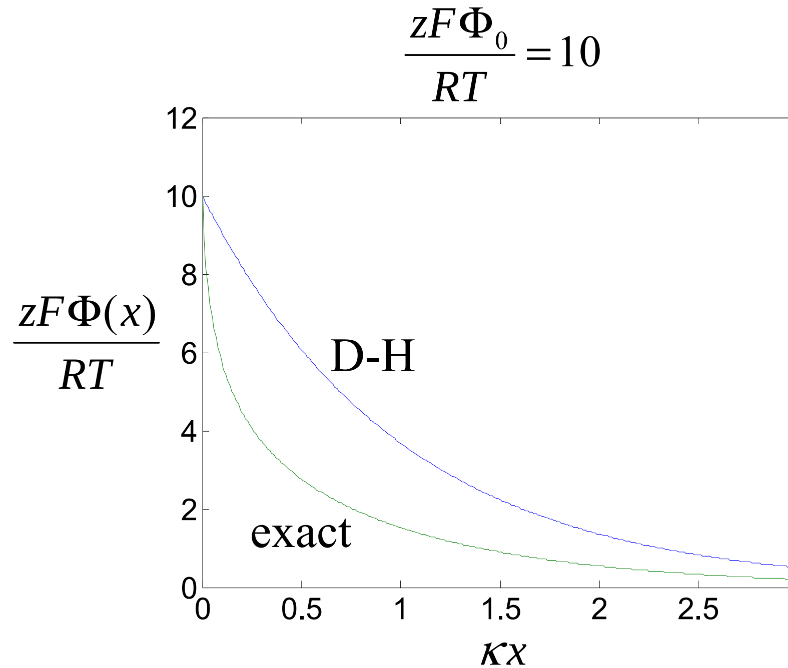


- (Quasi) Electroneutrality Approximation
  - It is an approximation.
    - Valid only after the relaxation time constant τ, and outside of the Debye length (κ^{-1}, to be discussed)
  - Not valid when there is a discontinuity
    - Valid only within a single medium (σ=constant)
    - Boundary of the medium could carry (surface) free charge
  - No inter-charge interaction in liquid media

