MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING
CAMBRIDGE, MASSACHUSETTS 02139

## 2.29 NUMERICAL FLUID MECHANICS— SPRING 2007

# Problem Set 3

Totally 120 points
Posted 04/03/07, due Thursday 4 p.m. 04/19/07, Focused on Lecture 8 to 17

**Problem 3.1 (15 points):**

Consider the following system of equations:

$$Ax = b, \quad A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 8 & 0 \\ -1 & 0 & 4 \end{bmatrix}, \; b = \begin{bmatrix} 0 \\ 8 \\ 4 \end{bmatrix}$$

a) Cholesky factorize A (Note that A is positive definite).
b) Find an LU factorization form for A.
c) Use LU factorization of A to find x.
d) Compute the x by two iterations of successive over-relaxation scheme. Use relaxation parameter $\omega = 1.5$ and initial guess of zero.
e) Compute the solution by 4 iterations of conjugate gradient method.

**Problem 3.2 (10 points): Polynomial Interpolation**

Consider the below (x,y) pairs:

$$x = \begin{bmatrix} -2 \\ 0 \\ 1 \\ 2 \end{bmatrix}, \quad y = f(x) = \begin{bmatrix} 2 \\ 0 \\ 1 \\ -2 \end{bmatrix}$$

a) Find the Lagrange polynomial for above points.
b) Interpolate that polynomial at x=-1.
c) Find the ordered polynomial for above points with Newton's formula.

d) Interpolate the ordered polynomial at x=-1.
e) Find the 3rd order interpolating polynomial with forming a linear system of equations.
f) Interpolate the above polynomial at x=-1.

**Problem 3.3 (35 points): Streamlines**

For a uniform inviscid flow passing a sphere with radius "R", the potential field is given by:

$$\phi(r,\theta) = U(r + \frac{R^3}{2r^2})\cos(\theta)$$

Here U is the far field velocity and the far field pressure is zero.

a) Find the velocity field.
b) Find the tangential and normal acceleration of fluid particles.
c) Find the analytical form of the streamline that passes through arbitrary point of $(r_0,\theta_0)$. Simplify the relation for the case when $\theta_0 = \frac{\pi}{2}$.
d) Derive an analytical differential equation for the distance increment $ds = ds(r,\theta)$ traveled by a particle fluid at a given position from its velocity components. Note that "s" is the path length traveled by fluid particle.

Now do the following for $r_0 = 1.01R, 1.1R, 1.5R, 3R$ :

e) Integrate the streamline differential equation as well as the path length differential equation. For the integration use the fixed step size $\Delta\theta = 0.05$ and continue as long as $\theta \le \pi - \Delta\theta$ *and* $r \le 10R$. Assume that $s(\frac{\pi}{2}) = 0$.
f) Plot the analytical form of streamlines as well as the numerical form obtained in previous part.
g) Plot both "r($\theta$)" and "s($\theta$)" for each streamline.
h) Fit a series of splines to your s($\theta$) discrete points computed at part "e". Then differentiate your fit two times to compute the tangential acceleration and compare it with the analytical value at the same $\theta$.

**Problem 3.4 (60 Points): Textbook problems**

Solve the below problems from "Chapara and Canale" textbook. Note that you can use MATLAB functions whenever possible.

- 11.18, 11.20
- 13.8,13.9, 13.11
- EXTRA CREDIT: 13.19 (5 Points)

- 14.8, 14.12
- 17.12, 17.29
- 18.4
- EXTRA CREDIT: 18.9 (5 Points)
- 19.18
- 20.19
- 21.7
- 22.9 only part b, 22.13
- 23.19
- EXTRA CREDIT: 23.26 (5 Points)