Causality Assignment

The nine primitive elements may be used to represent energy storage and power flow within an energetic system. The energetic interactions represented by the elements constrain (but do not uniquely determine) the mathematical operations which may be used to describe the system behavior. The causality assignment procedure identifies those constraints. The procedure is repeated below (this time including the gyrator and transformer juction elements).

1. All elements in a bond graph which are constrained to have an unique causality must have the appropriate causality assigned and denoted by a causal stroke at the appropriate end of the associated bond.

    1a. Active one-port elements (e.g. power sources) are *always* constrained to an unique causality.

    1b. Remember that in some cases passive one-port elements may be constrained to an unique causality because of their constitutive equations; for example, this is often true of resistors.

2. Examine the junction elements connected to all elements with causal assignments. Propagate causal assignments through the junction structure until no further causal assignments can be made.

    2a. If a one-junction has its unique flow determined by one of its bonds, then all other bonds must be assigned flow output causality.

    2b. If a zero-junction has its unique effort determined by one of its bonds, then all other bonds must be assigned effort output causality.

    2c. If a transformer has a flow input determined on one of its bonds, then a flow output must be assigned to its other bond. Equivalently, if an effort input is determined on one of its bonds, then an effort output must be assigned to its other bond.

    2d. If a gyrator has a flow input determined on one of its bonds, then an effort output must be assigned to its other bond. Conversely, if an effort input is determined on one of its bonds, then a flow output must be assigned to its other bond.

3. Choose one of the energy storage elements which does not already have a causal assignment (it doesn't matter which one) and assign it integral causality (effort input to an inertia, flow input to a capacitor).

    3a. Return to step 2 and propagate this causal assignment through the junction structure until no further causal assignments can be made.

    3b. Repeat steps 3 and 3a until all energy-storage elements have a causal assignment.

4. Any elements remaining which have not been given a causal assignment are dissipators which are indifferent to their causal assignment. Choose one or those dissipators (it doesn't matter which one), assign it an arbitrary causality

    4a. Return to step 2 and propagate this causal assignment through the junction structure until no further causal assignments can be made.

    4b. Repeat steps 4 and 4a until all elements have a causal assignment.

If any of the steps in this procedure cannot be completed without violating the causal constraints imposed by the junction structure, the model is operationally inconsistent. At least one quantity has been determined by more than one relationship. This is an error which must be corrected.

In addition to uncovering logical inconsistencies in the system model, causality assignment also serves to identify dependent and independent energy storage elements. If, in the process, any energy storing element is assigned derivative causality, then that is a dependent storage element. Its stored energy is determined by the variables associated with the element from which the causal propagation began.

Derivative causality on an energy storage element is not an error but it can have undesirable consequences; it may lead to extensive algebra in deriving state equations; it may also lead to numerical difficulties when simulating system behavior on a computer. If one or more energy storage elements in a model have derivative causality, you may want to modify the model to eliminate the dependencies, but this is not essential; indeed it is not always possible.

<u>Causality Assignment and State Variables</u>

Models composed of assemblies of the nine primitive elements are state-determined. To derive state equations, state variables must be chosen. In general, state variables required to describe a system are not unique; many equivalent sets may be identified and the best choice depends on the problem under consideration.

For an energetic system the state variables must at least uniquely define the energy stored in the system. The minimum number of state variables required is determined by the number of independent energy storage elements in the system model. Those energy-storage elements which have been assigned integral causality are independent. The energy stored in the element at a given time may be prescribed as an independent initial condition and gives rise to an independent state variable.