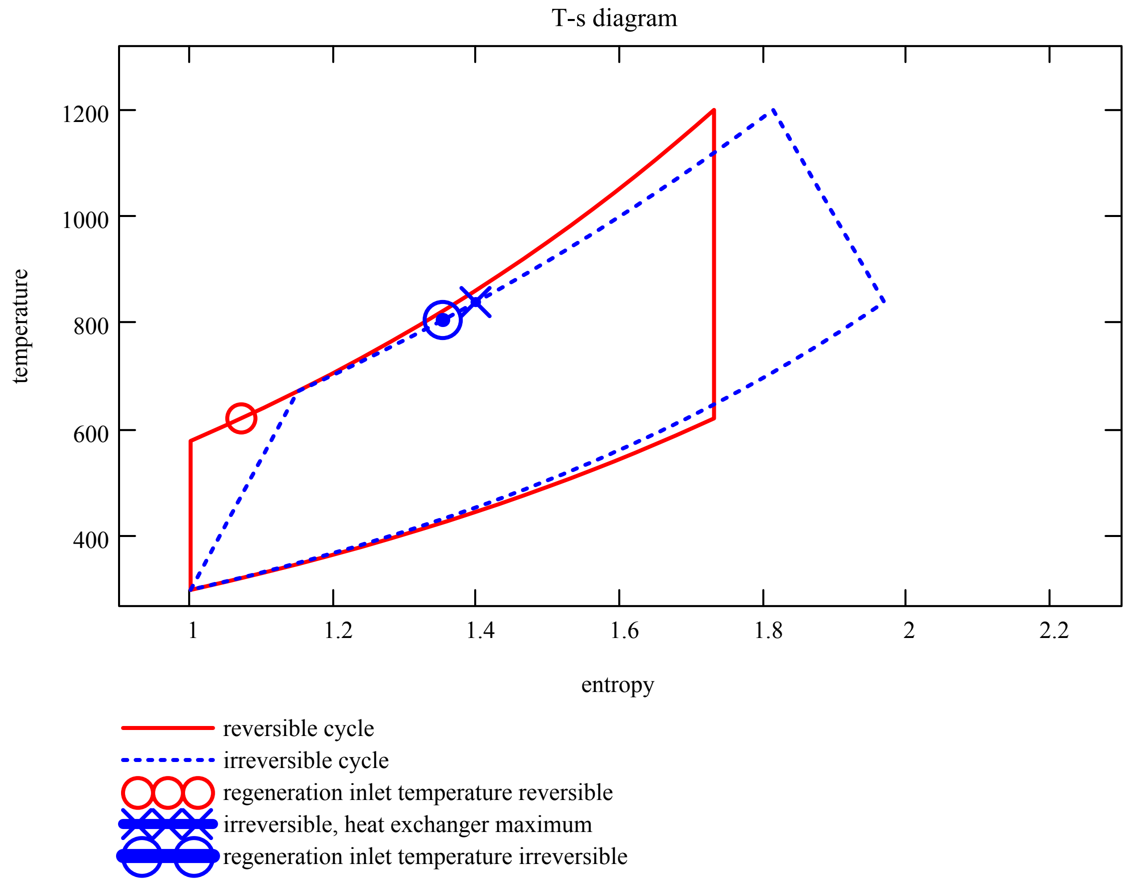
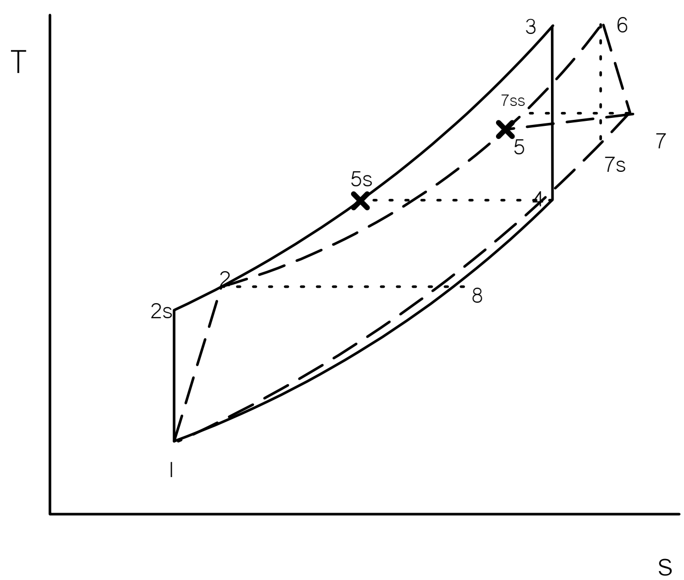
Regeneration Brayton cycle - irreversible

