

AMPLIFIERS

A circuit containing only capacitors, amplifiers (transistors) and resistors may resonate.

A circuit containing only capacitors and resistors may not.

Why does amplification permit resonance in a circuit with only one kind of storage element?

Amplification arises from static behavior, not dynamics and energy storage.

Amplification is fundamentally dissipative.

$$P_{\text{output}} \leq P_{\text{supply}}$$

$$P_{\text{output}} = P_{\text{supply}} + P_{\text{input}} - P_{\text{dissipated}}$$

Amplifiers are basically multiport resistors

How does the addition of a (multiport) dissipator enable resonance?

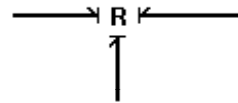
WE WILL SEE THAT

- **Amplification is fundamentally a non-equilibrium phenomenon.**
- **Resistors far from equilibrium may contain a "hidden" junction structure that includes a gyrator.**
- **This gyrator can cause resonance with only one kind of storage element.**

NODICITY

Electrical amplifiers are nodic.

Assuming conductance causality



the equations of the three-port resistor may be written as follows.

$$f_1 = \Gamma_1(e_1, e_2, e_3)$$

$$f_2 = \Gamma_2(e_1, e_2, e_3)$$

$$f_3 = \Gamma_3(e_1, e_2, e_3)$$

Nodicity means that the efforts and flows at the ports are constrained so that they satisfy two conditions:

(1) Continuity of flow: The sum of flows into the system is zero. This means that the element behaves as a node characterized by a Kirchhoff current law.

(Note the implicit assumption of the "power positive in" sign convention)

(2) Relativity of effort: Each flow depends only on the difference of applied efforts. The same effort may be added to all inputs without changing the output.

The constitutive equations of a *nodic* three-port resistor may be written as follows.

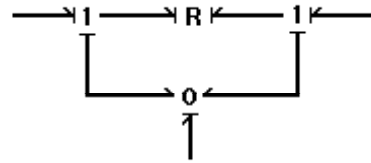
$$f_1 = \Gamma_1[(e_1 - e_3), (e_2 - e_3)]$$

$$f_2 = \Gamma_2[(e_1 - e_3), (e_2 - e_3)]$$

$$f_3 = -(f_1 + f_2)$$

Nodicity implies that the behavior of the element is independent of any absolute reference frame.

A *nodic* three-port (n-port) contains an “embedded” two-port (n-1-port)



characterized by *two* constitutive equations (not three).

EXAMPLE: SEMI-CONDUCTOR DIODE.

A semi-conductor diode has two ports (two wires) but, in common with all electronic devices it is well described as a nodic element and is characterized quite accurately by a single constitutive equation such as

$$i = I_s [e^{(e_1 - e_2)/V_t} - 1]$$

where

V_t : thermal voltage = $kT/q = 25.3$ mV at 20°C

k : Boltzmann's constant

T : absolute temperature

q : charge on the electron

I_s : reverse saturation current

Note that the element satisfies the conditions for nodicity even though both the thermal voltage and the reverse saturation current depend on absolute temperature.

EMBEDDED JUNCTION STRUCTURES

In general a multi-port resistor may contain a "hidden" junction structure coupling the power flows on its ports.

A two-port resistor may contain a "hidden" or "embedded" gyrator.

Without loss of generality assume $e_3 = 0$.

The constitutive equations of the two-port

$$f_1 = \Gamma_1(e_1, e_2)$$

$$f_2 = \Gamma_2(e_1, e_2)$$

$$\mathbf{f} = \Gamma(\mathbf{e})$$

are restricted to the first and third quadrants.

The general form may be replaced by

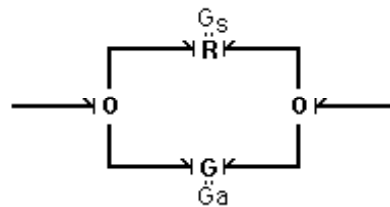
$$\mathbf{f} = \mathbf{G}(\mathbf{e}) \mathbf{e}$$

where \mathbf{G} is a 2x2 conductance matrix.

