A driven pendulum for classroom demonstrations

Jan M. Skotheim '98

In this handout we describe the driven pendulum used for demonstrations in 12.006J/18.353J. Further details may be found in Blackburn et al.*. There are also a few relevant comments in Baker and Gollub, pp. 135–137. Their Figure 6.2, which is an exploded view of the experimental pendulum, is found at the end of this handout (see Fig. 4).

The equation of motion for a driven pendulum is

$$I\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + mgr\sin(\theta) = T_0\sin(\omega t), \tag{1}$$

where I, m and r are the moment of inertia, mass and length of the pendulum, b is the damping coefficient, g is gravity and T_0 and ω are the amplitude and frequency of the driving torque. The first term is known as the inertial term, the second is the damping term, the next is due to gravity and the final is the driving term. The inertial and gravitational term should be familiar to you from introductory mechanics, however, the origin of the other two may require a bit of explanation.

I. DAMPING FORCE

The fact that the damping force is proportional to the angular velocity of the pendulum is not obvious. The pendulum uses Eddy currents to achieve this damping. A magnet is connected to the pendulum in such a way that when the pendulum moves at a certain angular velocity the magnet moves with the same angular velocity. A copper plate placed next to the magnet experiences a nonuniform magnetic field because the magnet is made of 8 individual magnets placed in a ring with alternating north and south faces next to each other (see Fig. 1 and Fig. 2).

The change in magnetic field with respect to time caused by the rotation of the magnet induces a voltage in the copper plate which in turn leads to currents in the copper plate. These currents produce magnetic fields of their own that interact with the magnetic field of the magnet producing a force. Using basic electromagnetism and ignoring constants it follows that

$$\omega \propto \frac{dB}{dt} \propto V \propto I \propto B_p \propto F, \tag{2}$$

where ω is the angular frequency of the magnet, B, V and I are the magnetic field, voltage and current in the copper plate, B_p is the magnetic field produced by the current in the copper and F is the force experienced by the magnet. So,

---

*J. Blackburn, S. Vik, B. Wu, and H. Smith, A driven pendulum for studying chaos, *Rev. Sci. Instrum.* **60**, 422-426, 1989.

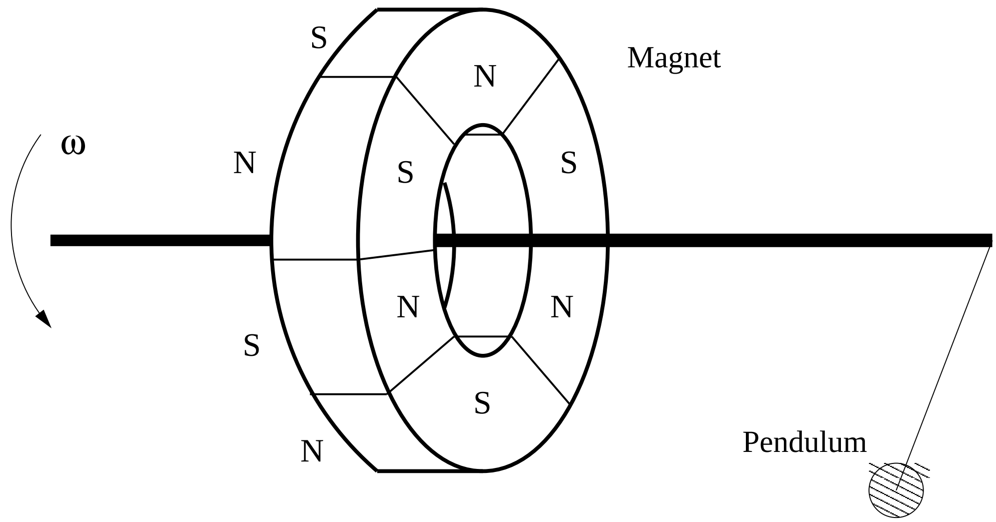

FIG. 1.

$$\omega \propto F. \tag{3}$$

However, this does not give the direction of the force. The force on the magnet must be in the opposite of the direction of motion by the principle of energy conservation. Since no energy is being put into the system the force cannot act to speed up the magnet since this would leave the system in a higher energy state than previously. Hence, the force must act to damp the motion and has the form

$$F_{damping} = b\omega, \tag{4}$$

where b is a constant called the damping coefficient.

II. DRIVING FORCE

The principles behind the driving of the pendulum are very similar to those behind the damping. In this case, there are copper coils placed next to the magnet (see Figure 3). A controlled sinusoidal voltage is applied to the coils producing a current in the coil. This current in turn produces a magnetic field which interacts with the field of the magnet producing a force on the magnet. So,

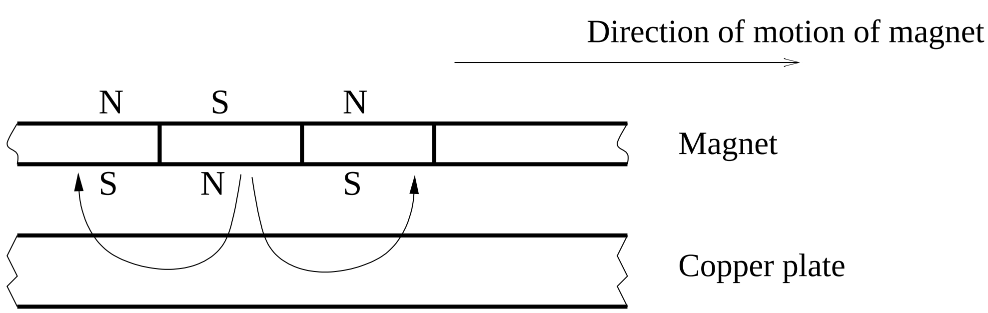

FIG. 2.

