Review of last lecture

1. ε_λ' , ε_λ , ε' , ε
2. α_λ' , α_λ , α' , α
3. ρ_λ'' , $\rho_\lambda'^{\cap}$, $\rho_\lambda^{\cap\prime}$
   τ_λ''

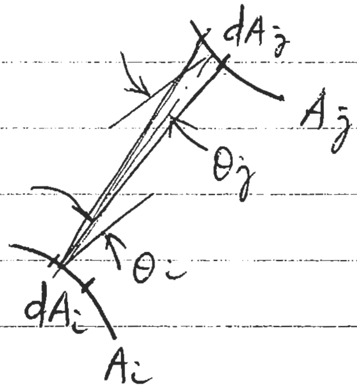
diffuse-gray surface
- emitter
- reflector

4. Energy balance : $\rho + \alpha + \tau = 1$

$$\rho_\lambda'^{\cap} + \alpha_\lambda' + \rho_\lambda'^{\cap} = 1$$

5. Kirchoff's Law $\alpha_\lambda' = \varepsilon_\lambda'$

6.



diffuse-gray surfaces

$$F_{dA_i-dA_j} = \frac{\cos\theta_i \cos\theta_j}{\pi S^2} dA_j .$$

$$= \frac{\text{power intercepted by } dA_j}{\text{total power leaving } dA_i}$$

$$dF_{dA_i - dA_j} = \frac{\cos\theta_i \cos\theta_j}{\pi S^2} dA_j.$$

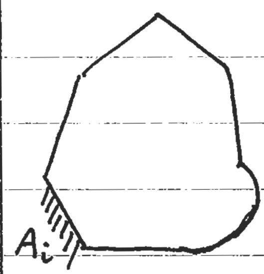
$$\Rightarrow \quad dA_i \, dF_{dA_i - dA_j} = dA_j \cdot dF_{dA_j - dA_i}$$

law of reciprocity.

$$F_{dA_i - A_j} = \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi S^2} dA_j$$

$$F_{A_i - A_j} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi S^2} dA_i \, dA_j.$$

$$A_i F_{A_i - A_j} = A_j F_{A_j - A_i}$$

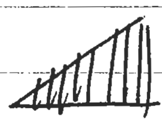


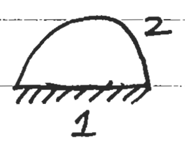
$$\sum_{j=1}^{n} A_i F_{A_i - A_j} = 1 \Rightarrow \sum_{j=1}^{n} F_{A_i - A_j} = 1$$

summation rule.

- Evaluation of View factors

Simple cases



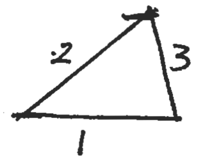


$F_{12} = 1$

$A_2 F_{21} = A_1 F_{12}$

$F_{21} = \frac{A_1}{A_2}$

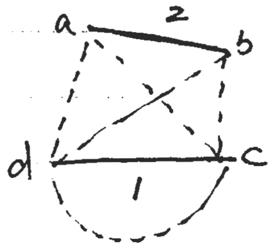
$F_{22} = 1 - F_{21} = 1 - \frac{A_1}{A_2}.$



$$F_{12}+F_{13}=1 \quad , \quad A_1F_{12}=A_2F_{21}$$
$$F_{21}+F_{23}=1 \quad , \quad A_2F_{23}=A_3F_{32}$$
$$F_{31}+F_{32}=1 \quad , \quad A_1F_{13}=A_3F_{31}$$

Solve :

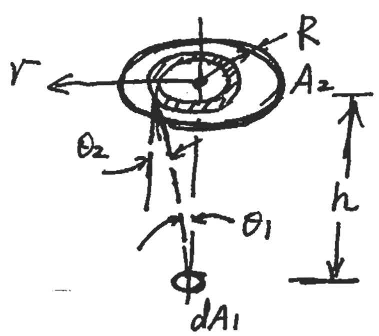
$$F_{12}=\frac{A_1+A_2-A_3}{2A_1}$$



↑ of concave

$$F_{12}=\frac{(ac+bd)-(ad+bc)}{2A_1}$$ Hottel string rule.

Direct Integration :



$$F_{12}=\int_{A_2}\frac{\cos\theta_1\cos\theta_2}{\pi S^2}\,dA_2$$

$$dA_2=2\pi r\,dr.$$

$$\cos\theta_1=\cos\theta_2=\frac{h}{\sqrt{h^2+r^2}}$$

$$F_{12}=\int_0^R\frac{h^2}{\pi(h^2+r^2)^2}\,2\pi r\,dr$$

$$=h^2\int_0^R\frac{dr^2}{(h^2+r^2)^2}$$

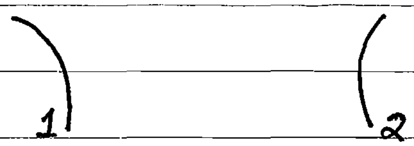
$$=h^2\left(-\frac{1}{h^2+r^2}\right)_0^R=h^2\left(\frac{1}{h^2}-\frac{1}{h^2+R^2}\right)$$