The matrix \mathbf{G} may be decomposed into symmetric and anti-symmetric components.

$$\mathbf{G} = (\mathbf{G} + \mathbf{G}_t)/2 + (\mathbf{G} - \mathbf{G}_t)/2 = \mathbf{G}_s + \mathbf{G}_a$$

The anti-symmetric component describes an ideal power-continuous gyrator which is "embedded" in the resistive two-port.



Power must be dissipated and not generated.

The total power dissipated by the element is:

$$P_{\text{dissipated}} = \mathbf{e}^t \mathbf{f} = \mathbf{e}^t \mathbf{G} \mathbf{e} = \mathbf{e}^t \mathbf{G}_s \mathbf{e} + \mathbf{e}^t \mathbf{G}_a \mathbf{e}$$

but

$$\mathbf{e}^t \mathbf{G}_a \mathbf{e} = 0$$

$$P_{\text{dissipated}} = \mathbf{e}^t \mathbf{G}_s \mathbf{e}$$

Power dissipation depends only on the *symmetric* component of \mathbf{G} .

EXAMPLE: NPN TRANSISTOR

Consider a typical NPN transistor, Motorola's 2N4400. From the specification sheets, the AC small-signal characteristics at 1 KHz are as follows (operating conditions: $i_c = 1\text{mA}$, $v_{ce} = 10\text{ Vdc}$)

Forward current gain: $h_{fe} = 250$

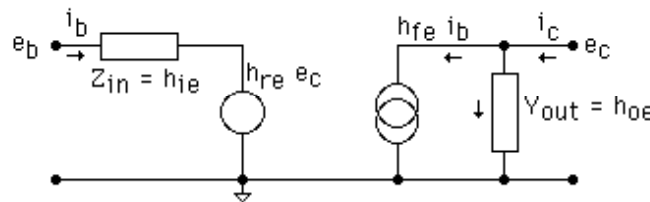
Voltage feedback ratio: $h_{re} = 8 \times 10^{-4}$

Input impedance: $h_{ie} = 7.5\text{ K}\Omega$

Output conductance: $h_{oe} = 30\ \mu\text{mho}$

The subscript e on the "h-parameters" refers to the common-emitter configuration used in a typical amplifier circuit.

They are based on a "controlled-source" model of the transistor.



The corresponding model equations are

$$i_c = h_{fe} i_b + h_{oe} e_c$$

$$e_b = h_{ie} i_b + h_{re} e_c$$

Rearranging into matrix form,

$$\begin{bmatrix} i_c \\ e_b \end{bmatrix} = \begin{bmatrix} h_{oe} & h_{fe} \\ h_{re} & h_{ie} \end{bmatrix} \begin{bmatrix} e_c \\ i_b \end{bmatrix}$$

Note that the gain terms, h_{fe} and h_{re} , appear in the off-diagonal positions.

This matrix may be partially inverted and rearranged into a conductance form as follows.

$$i_c = (h_{oe} - h_{fe}h_{re}/h_{ie}) e_c + (h_{fe}/h_{ie}) e_b$$

$$i_b = -(h_{re}/h_{ie}) e_c + (1/h_{ie}) e_b$$

$$\begin{bmatrix} i_c \\ i_b \end{bmatrix} = \begin{bmatrix} Y_{cc} & Y_{cb} \\ Y_{bc} & Y_{bb} \end{bmatrix} \begin{bmatrix} e_c \\ e_b \end{bmatrix}$$

Equating terms:

$$Y_{cc} = h_{oe} - h_{fe}h_{re}/h_{ie}$$

$$Y_{cb} = h_{fe}/h_{ie}$$

$$Y_{bc} = -h_{re}/h_{ie}$$

$$Y_{bb} = 1/h_{ie}$$

This is a two-port nodic resistor.

NOTE:

**The controlled-source representation is widely used
this conductance representation is not.**

**The conductance representation reveals that the h-parameters are
constrained**

the conductance matrix must be positive definite.

The controlled-source model hides this constraint.