An actual gas turbine differs from the ideal due to inefficiencies in the turbines and compressors and pressure losses in the flow passages (heat exchangers in closed cycle). The T - s diagram may be as shown:

static data for plot





state - reversible process
1 - start
2s - reversible compressor outlet
3 - outlet of heat addition $T_3 = T_{max}$
4 - outlet of turbine
5s - inlet to regenerator $T_{5s} = T_4$

irreversible
1 - start
2 - irreversible compressor outlet
6 - outlet of heat addition $T_6 = T_{max}$
4 - outlet of turbine
5 - inlet to regenerator $T_5 = T_7$

irreversible processes can be described by some efficiencies and heat transfer effectiveness:

N.B. the efficiencies are defined wrt irreversible overall cycle

turbine efficiency $\eta_t = \dfrac{h_6 - h_7}{h_6 - h_{7s}} = \dfrac{T_6 - T_7}{T_6 - T_{7s}}$ $\eta_t := 0.8$

compressor efficiency $\eta_c = \dfrac{h_{2s} - h_1}{h_2 - h_1} = \dfrac{T_{2s} - T_1}{T_2 - T_1}$ $\eta_c := 0.78$

heat exchanger effectiveness $\varepsilon = \dfrac{T_5 - T_2}{T_{7ss} - T_2}$ $\varepsilon := 94\%$

pressure loss in heater $p_6 = p_3 - \delta p_H = p_3 \cdot \left(1 - \dfrac{\delta p_H}{p_3}\right)$ delta_p_over_p_H := 5%

pressure loss (increase) in cooler, relative to p_1 $p_7 = p_1 + \delta p_L = p_1 \cdot \left(1 + \dfrac{\delta p_L}{p_1}\right)$ delta_p_over_p_L := 3%

we will combine these as follows as for efficiency only Δp across turbine matters:

$$\frac{p_6}{p_7} = \frac{p_3 \cdot \left(1 - \dfrac{\delta p_H}{p_3}\right)}{p_1 \cdot \left(1 + \dfrac{\delta p_L}{p_1}\right)} = \frac{p_2}{p_1} \cdot (1 - \delta p\%)$$

$$\text{delta\_p\_over\_p} := 1 - \left(\frac{1 - \text{delta\_p\_over\_p\_H}}{1 + \text{delta\_p\_over\_p\_L}}\right)$$

this combines losses into effect on turbine delta_p_over_p = 7.767 %

for these calculations

taking advantage of constant c_{po}

$\gamma := 1.4$ $\text{power} := \dfrac{\gamma - 1}{\gamma}$ $T_1 := 300$ $T_{max} := 1200$ maximum $T_3 := T_{max}$ $T_6 := T_{max}$

$N_c = 1$ one compressor no intercooling $pr := 1.3, 1.4 .. 5$ start with 1+ as $\eta = 1$ mathematically

reversible relationships are developed in brayton_cycle_summary.mcd (may be 2005)

reversible ....

$T_{2s}(pr) := pr^{power} \cdot T_1$ $T_{2s}(2) = 365.704$

$T_4(pr) := \left(\dfrac{1}{pr}\right)^{power} \cdot T_3$ $T_4(2) = 984.402$

irreversible .....

$T_2(pr) := T_1 + \dfrac{T_{2s}(pr) - T_1}{\eta_c}$ $T_2(2) = 384.236$

p6_over_p7(pr) := pr·(1 − delta_p_over_p)

p6_over_p7(2) = 1.845

reversible turbine calc in irreversible cycle ... $T_{7s}(pr) := T_6 \cdot \left(\dfrac{1}{\text{p6\_over\_p7}(pr)}\right)^{power}$ $T_{7s}(2) = 1007$