Electroneutrality





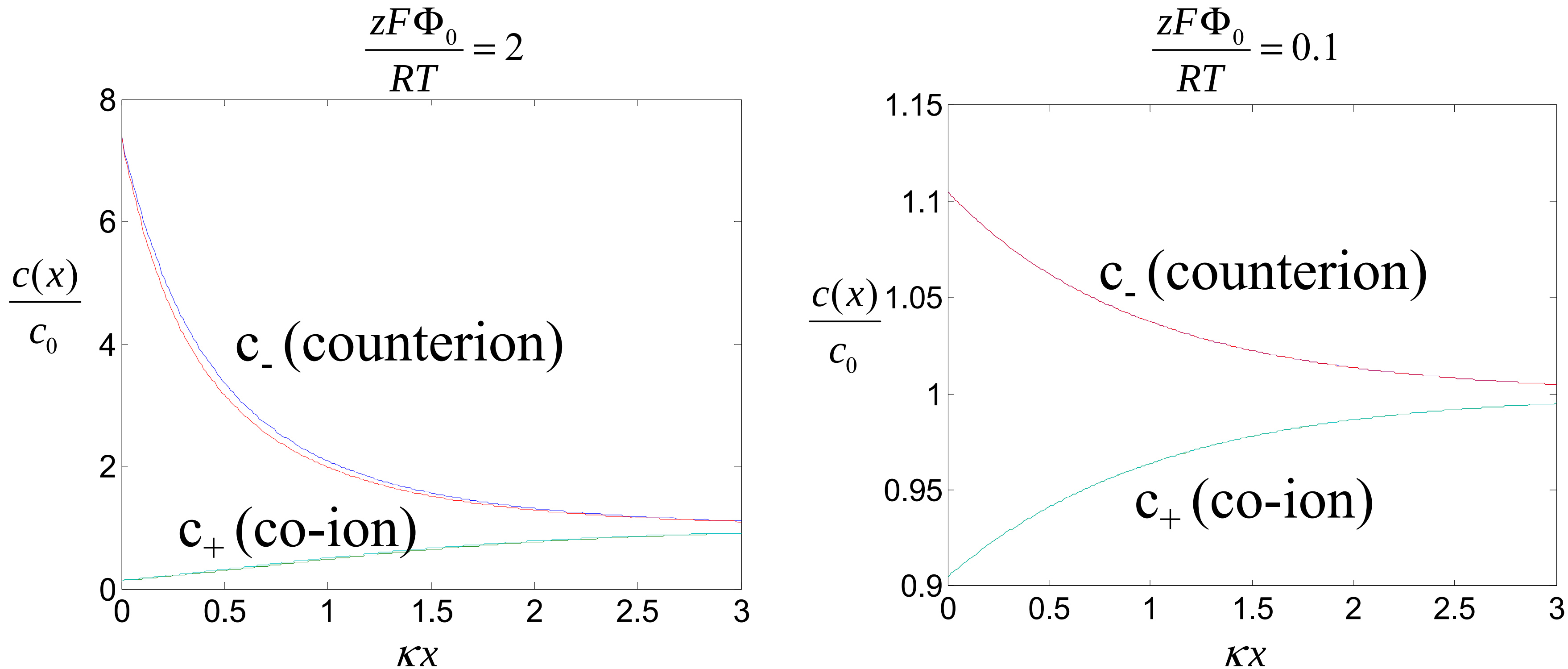


Exact solution

$$\Phi(x) = \frac{2RT}{zF} \ln \left[\frac{1 + e^{-\kappa x} \tanh\left(\frac{zF\Phi_0}{4RT}\right)}{1 - e^{-\kappa x} \tanh\left(\frac{zF\Phi_0}{4RT}\right)} \right], \quad \kappa = \left(\frac{2z^2F^2c_0}{\varepsilon RT} \right)^{1/2}$$

Debye-Huckel approximation

$$\Phi(x) = \Phi_0 e^{-\kappa x} \qquad \text{When} \qquad zF\Phi_0 << RT$$



When $zF\Phi_0 \ll RT \quad (ze\Phi_0 \ll kT)$

thermal energy >> electrical potential energy
(diffusion dominates.)

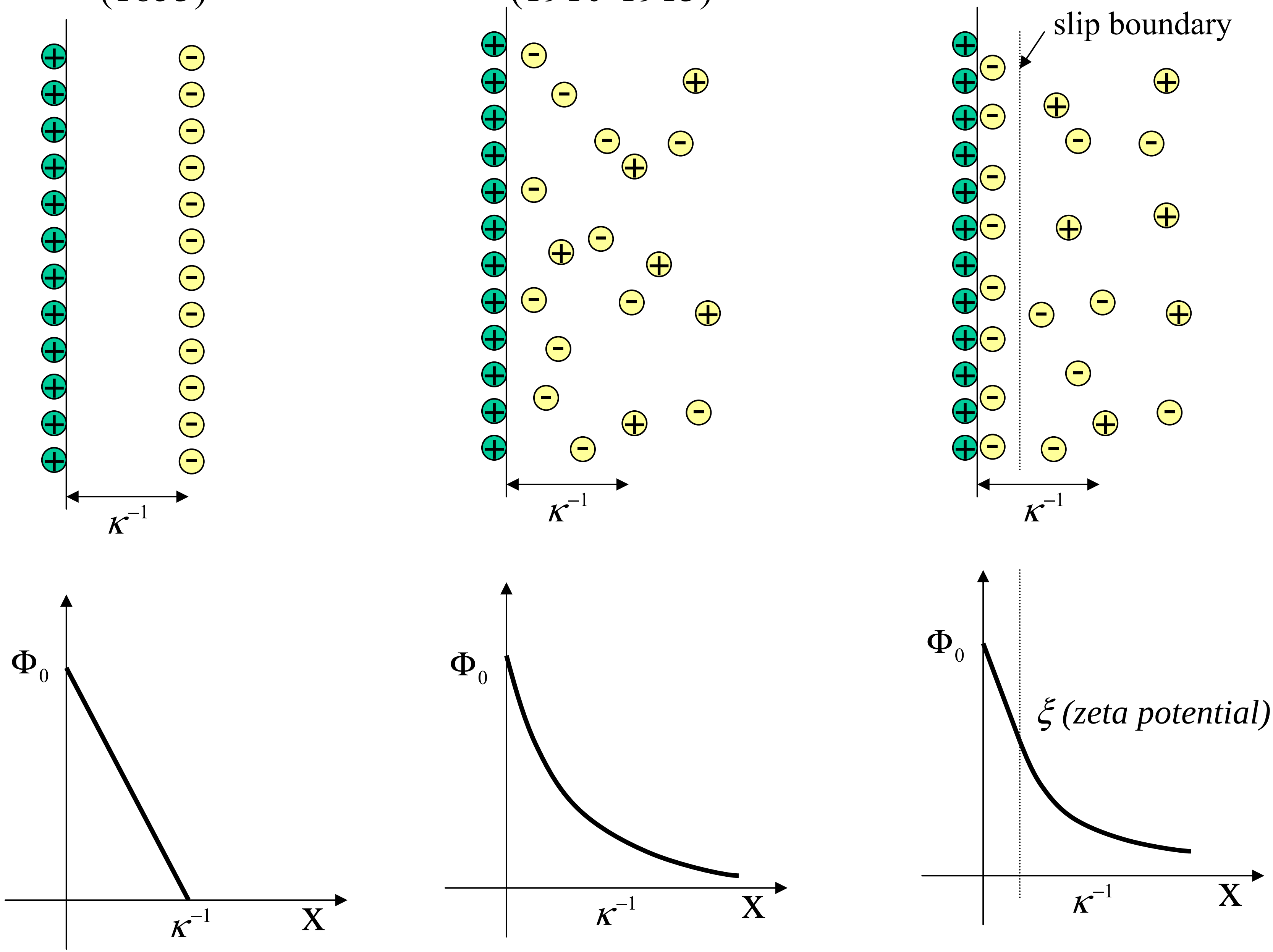
When $zF\Phi_0 \gg RT \quad (ze\Phi_0 \gg kT)$