EVALUATE PARAMETERS:

$$Y_{cc} = 3.33 \mu\text{mho}$$

$$Y_{cb} = 33,333 \mu\text{mho}$$

$$Y_{bc} = -0.107 \mu\text{mho}$$

$$Y_{bb} = 133.3 \mu\text{mho}$$

In μmho units:

$$\begin{bmatrix} i_c \\ i_b \end{bmatrix} = \begin{bmatrix} 3.33 & 33333 \\ -0.107 & 133.3 \end{bmatrix} \begin{bmatrix} e_c \\ e_b \end{bmatrix}$$

Note the extreme asymmetry of the conductance matrix.

Its symmetric and anti-symmetric components are:

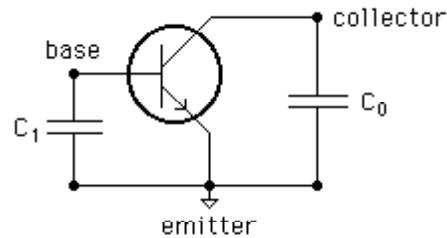
$$\begin{bmatrix} i_c \\ i_b \end{bmatrix} = \begin{bmatrix} 3.33 & 16666 \\ 16666 & 133.3 \end{bmatrix} \begin{bmatrix} e_c \\ e_b \end{bmatrix} + \begin{bmatrix} 0 & 16667 \\ -16667 & 0 \end{bmatrix} \begin{bmatrix} e_c \\ e_b \end{bmatrix}$$

The symmetric component is purely dissipative.

The anti-symmetric component describes a gyrational coupling between input (base) and output (collector).

Because of this gyrational coupling, a resonant circuit may be constructed using this transistor, two capacitors and no inductors.

Consider connecting a capacitor $C_c = 1000 \text{ pF}$ across the output (collector to grounded emitter) and a capacitor $C_b = 0.1 \text{ }\mu\text{F}$ across the input (base to grounded emitter) as shown.



Time-differentiating the constitutive equations for these capacitors results in the following.

$$\frac{d}{dt} e_c = \frac{1}{C_c} i_c$$

$$\frac{d}{dt} e_b = \frac{1}{C_b} i_b$$

Substituting for the collector and base currents using the transistor conductance matrix, state equations for this linear system may be written as follows.

$$\frac{d}{dt} \begin{bmatrix} e_c \\ e_b \end{bmatrix} = \begin{bmatrix} -Y_{cc}/C_c & -Y_{cb}/C_c \\ -Y_{bc}/C_b & -Y_{bb}/C_b \end{bmatrix} \begin{bmatrix} e_c \\ e_b \end{bmatrix}$$

The characteristic equation for this system is

$$s^2 + (Y_{cc}/C_c + Y_{bb}/C_b) s + (Y_{cc}Y_{bb} - Y_{bc}Y_{cb})/C_cC_b = 0$$

The parameters listed above yield

$$\omega_n = 6.32 \text{ KHz}$$

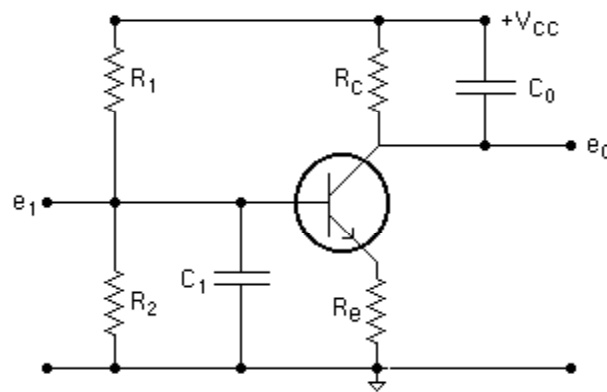
and

$$\zeta = 0.37$$

BIASING

The AC parameters of the transistor are only valid if the device is properly "biased" at DC.

An appropriate network of resistors must be added. A typical common-emitter biasing circuit with "emitter degeneration" (an emitter resistor) is shown.



These resistors add to the damping in the circuit and may suppress resonance. but the central point still holds: two capacitors and a transistor can make a resonant circuit because the transistor contains a "hidden" gyrator.

