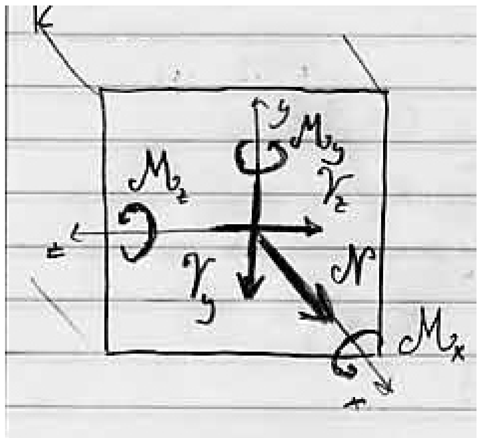
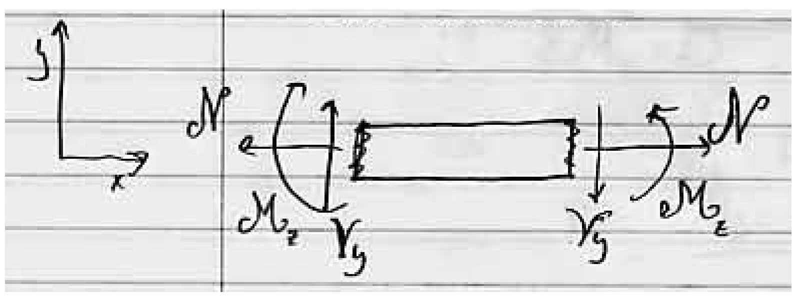
2.001 - MECHANICS AND MATERIALS I
Lecture #6
9/27/2006
Prof. Carol Livermore

Recall:

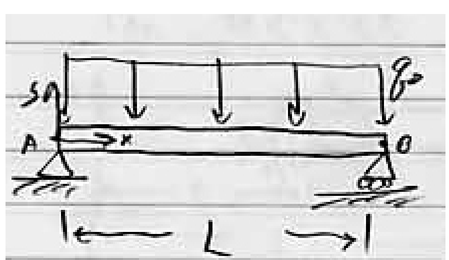


Sign Convention



Positive internal forces and moments shown.

Distributed Loads

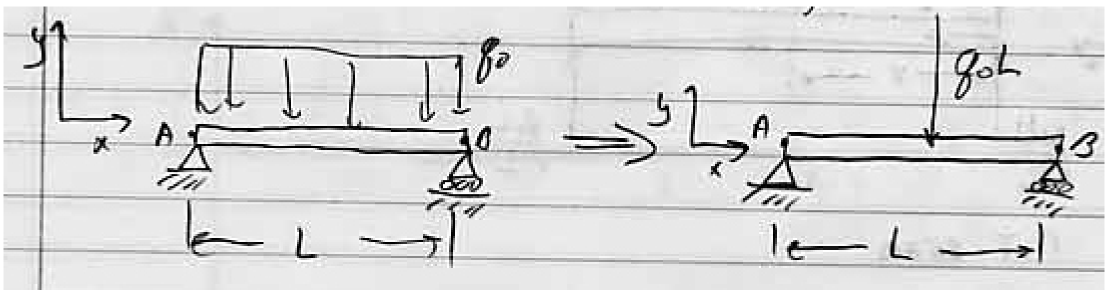


$$q(x) = q_0$$

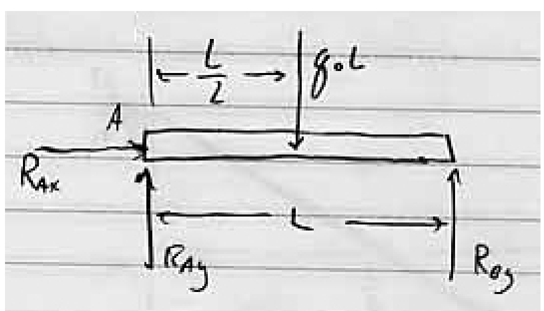
$$F_{\text{Total Loading}} = \int_0^L q(x)dx = \Big[q_0 x\Big]_0^L = q_0 L$$

$$M_{A_{\text{Distributed Load}}} = \int_0^L q(x)xdx = \left[\frac{q_0 x^2}{2}\right]_0^L = \frac{q_0 L^2}{2}$$