$$V(t) = V_0 \sin(\omega t) \propto I(t) \propto B(t) \propto F(t), \tag{5}$$

where $V(t)$ is the voltage applied to the coil, V_0 is a constant, $I(t)$ is the current in the coil, $B(t)$ is the magnetic field experienced by the magnet due to the coil and $F(t)$ is the force on the magnet. So, the driving force can be expressed as

$$F_{driving} = A \sin(\omega t), \tag{6}$$

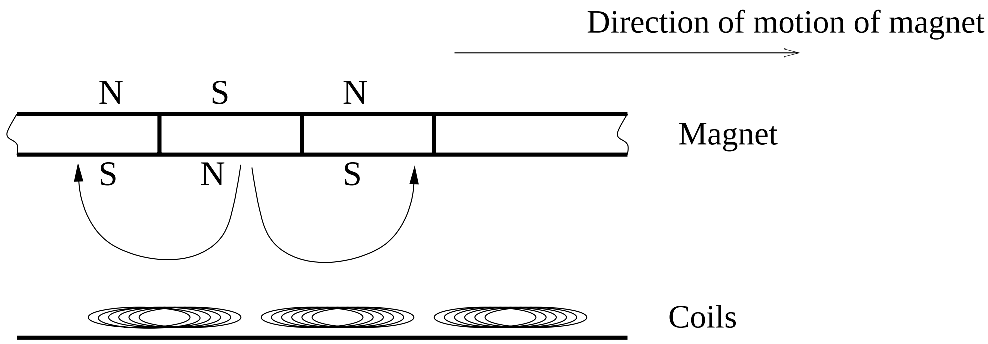

FIG. 3.

where A is a constant proportional to V_0. It is important to note that there are three coils and that the coils switch on and then off in turn so that there is only one coil with current at any given time. This is so the magnet always appears the same above the coil having current. This prevents a reversal in the force caused by rotation of the magnet.

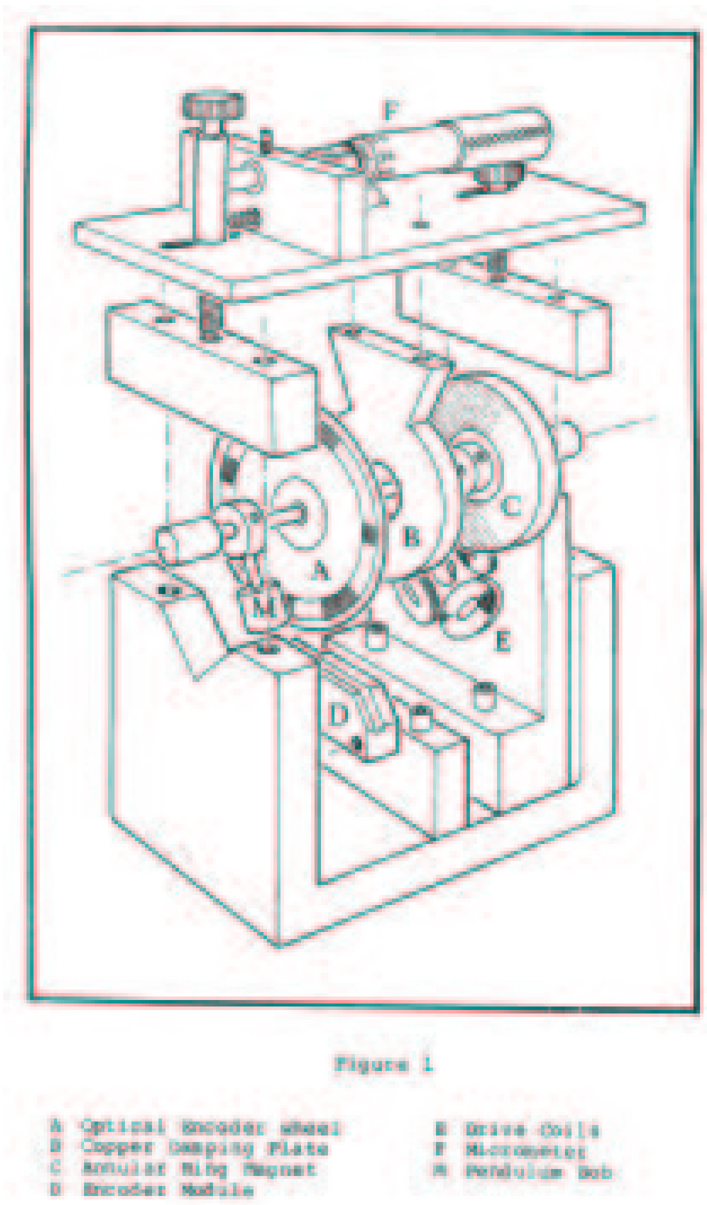

FIG. 4. The experimental pendulum, lifted from Baker and Gollub