$T_7(pr) := T_6 - (T_6 - T_{7s}(pr)) \cdot \eta_t$ $T_7(2) = 1046$

at this point we can compute the thermal efficiency without regeneration

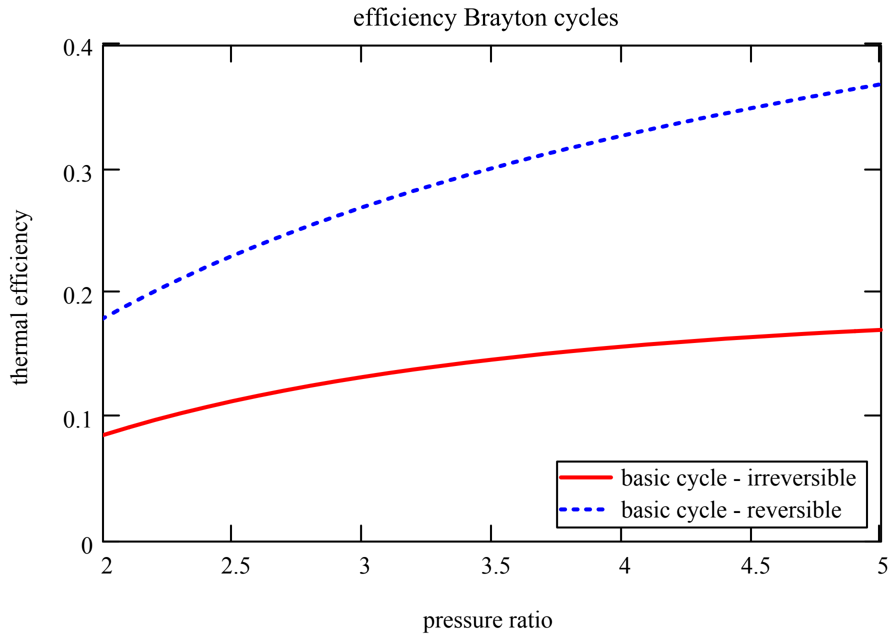
reversible | irreversible | rev | irrev

$$\eta_{th} = \frac{w_{net}}{q_H} = \frac{w_t + w_c}{q_H} = \frac{T_3 - T_4 - (T_{2s} - T_1)}{T_3 - T_{2s}} = \left[\frac{T_6 - T_7 - (T_2 - T_1)}{T_6 - T_2}\right]$$

$$Q_H = T_3 - T_{2s} = (T_6 - T_2)$$

so thermal efficiency becomes

$$\eta_{th\_basic\_rev}(pr) := \frac{T_3 - T_4(pr) - (T_{2s}(pr) - T_1)}{T_3 - T_{2s}(pr)} \qquad \eta_{th\_basic\_irr}(pr) := \frac{T_6 - T_7(pr) - (T_2(pr) - T_1)}{T_6 - T_2(pr)}$$



$\eta_t = 0.8$

$\eta_c = 0.78$

$\varepsilon = 0.94$

delta_p_over_p = 7.8%

with regeneration, all the states are the same with

reversible - regen inlet temperature | irreversible ...

$$T_{5s} := T_4$$

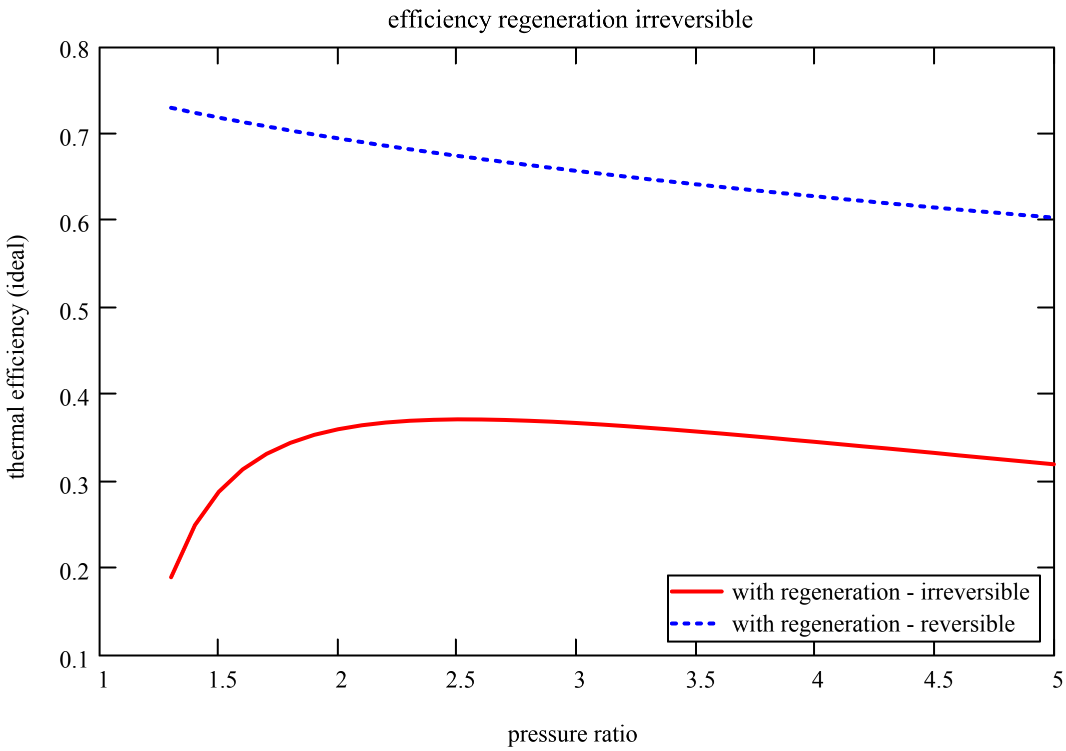
$$T_5(pr) := T_2(pr) + \varepsilon \cdot (T_7(pr) - T_2(pr)) \qquad T_5(2) = 1006$$