$$=\frac{R^2}{h^2+R^2}$$

Table

Numerical

- Radiation Heat Transfer Between Black surfaces

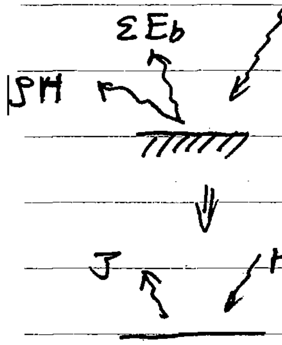


$$Q_{1\to 2} = F_{12}\ A_1 E_{b1}(T_1)$$
$$Q_{2\to 1} = F_{21}\ A_2 E_{b2}(T_2)$$

$$Q_{12} = Q_{1\to 2} - Q_{2\to 1}$$
$$= A_1 F_{12}\, \sigma T_1^4 - A_2 F_{21}\, \sigma T_2^4$$
$$= A_1 F_{12}\, \sigma (T_1^4 - T_2^4)$$

Sanity check $T_1 = T_2$ $Q_{12} = 0$

- Diffuse-gray surfaces : Radiosity & Irradiation



H (all incoming per unit area, isotropic) (sometime q^-)

radiosity $J = \varepsilon E_b + \rho H$

$(J = \pi I)$ (q^+)

Surface heat flux

$$q = J - H = \varepsilon E_b - \alpha H.$$

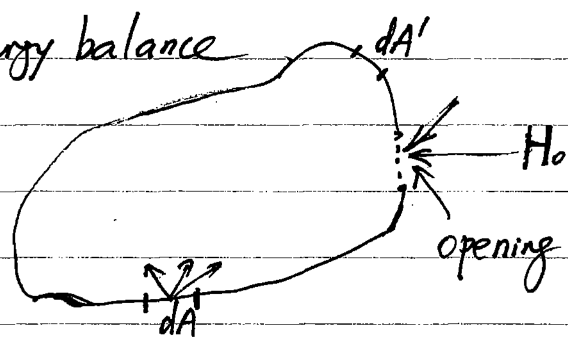
viewed from outside — view from inside

$H = J - q$

$q = \varepsilon E_b - \alpha J + \alpha q$

$$\boxed{q = \frac{\varepsilon(E_b - J)}{1-\varepsilon}}$$

- Energy balance



← viewed from immediate inside

$$q(\vec{r}) = \varepsilon(\vec{r})E_b(\vec{r}) - \alpha(\vec{r})H(\vec{r})$$
$$= J(\vec{r}) - H(\vec{r})$$

$\Rightarrow J(\vec{r}) = \varepsilon(\vec{r})E_b(\vec{r}) + \rho(\vec{r})H(\vec{r})$

$q(\vec{r}) = \frac{\varepsilon(\vec{r})}{1-\varepsilon(\vec{r})}[E_b - J]$

L viewed from outside of surface

$$dA\, H(\vec{r}) = \int J(\vec{r}')\, dF_{dA'-dA} \cdot dA' + H_o(\vec{r})\, dA$$

L external arriving at dA

$\Downarrow$

$dF_{dA-dA'} \cdot dA$

$\Rightarrow$

$$H(\vec{r}) = \int_A J(\vec{r}')\, dF_{dA-dA'} + H_o(\vec{r})$$

$$q(\vec{r}) = \varepsilon(\vec{r})E_b(\vec{r}) - \alpha(\vec{r})\left[\int_A J(\vec{r}')\, dF_{dA-dA'} + H_o(\vec{r})\right]$$

or

$$J(\vec{r}) = \varepsilon(\vec{r})E_b(\vec{r}) + \rho(\vec{r})\left[\int_A J(\vec{r}')\, dF_{dA-dA'} + H_o(\vec{r})\right]$$

$\Uparrow$ if $T(\vec{r})$ is known.

$\Uparrow$ Integral equation to solve for $J(\vec{r})$

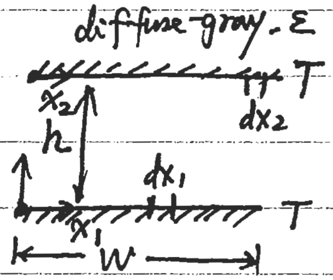
If $E_b(\vec{r})$ is unknown, but $q(\vec{r})$ is known, substitute $q = \frac{\varepsilon(E_b - J)}{1-\varepsilon}$.