Lump: (Equivalent Forces)



FBD



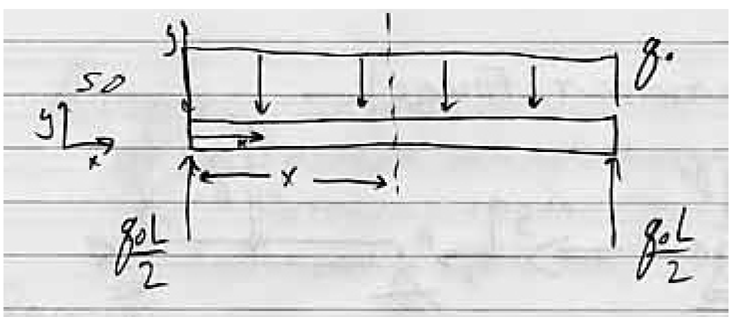
$$\sum F_x = 0$$
$$R_{A_x} = 0$$

$$\sum F_y = 0$$
$$R_{A_y} + R_{B_y} - q_0 L = 0$$

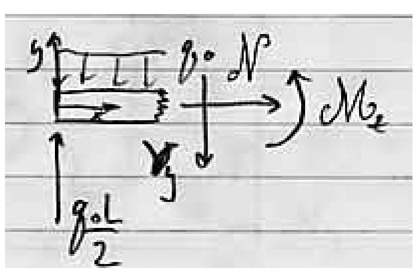
$$\sum M_A = 0$$
$$-\frac{L}{2} q_0 L + R_{B_y} L = 0$$
$$R_{B_y} = \frac{q_0 L}{2}$$

Substitute:

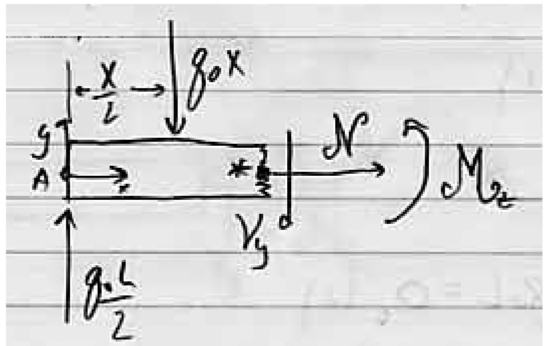
$$R_{A_y} + \frac{q_0 L}{2} - q_0 L = 0$$
$$R_{A_y} = \frac{q_o L}{2}$$



Cut beam.



Lump:

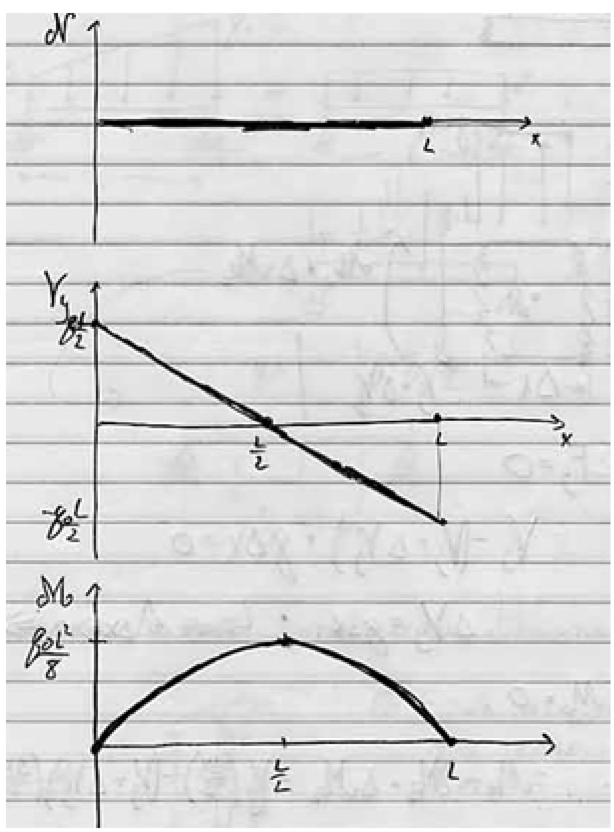


$$\sum F_x = 0$$
$$N = 0$$

$$\sum F_y = 0$$
$$\frac{q_0 L}{2} - q_0 x - V_y = 0 \Rightarrow V_y q_0 \left(\frac{L}{2} - x \right)$$

$$\sum M_* = 0$$
$$-\frac{q_0 L}{2} x + q_0 x \left(\frac{x}{2} \right) + M_z = 0$$
$$M_z = q_0 \left(\frac{L}{2} x - \frac{x^2}{2} \right)$$

Plot

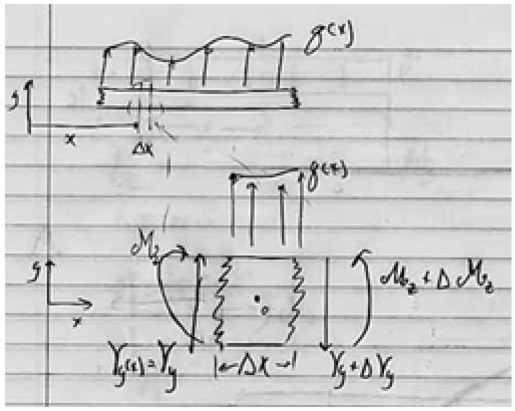


Sanity Check

Pinned and Pinned on rollers at end

Cannot support moment M at ends

Can support x and y loads $\Rightarrow$ OK



$$\sum F_y = 0$$

$$V_y - (V_y + \delta V_y) + q\delta x = 0$$

$$\delta V_y = q\delta x$$

Limit as $\delta x \to 0 \Rightarrow \frac{qdV_y}{dx}$

$$\sum M_0 = 0$$

$$-M_z + M_z + \delta M_z - V_y\left(\frac{\delta x}{2}\right) - (V_y + \delta V_y)\left(\frac{\delta x}{2}\right) = 0$$

$$\delta M_z - 2V_y\frac{\delta x}{2} - \delta V_y\frac{\delta x}{2}$$

$$\delta M_z = V_y\delta x - \delta V_y\frac{\delta x}{2}$$

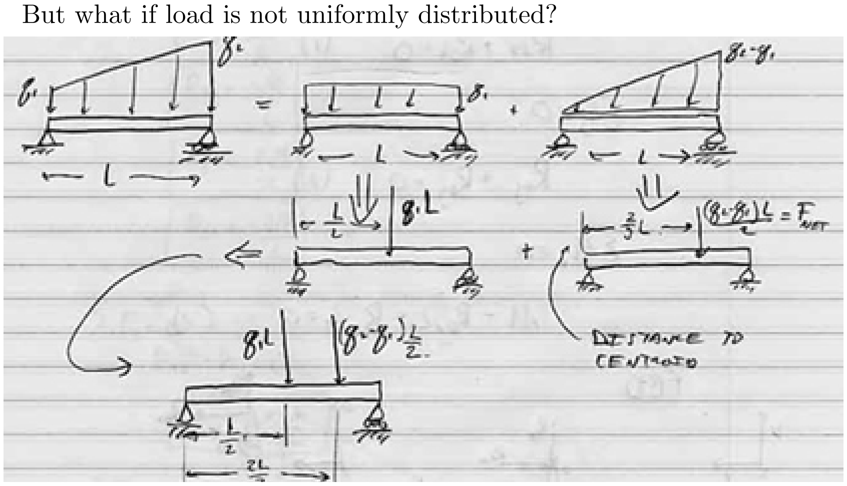
$$V_y = \frac{\delta M_z}{\delta x} + \frac{\delta V_y}{2}$$