Why does "biasing" matter?

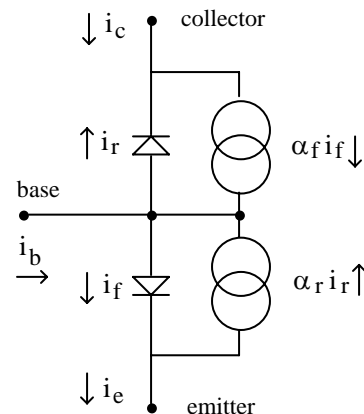
The device only amplifies if operated far from equilibrium

LARGE-SIGNAL TRANSISTOR BEHAVIOR: THE EBERS-MOLL MODEL.

(describes static characteristics only – ignores important dynamics such as charge storage on the junction regions)

The Ebers-Moll model describes a transistor as a pair of coupled back-to-back diodes.

The coupling is usually represented as a pair of current-controlled current sources.



For the sign convention shown, current continuity yields

$$i_e = i_f - \alpha_r i_r$$

$$i_c = \alpha_f i_f - i_r$$

The currents i_f and i_r refer to the forward and reverse currents of the base-to-emitter and base-to-collector diodes respectively.

Each is related to the corresponding voltage drop through the usual exponential diode relation.

$$i_f = I_{es}[e^{(e_b - e_e)/V_t} - 1]$$

$$i_r = I_{cs}[e^{(e_b - e_c)/V_t} - 1]$$

Physical constraints of the transistor require that $\alpha_r I_{cs} = \alpha_f I_{es}$. For a typical transistor:

$$\alpha_f = 0.99$$

$$\alpha_r = 0.01$$

$$I_{es} = 2.0 \times 10^{-15} \text{ amps}$$

$$V_t = 25.3 \times 10^{-3} \text{ volts}$$

An NPN transistor is usually operated in the "active" region with the emitter diode strongly forward biased (i.e. conducting with $e_b \gg e_e$) and the collector diode strongly reverse biased (i.e. not conducting with $e_c \gg e_b$).

The collector diode current is small and insensitive to changes in the voltage drop from collector to base. As a result,

$$i_e \approx i_f - \alpha_r I_{CS} \approx i_f$$

$$i_e \approx I_{es} [e^{(e_b - e_e)/Vt} - 1]$$

The collector current is primarily determined by its coupling to the emitter current

$$i_c \approx \alpha_f i_f \approx \alpha_f i_e$$

Applying current continuity to the entire network (i.e. treating it as a node) we may relate collector current to base current as follows.

$$i_e = i_b + i_c$$

$$i_c \approx \alpha_f (i_b + i_c)$$

$$i_c \approx \frac{\alpha_f}{1-\alpha_f} i_b \approx \beta i_b$$

where β is the transistor forward current gain.

In practice β may vary between 50 to 1000, but α only changes from 0.98 to 0.999.

Thus to a close approximation the collector current is

$$i_c \approx I_{es} [e^{(e_b - e_e)/V_t} - 1]$$

NOTE:

this model describes the transistor by a relation between an output (collector) current and an input voltage drop (from emitter to base)

This again shows the fundamentally gyrational character of the transistor.

AMPLIFICATION IS A NON-EQUILIBRIUM PHENOMENON

Linearizing the Ebers-Moll model reveals an interesting property of amplifiers in general:

operated near equilibrium, amplifiers cease to amplify.

Consider the grounded-emitter configuration described above.

The elements of the locally linear conductance matrix are as follows.

$$Y_{cc} = \partial i_c / \partial e_c$$

$$Y_{cb} = \partial i_c / \partial e_b$$

$$Y_{bc} = \partial i_b / \partial e_c$$

$$Y_{bb} = \partial i_b / \partial e_b$$

We may use the current continuity conditions to evaluate these.

