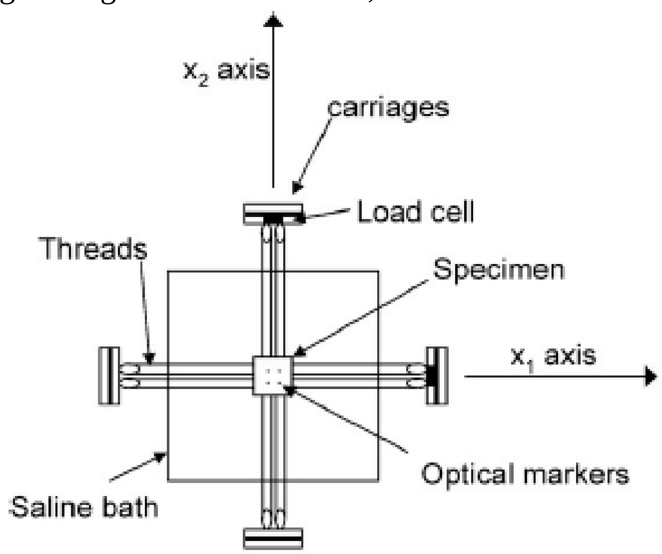
Massachusetts Institute of Technology
Biological Engineering Division
Department of Mechanical Engineering
Department of Electrical Engineering and Computer Science

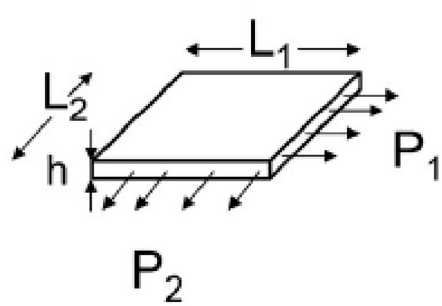
2.797/20.310/3.0536.024, Fall 2006
MOLECULAR, CELLULAR, & TISSUE BIOMECHANICS

Problem Set # 6 | Issued: 11/4/06
Due: 11/9/06

Problem 1

A small specimen of an isotropic homogeneous linear elastic (properties E and ν) membrane is tested in biaxial tension as indicated in the figure below. The dimensions of the specimen in its unloaded configuration are L_1 x L_2 (in plane) x h (thickness). The testing configuration is designed so that the in-plane loads (P_1 and P_2) are uniformly distributed along the edges of the membrane, as indicated.





(a) Obtain expressions for all the components of the stress tensor.

(b) Obtain expressions for all the components of the strain tensor.

(c) Obtain expressions for the dimensions of the loaded membrane.

(d) With the following values of the data:

L_1 = 1cm
L_2 = 2cm
h = 0.5 mm
E = 2MPa
ν = 0.4

the deformed dimensions of the specimen where found to be 1.1cm (along the 1 axis) x 2.05 cm (along the 2-axis). What are the magnitudes of the applied loads P_1 and P_2? What is the corresponding value of the thickness of the deformed specimen?

Problem 2

An experimentalist measures the relationship between transmural (inside minus outside) pressure (p_{tm}) and cross-sectional area A for an artery. When the artery is unloaded (p_{tm} = 0) the wall thickness is measured to be h_0 and the radius is R_0 ($h_0/R_0 << 1$). The experimentalist finds that the data can be well fit by the following relationship:

$$p_{tm} = c_1\left[\exp\left(c_2\frac{A-A_0}{A_0}\right)-1\right]$$

where A is the area corresponding to the level p_{tm} of transmural pressure, and $A_0 = \pi R_0^2$.

a) Relate the argument of the exponential to the strain in the arterial wall in the circumferential direction, $\epsilon_{\theta\theta}$.

b) Relate p_{tm} to circumferential stress, $\sigma_{\theta\theta}$.

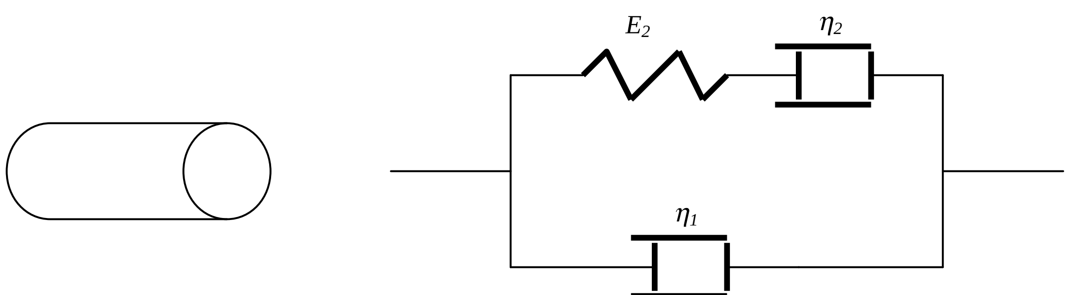
c) Show that the $\sigma_{\theta\theta}$- $\epsilon_{\theta\theta}$. relationship is consistent with the uniaxial σ- ϵ response of an exponentially-stiffening material:

$$\sigma = A\left[\exp(B\varepsilon)-1\right]$$

You may assume for all of the above, that the axial stress and radial stress are negligible.

Problem 3

In class we derived the equations governing the relationship between strain and stress for the Maxwell and Voigt viscoelastic models, and will discuss the standard linear solids model on Tuesday. To reinforce your understanding of these models and how to work with them, consider the following "Kamm model" (it probably has someone's name associated with it, but I haven't seen it mentioned!):



a) Write the equilibrium equations, compatibility expressions, and derive a single differential equation relating the overall stress σ to the overall strain ε, plus their time derivatives.

b) Calculate the creep compliance *J(t)* and the relaxation modululs *G(t)*, both as functions of time, for this system.

Problem 2

The bulk modulus, *K*, of a linear elastic material describes the response of the material to changes in volume. The volumetric strain, ϵ_V, is defined as: $\epsilon_V = \delta V/V_0$, and, for small strains, it can be obtained from the components of the strain tensors as: $\epsilon_V = \epsilon_{11} + \epsilon_{22} + \epsilon_{33}$. Similarly, the hydrostatic stress, σ_h, is defined as: $\sigma_h = 1/3(\sigma_{11} + \sigma_{22} + \sigma_{33})$. Notice that, with this definition, for the case of pure hydrostatic pressure of magnitude *p*, the hydrostatic stress is $\sigma_h = - p$.

The bulk modulus is defined as $K = \sigma_h/\epsilon V$ (similar to the Young's modulus definition, $E=\sigma_{11}/\epsilon_{11}$ in uniaxial tension, but now we look at the volumetric components instead of the axial components).

a) We want to obtain an expression for K in terms of E and ν. To do this, consider a problem in which a specimen of isotropic linear elastic connective tissue is suspended in a fluid at pressure p. What is the stress state in the tissue (all components of the stress tensor σ)? Use the stress-strain relations to obtain the corresponding components of the strain tensor ϵ. Obtain an expression for K in terms of E and ν.

b) From the relations between elastic constants given in class (third slide of lecture 3), obtain an expression for ν in terms of the shear and bulk modulus. For materials that are almost incompressible, $K>>G$. What is the value that ν approaches for this class of materials?

c) Describe how you would go about obtaining E and ν for an isotropic linear elastic material using a tensile test, and a hydrostatic compression test like the one in the figure: draw labeled sketches of your testing configurations. What forces or displacements would you apply? What forces or displacements would you measure? What would you do with your data to get E and ν?