with regeneration
reversible | irreversible | rev | irrev

$$\eta_{th\_ic} = \frac{w_{net}}{q_H} = \frac{w_t + w_c}{q_H} = \frac{T_3 - T_4 - (T_{2s} - T_1)}{T_3 - T_{5s} = T_4} = \left[\frac{T_6 - T_7 - (T_2 - T_1)}{T_6 - T_5}\right]$$

$$Q_H = T_3 - T_{5s} = (T_6 - T_5)$$

$$\eta_{th\_reg\_rev}(pr) := 1 - \frac{T_{2s}(pr) - T_1}{T_3 - T_3 \cdot \left(\frac{1}{pr}\right)^{power}} \qquad \eta_{th\_reg\_irr}(pr) := \frac{T_6 - T_7(pr) - (T_2(pr) - T_1)}{T_6 - T_5(pr)}$$



also look at magnitude of compressor work compared to turbine, say for pr = 2 (since these states are the same for w & w/o regeneration, the work is also the same

$$ratio_{rev} = \frac{work_{comp}}{work_{turb}} = \left(\frac{T_{2s} - T_1}{T_3 - T_4}\right) \qquad ratio_{irr} = \frac{work_{comp}}{work_{turb}} = \left(\frac{T_2 - T_1}{T_6 - T_7}\right)$$

$$ratio_{rev}(pr) := \frac{T_{2s}(pr) - T_1}{T_3 - T_4(pr)} \qquad ratio_{irr}(pr) := \frac{T_2(pr) - T_1}{T_6 - T_7(pr)}$$

$ratio_{rev}(2) = 30.5\,\%$ $\qquad ratio_{irr}(2) = 54.7\,\%$

Intercooled Irreversible (and reversible)

$T_{2s} := 0$ $\quad T_2 := 0$ $\quad T_7 := 0$ $\quad T_{7s} := 0$ $\quad T_4 := 0$ $\quad$ reset to insure calculation

parameters from above ...

$\gamma = 1.4$ $\qquad power = 0.286$ $\qquad T_1 = 300$ $\qquad T_6 = 1.2 \times 10^3$ $\qquad$ maximum $\qquad$ for these calculations

$N := 1$ $\qquad$ one stage intercooling two compressors

$\eta_t = 0.8$ $\qquad \eta_c = 0.78$ $\qquad delta\_p\_over\_p = 7.767\,\%$ $\qquad$ efficiencies from above ...

$pr := 1.1, 1.2 .. 5$ $\qquad$ range for pressure ratio $\qquad$ assuming equal pressure ratios across multiple compressors, the ratio for each is ... $\qquad r_c(pr, N) := pr^{\frac{1}{N+1}}$

reversible .....

temperature out of all compressors (isentropic)
intercooling occurs along p = constant to same T_1. Subsequent compressions are at the same ratio so temperatures after each compression are the same.

$$T_{2s}(pr, N) := r_c(pr, N)^{power} \cdot T_1$$

$T_{2s}(2,1) = 331.227$ irreversible ... all compressors $T_2(pr,N) := T_1 + \frac{T_{2s}(pr,N) - T_1}{\eta_c}$ $T_2(2,1) = 340.034$