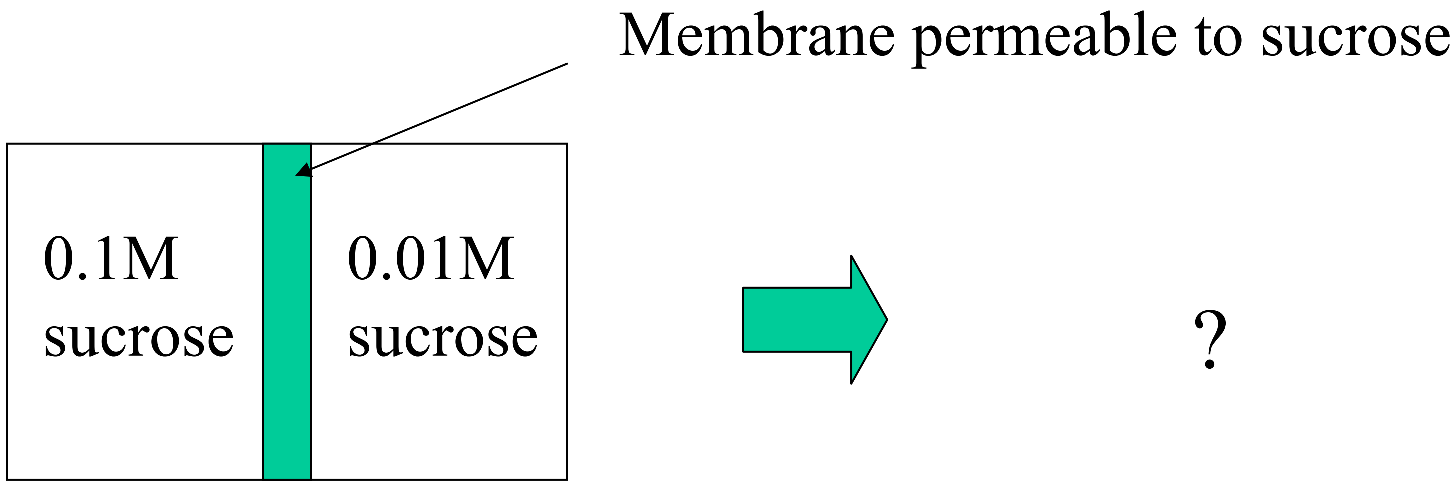
thermal energy << electrical potential energy
(drift dominates. significant charge accumulation)

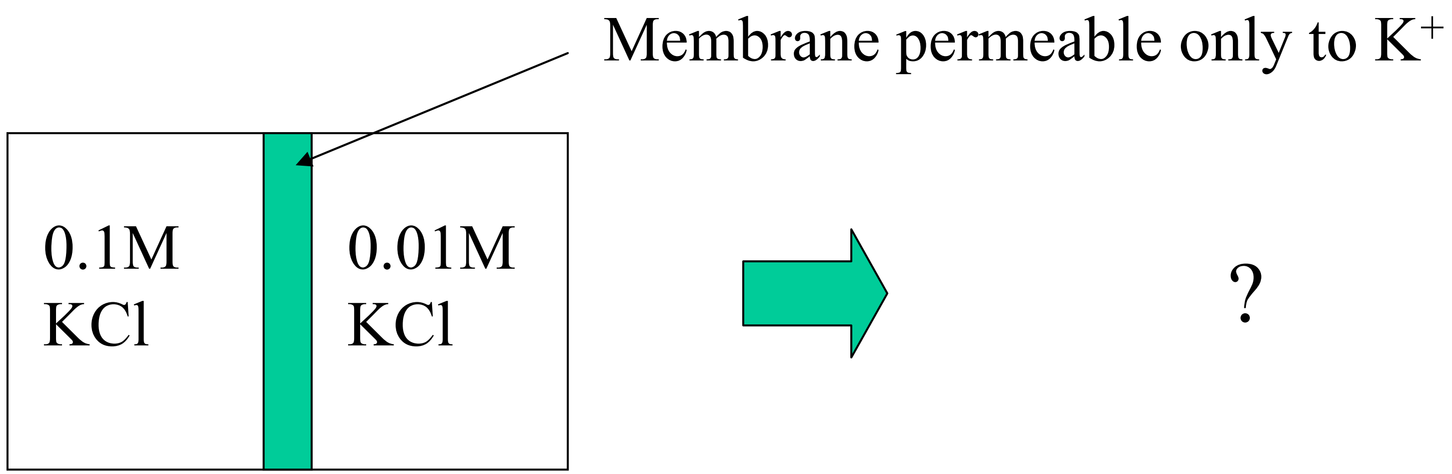
Helmholtz model (1853)