$$d i_c = \alpha_f d i_f - d i_r$$

$$d i_e = d i_f - \alpha_r d i_r$$

$$d i_b = d i_e - d i_c = (1 - \alpha_f) d i_f + (1 - \alpha_r) d i_r$$

From the diode equations,

$$\partial i_f / \partial e_c = 0$$

$$\partial i_f / \partial e_b = I_{es} [e^{(e_b - e_e) / V_t}] \frac{1}{V_t}$$

$$\partial i_r / \partial e_c = I_{cs} [e^{(e_b - e_c) / V_t}] \frac{-1}{V_t}$$

$$\partial i_r / \partial e_b = I_{cs} [e^{(e_b - e_c) / V_t}] \frac{1}{V_t}$$

Substituting

$$\partial i_c / \partial e_c = I_{cs} [e^{(e_b - e_c)} / V_t] \frac{1}{V_t}$$

$$\partial i_c / \partial e_b = \alpha_f I_{es} [e^{(e_b - e_e)} / V_t] \frac{1}{V_t} - I_{cs} [e^{(e_b - e_c)} / V_t] \frac{1}{V_t}$$

$$\partial i_b / \partial e_c = -(1 - \alpha_r) I_{cs} [e^{(e_b - e_c)} / V_t] \frac{1}{V_t}$$

$$\begin{aligned} \partial i_b / \partial e_b &= (1 - \alpha_f) I_{es} [e^{(e_b - e_e)} / V_t] \frac{1}{V_t} \\ &\quad + (1 - \alpha_r) I_{cs} [e^{(e_b - e_c)} / V_t] \frac{1}{V_t} \end{aligned}$$

Normal operating conditions (denoted by subscript n) are defined by $e_e = 0$, $e_b > e_e + V_t$ and $e_c \gg e_b$.

Under those conditions, $e^{(e_b - e_c)} / V_t$ is very close to zero, whereas $e^{(e_b - e_e)} / V_t$ is a positive number greater than unity (typically very large) whose exact value is strong function of e_b .

Evaluating,

$$Y_{cc} = \partial i_c / \partial e_c \approx 0$$

$$Y_{cb} = \partial i_c / \partial e_b \approx \alpha_f \frac{I_{es}}{V_t} e^{e_b / V_t} \Big|_n$$

$$Y_{bc} = \partial i_b / \partial e_c \approx 0$$

$$Y_{bb} = \partial i_b / \partial e_b \approx (1 - \alpha_f) \frac{I_{es}}{V_t} e^{e_b / V_t} \Big|_n$$

Thus, under normal operating conditions far from equilibrium, the locally linearized conductance matrix is strongly asymmetric as we saw before.

Equilibrium conditions are defined by $e_e = e_b = e_c = 0$.

At equilibrium, both exponentials are equal to unity.

Evaluating,

$$Y_{cc} = \partial i_c / \partial e_c = \frac{I_{cs}}{V_t}$$

$$Y_{cb} = \partial i_c / \partial e_b = \alpha_f \frac{I_{es}}{V_t} - \frac{I_{cs}}{V_t}$$

$$Y_{bc} = \partial i_b / \partial e_c = -(1 - \alpha_r) \frac{I_{cs}}{V_t} = \alpha_r \frac{I_{cs}}{V_t} - \frac{I_{cs}}{V_t}$$

$$Y_{bb} = \partial i_b / \partial e_b = (1 - \alpha_f) \frac{I_{es}}{V_t} + (1 - \alpha_r) \frac{I_{cs}}{V_t}$$

Using the relation between the diode saturation currents, $\alpha_r I_{cs} = \alpha_f I_{es}$, we find that

$$Y_{cb} = \alpha_r \frac{I_{cs}}{V_t} - \frac{I_{cs}}{V_t} = Y_{bc}$$

CONCLUSION:

If the transistor is operated about equilibrium, the locally linearized conductance matrix becomes symmetric.

As a result, the transistor behavior is purely dissipative without any gyrational coupling between input and output.

Relating this to the controlled-source model of the transistor,

$$h_{fe} = Y_{cb}/Y_{bb}$$

$$h_{re} = -Y_{bc}/Y_{bb}$$

– the forward current gain is identical to the reverse voltage gain.

OPERATED ABOUT EQUILIBRIUM,

the embedded gyrator disappears

the transistor fails to amplify