$p6\_over\_p7(pr) := pr \cdot (1 - delta\_p\_over\_p)$

$T_4(pr) := T_3 \cdot \left(\frac{1}{pr}\right)^{power}$ $T_4(2) = 984.402$ $T_{7s}(pr) := T_6 \cdot \left(\frac{1}{p6\_over\_p7(pr)}\right)^{power}$ $T_{7s}(2) = 1.007 \times 10^3$

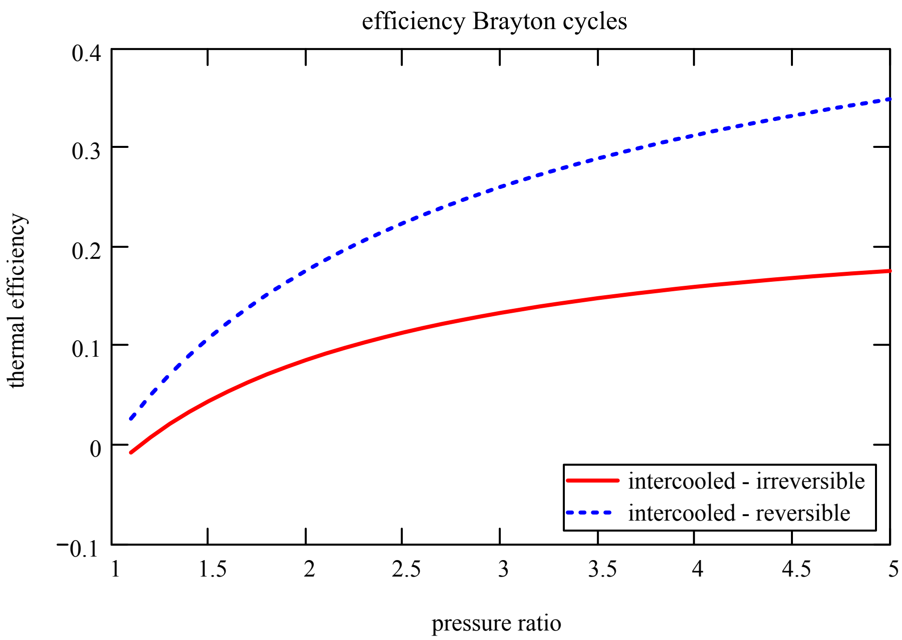
$T_7(pr) := T_6 - (T_6 - T_{7s}(pr)) \cdot \eta_t$ $T_7(2) = 1.046 \times 10^3$

reversile ....   irreversible

$$\eta_{th\_ic} = \frac{w_{net}}{q_H} = \frac{w_t + w_c}{q_H} = \frac{T_3 - T_4 - (N+1)\cdot(T_{2s} - T_1)}{T_3 - T_{2s}} = \frac{T_6 - T_7 - (N+1)\cdot(T_2 - T_1)}{T_6 - T_2}$$

$$\eta_{th\_ic\_irr}(pr,N) := \frac{(T_6 - T_7(pr)) - (N+1)\cdot(T_2(pr,N) - T_1)}{T_6 - T_2(pr,N)}$$

$$\eta_{th\_ic\_rev}(pr,N) := \frac{(T_3 - T_4(pr)) - (N+1)\cdot(T_{2s}(pr,N) - T_1)}{T_3 - T_{2s}(pr,N)}$$



$\eta_t = 0.8$

$\eta_c = 0.78$

$delta\_p\_over\_p = 7.767\,\%$

$$ratio_{rev} = \frac{work_{comp}}{work_{turb}} = \frac{(N+1)\cdot(T_{2s} - T_1)}{T_3 - T_4}$$

$$ratio_{irr} = \frac{work_{comp}}{work_{turb}} = \frac{(N+1)\cdot(T_2 - T_1)}{T_6 - T_7}$$

$$ratio_{rev}(pr) := \frac{(N+1)\cdot(T_{2s}(pr,N) - T_1)}{T_3 - T_4(pr)}$$

$$ratio_{irr}(pr) := \frac{(N+1)\cdot(T_2(pr,N) - T_1)}{T_6 - T_7(pr)}$$

$ratio_{rev}(2) = 29\,\%$ $ratio_{irr}(2) = 52\,\%$

calculations with reheat and multiple turbines are similar and will not be done here. see brayton_plot.mcd for general calculations and plotting