Guoy-Chapman model (1910-1913)

Stern model (1924)

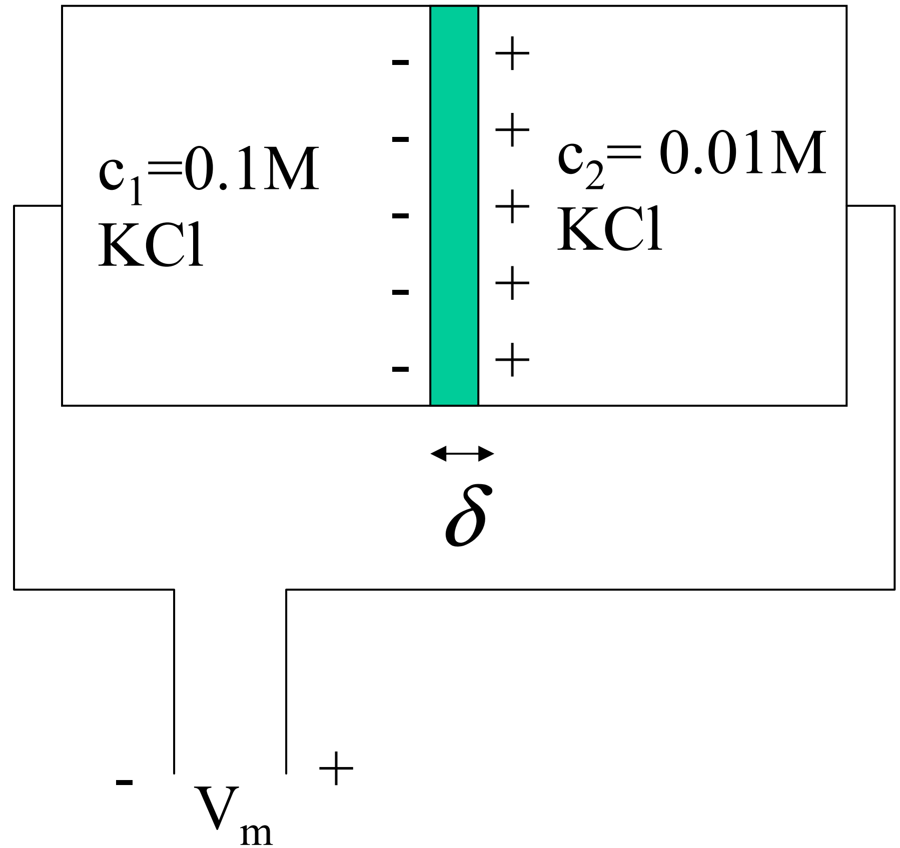



Membrane permeable to sucrose
0.1M sucrose
0.01M sucrose
?


Membrane permeable only to K+
0.1M KCl
0.01M KCl
?

Nernst Equilibrium Potential

Membrane permeable only to K^+



c: K+ concentration

$$-D\frac{dc}{dx}+E\cdot u\cdot c=0 \quad E=-\frac{d\Phi}{dx}$$

$$-D\frac{dc}{c}=u\cdot\frac{d\Phi}{dx}dx$$

$$-D\int_{x=0}^{x=\delta}\frac{dc}{c}=u\cdot\int_{x=0}^{x=\delta}\frac{d\Phi}{dx}dx$$

$$-D\ln\left(\frac{c_2}{c_1}\right)=u\left[\Phi(x=\delta)-\Phi(x=0)\right]$$

$$\Delta\Phi_{12}=\Phi_1-\Phi_2=\frac{D}{u}\ln\left(\frac{c_2}{c_1}\right)=\frac{RT}{zF}\ln\left(\frac{c_2}{c_1}\right)$$ Nernst Equilibrium potential

Diffusion of charged particles -> generate electric field
-> stops diffusion of ions