Take limit $\delta x \to 0$

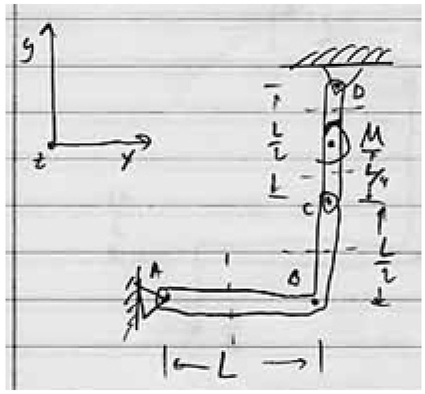
$$V_y = \frac{dM_z}{dx}$$

$$q = \frac{dV_y}{dx}$$

But what if load is not uniformly distributed?

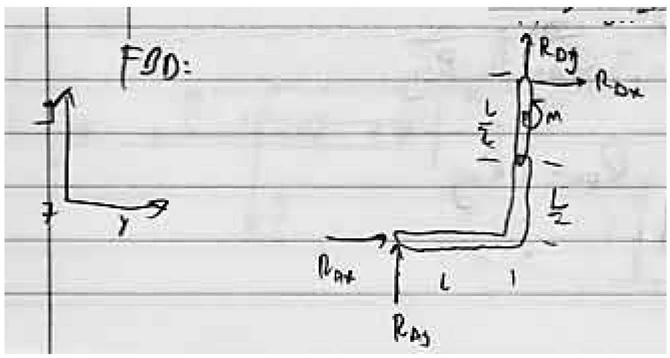


EXAMPLE: A more interesting structure



Q: Find all internal forces and moments in all members. Plot.

Find reactions at supports.
FBD

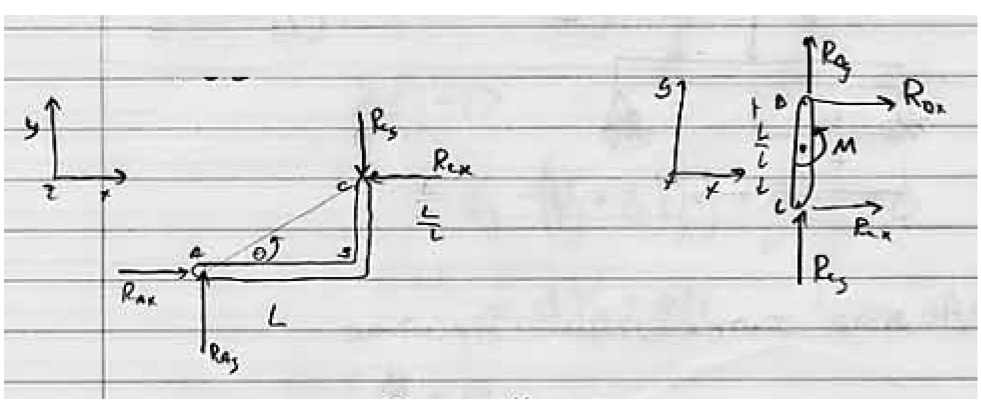


$$\sum F_x = 0$$
$$R_{A_x} + R_{D_x} = 0$$

$$\sum F_y = 0$$
$$R_{A_y} + R_{D_y} = 0$$

$$\sum M_A = 0$$
$$M - R_{D_x} L + R_{D_y} L = 0$$

FBD



This is a 2-force member.

$$\frac{R_{A_y}}{R_{A_x}} = \tan\theta = \frac{\frac{L}{2}}{L} = \frac{1}{2}$$

$$R_{A_x} = 2R_{A_y} \Rightarrow \frac{R_{A_x}}{R_{A_y}} = 2$$

$$\frac{R_{A_x}}{R_{D_x}} = -1$$

$$\frac{R_{A_y}}{R_{D_y}} = -1$$

$$\frac{R_{A_x}}{R_{D_x}} = \frac{R_{A_y}}{R_{D_y}}$$

$$\frac{R_{A_x}}{R_{A_y}} = \frac{R_{D_x}}{R_{D_y}}$$

So:

$$\frac{R_{A_x}}{R_{D_y}} = 2 \Rightarrow R_{D_x} = 2R_{D_y}$$

So:

$$R_{D_y} = \frac{M}{L}$$

$$R_{A_y} = -\frac{M}{L}$$

$$R_{D_x} = \frac{2M}{L}$$

$$R_{A_x} = \frac{-2M}{L}$$

$$\sum F_x = 0$$

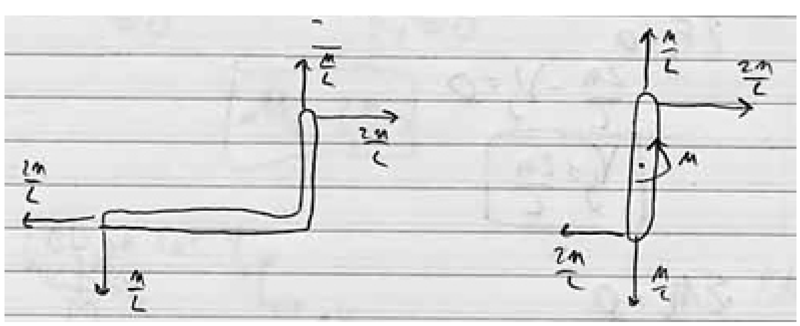
$$R_{A_x} - R_{C_x} = 0$$

$$R_{C_x} = \frac{-2M}{L}$$

$$\sum F_y = 0$$

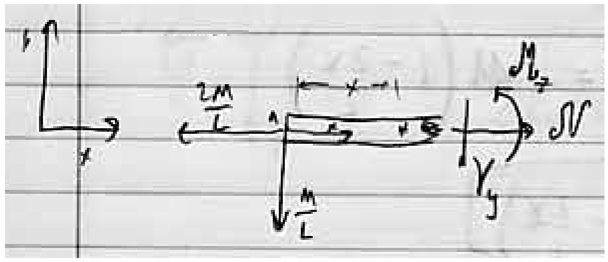
$$R_{A_y} - R_{C_y} = 0$$

$$R_{C_y} = \frac{-M}{L}$$



Take cut

FBD



$$\sum F_x = 0$$

$$N - \frac{2M}{L} = 0$$

$$N = \frac{2M}{L}$$

$$\sum F_y = 0$$

$$-\frac{M}{L} - V_y = 0$$

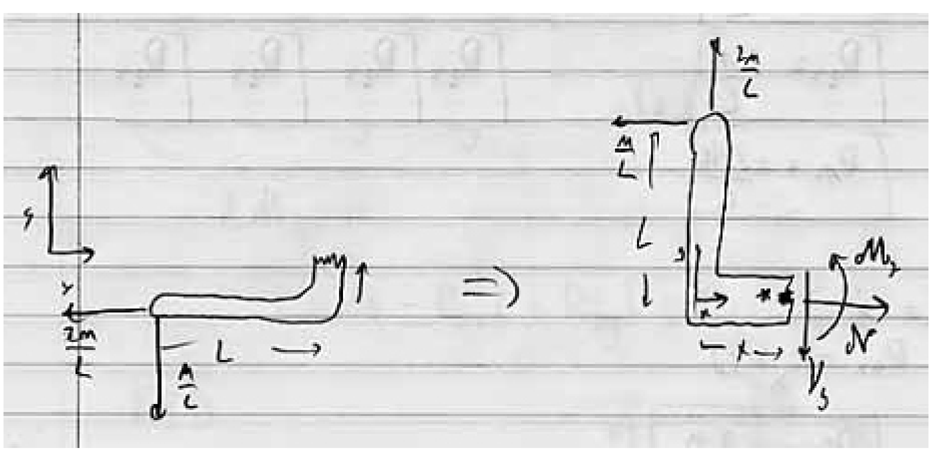
$$V_y = \frac{-M}{L}$$

$$\sum M_* = 0$$

$$\frac{M}{L}x + M_z = 0$$

$$M_z = \frac{-M}{L}x$$

FBD of Cut 2



$$\sum F_x = 0$$

$$-\frac{M}{L} + N = 0$$

$$N = \frac{M}{L}$$

$$\sum F_y = 0$$

