Massachusetts Institute of Technology
Department of Mechanical Engineering

2.160 Identification, Estimation, and Learning
Spring 2006

Problem Set No. 2
Out: February 22, 2006 Due: March 1, 2006

Problem 1
A stationary random process *X(t)* has a mean value of *m* and an autocorrelation function of the form:

$$R_X(\tau) = \sigma^2 e^{-\beta|\tau|}$$

Another random process *Y(t)* is related to *X(t)* by the deterministic equation:

$$Y(t) = a\,X(t) + b$$

where *a* and *b* are known constants.

(a). What is the autocorrelation function for *Y(t)*?
(b). What is the cross-correlation function $R_{XY}(\tau)$?

Problem 2
Two random processes are defined by

$$X(t) = A\sin(\omega t + \theta)$$
$$Y(t) = B\sin(\omega t + \theta)$$

where θ is a random variable with uniform distribution between 0 and 2π, and ω is a known constant. The *A* and *B* coefficients are both normal random variables $N(0, \sigma^2)$, and are correlated to each other with a correlation coefficient ρ. Show that the cross-correlation function $R_{XY}(\tau)$ is given by:

$$R_{XY}(\tau) = \frac{1}{2}\rho\sigma^2\cos(\omega t)\,.$$

Assume *A* and *B* are independent of θ.

Problem 3
Wearable medical sensors monitor a patient's health conditions anytime, anywhere, and continuously. These sensors are expected to revolutionize health care services, including diagnosis and treatment in the home for diabetes, hypertension, and pulmonary disorders. However, the wearable sensors must be robust against disturbances, in particular, the motion of the wearer, since the patients move around for daily activities

rather than staying in a hospital. Wearable sensors often cause false signals due to motion artifact.

Adaptive Noise Cancellation is a powerful tool for coping with the motion artifact problem. Figure 1 shows a motion-tolerant wearable sensor using adaptive noise cancellation. The sensor is a ring photoplethysmograph (PPG) sensor measuring pulse and oxygen level at the base of the finger. The PPG sensor is equipped with a MEMS accelerometer to measure the acceleration of the finger. Although the PPG sensor signal is corrupted with noise due to the finger motion, it can be recovered by using the finger acceleration signal as a noise reference and canceling out the interference caused by the motion.

Figure 2 shows the block diagram of the motion-tolerant PPG ringsensor with a MEMS accelerometer. The true PPG signal, denoted y_o, is mixed with a distorted signal w, when detected by the PPG sensor. An adaptive filter estimates the dynamics of the distortion process, and produces the estimate of the distorted signal $\hat{w}$ in response to the measured acceleration a. The estimated distortion $\hat{w}$ is then subtracted from the PPG sensor output y.

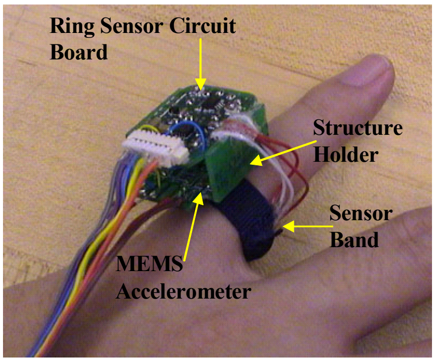

Figure 1 MEMS accelerometer collocated with PPG ring sensor
Courtesy of Prof. Asada. Used with permission.

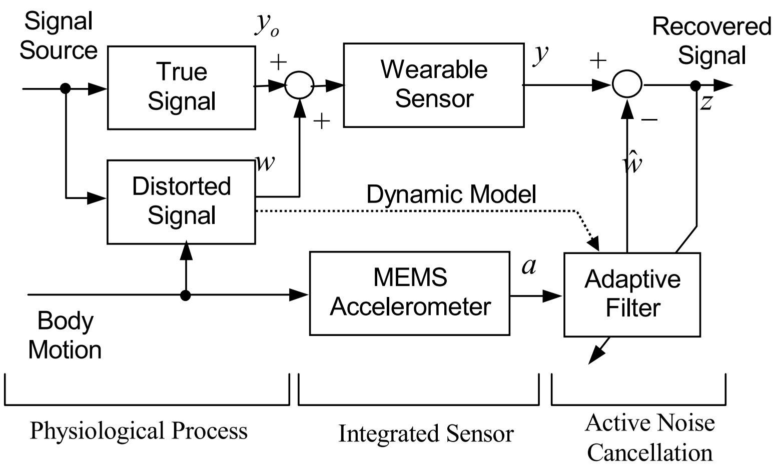

Figure 2 Block diagram of active noise cancellation of PPG signal using MEMS accelerometer

The adaptive filter comprises a dynamic model predicting how the distorted signal component is generated in response to the body acceleration. We assume that this interference dynamics relating the distorted signal $w(t)$ to the finger acceleration $a(t)$ is given by the following FIR model:

$$w(t) = b_1 a(t-1) + b_2 a(t-2) + \cdots + b_m a(t-m)$$

The model parameters, i.e. the FIR parameter vector : $\theta = [b_1 \quad \ldots \quad b_m]^T \in R^m$, are estimated such that

$$\hat{\theta} = \arg\min_{\theta} E[\{w(t) - \varphi^T(t)\cdot\theta\}^2] = \arg\min_{\theta} E[z(t:\theta)^2]$$

as discussed in class. An important assumption in the above formulation is that the interference *w(t)* is strongly correlated to noise, i.e. the finger acceleration *a(t)*, and that **the true signal $y_o(t)$ is uncorrelated with the acceleration *a(t)*.**

This parameter estimation can be performed with various estimation algorithms, including the Recursive Least Squares (RLS). This adaptive filtering method does not assume prior knowledge about the values of FIR parameters, but it continuously adjusts the model parameters to minimize the error between z and y_o through minimizing the power in z.

a). Using MATLAB, implement the above Adaptive Noise Cancellation method based on the recursive least-squares algorithm with an exponential forgetting factor α:

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{P_{t-1}\varphi(t)}{\alpha + \varphi^T(t)P_{t-1}\varphi(t)}\{y(t) - \varphi^T(t)\hat{\theta}(t-1)\}$$

$$P_t = \frac{1}{\alpha}[P_{t-1} - \frac{P_{t-1}\varphi(t)\varphi^T(t)P_{t-1}}{\alpha + \varphi^T(t)P_{t-1}\varphi(t)}]$$

where $0 < \alpha < 1$.

b). Test out your MATLAB code using data provided in assignments section. Both true and corrupted PPG signals as well as the acceleration signal are stored in the data files. Download these files and run your program. Compare the result with the true PPG signal. Vary the forgetting factor and discuss the results.

The following is for your extra credit. It is not mandatory, but is recommended if you are particularly interested in this technique.

c). Create your own data for the true PPG and acceleration signals. Also create your own true model and its output in response to the acceleration. In generating the data, attempt to correlate the acceleration with the true PPG signal to a certain degree. (A jogger often synchronizes his/her stride with the heart beat. Then the PPG signal measuring the pulse may be correlated with the acceleration, i.e. the stride motion.) Thisviolates the fundamental assumption of the above method. Discuss how the noise cancellation performance will degrade as the noise reference, *a(t)*, is more correlated with the true signal y_o.