$$\frac{2M}{L} - V_y = 0$$

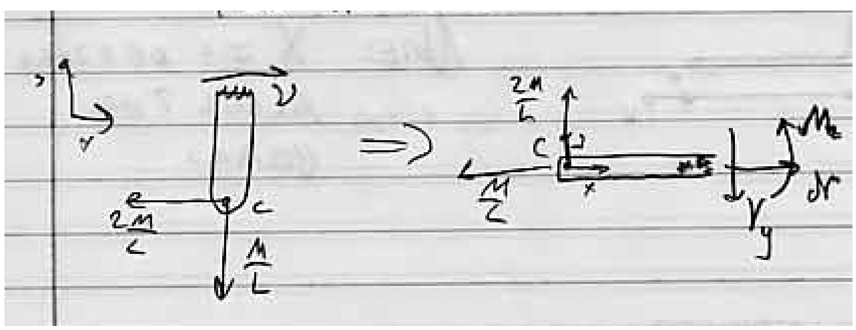
$$V_y = \frac{2M}{L}$$

$$\sum M_* = 0$$

$$M_z + \frac{M}{L}x - \frac{2M}{L}x = 0$$

$$M_z = -M\left(1 - \frac{2x}{L}\right)$$

FBD of Cut 3



$$\sum F_x = 0$$

$$-\frac{M}{L} + N = 0$$

$$N = \frac{M}{L}$$

$$\sum F_y = 0$$

$$\frac{2M}{L} - V_y = 0$$

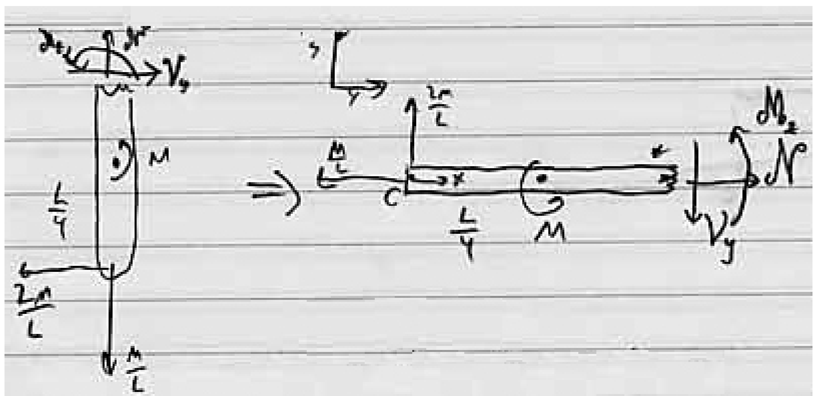
$$V_y = \frac{2M}{L}$$

$$\sum M_* = 0$$

$$M_z + \frac{2M}{L}x = 0$$

$$M_z = \frac{2M}{L}x$$

FBD of Cut 4



$$\sum F_x = 0$$

$$-\frac{M}{L} + N = 0$$

$$N = \frac{M}{L}$$

$$\sum F_y = 0$$

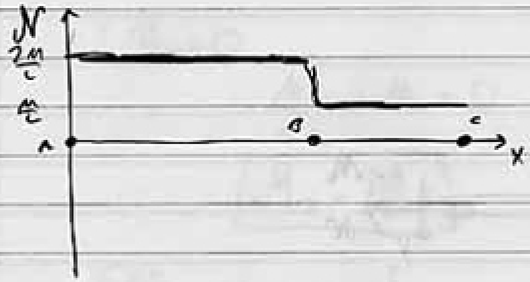
$$\frac{2M}{L} - V_y = 0$$

$$V_y = \frac{2M}{L}$$

$$\sum M_* = 0$$

$$M - \frac{2M}{L}x + M_z = 0$$

$$M_z = -M\left(1 - \frac{2x}{L}\right)$$



NOTE: X IS DEFINED ALONG THE BEAMS.