$$J(\vec{r}) = q(\vec{r}) + \int_A J(\vec{r}')\, dF_{dA-dA'} + H_o(\vec{r})$$

We can also eliminate $J(\vec{r})$ $\Rightarrow$

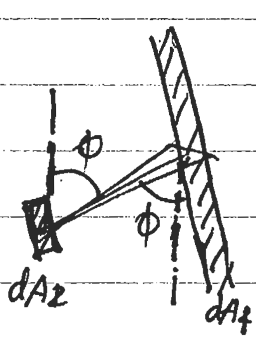
$$\frac{q(\vec{r})}{\varepsilon(\vec{r})} - \int_A \left(\frac{1}{\varepsilon(r')} - 1\right) q(\vec{r}')\, dF_{dA-dA'} + H_o(\vec{r})$$
$$= E_b(\vec{r}) - \int_A E_b(\vec{r}')\, dF_{dA-dA'}.$$

Example : local radiative flux $q(x)$



$$J_2(x_2) = \varepsilon \sigma T^4 + (1-\varepsilon)\int_0^W J_1(x_1)\, dF_{d2-d1}$$

Assuming no radiation from ambient entering the region (zero ambient).



$$\cos\phi = \frac{h}{\sqrt{h^2+(x_2-x_1)^2}}$$

$d\phi$ — for dF_{d2-d1}, dx_1 varying

$$dF_{d2-d1} = \frac{\cos\phi\, d\phi}{2}$$

$$d\phi = \frac{dx_1 \cos\phi}{\sqrt{h^2+(x_2-x_1)^2}}$$

$dx_1 \cos\phi$ projection of dx_1 into φ changing direction

$$J_2(x_2) = \varepsilon \sigma T^4 + (1-\varepsilon)\int_0^W \frac{h^2 J_1(x_1)}{2[h^2+(x_2-x_1)^2]^{\frac{3}{2}}}\, dx_1$$

Symmetry $J_1(x_1) = J_2(x_2)$

$$J_2(x_2) = \varepsilon \sigma T^4 + \frac{(1-\varepsilon)h^2}{2}\int_0^W \frac{J_2(x_1)}{[h^2+(x_2-x_1)^2]^{\frac{3}{2}}}\, dx_1$$

$$\xi = \frac{x}{h}, \quad j = \frac{J}{\sigma T^4}, \quad W = w/h$$

$$j(\xi) = \varepsilon + \frac{1-\varepsilon}{2}\int_0^W \underbrace{\frac{1}{[1+(\xi'-\xi)^2]^{\frac{3}{2}}}}_{\text{Kernel}}\, j(\xi')\, d\xi'$$

Integral equation

Fredholm equation of the 2nd kind

(1st kind ε=0, no extra term)

If $\varepsilon_1 \neq \varepsilon_2$, or $T_1 \neq T_2$, $\Rightarrow$ 2 coupled integral equations.

Solution method for integral equations

(1) Successive approximation
(2) variational calculus
(3) Kernel approximation
(4) Numerical quadrature

Numerical quadrature

$$\int_a^b f(\xi, \xi')d\xi' \approx (b-a) \sum_{j=1}^{N} w_j f(\xi, \xi_j)$$

weight coefficient
(quadrature coefficient)

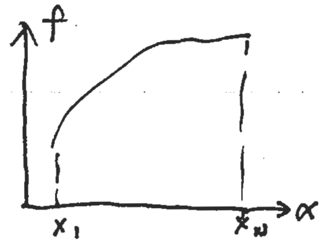
Apply to our equation

$$j(\xi) = \varepsilon + \frac{1-\varepsilon}{2} \varpi \sum_{j=1}^{N} w_j K(\xi, \xi_j) j(\xi_j)$$

Next set $\xi = \xi_1, \xi_2, \cdots, \xi_N$. $\Rightarrow$ a set of N equations to solve. $\Rightarrow$ Matrix inversion

Simplest.

Trapezoidal rule $\int_a^b f(x)dx = h\left[\frac{1}{2}f_1 + f_2 + \cdots + f_{N-1} + \frac{1}{2}f_N\right] + O(N^{-2})$



Simpson rule $-$ $O(N^{-4})$

equally spaced points

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Guassian Quadrature: both weight w_j & x_j

good if f is well approximated by polynomial

$$\int_a^b W(x) f(x)\, dx \approx \sum_{j=1}^{N} w_j f(x_j)$$

known functi — exact if $f(x)$ is polynomial of order N

Gauss-Legendre $W = 1$, $-1 < x < 1$

$$w_j = \frac{2}{(1-x_j^2)\,[P_N'(x_j)]^2}$$

$$(j+1) P_{j+1} = (2j+1)\, x P_j - j P_{j-1}$$

P_j — Legendre polynomial

Kernal Ill behaved:

at $\xi = \xi'$, $K \uparrow \infty$

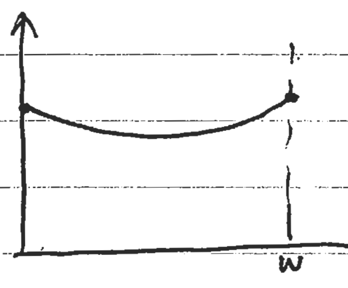
$$g(x) = f(x) + \int_a^b K(x, x')\, g(x')\, dx'$$

$$g(x) = f(x) + \int_a^b K(x, x')\,[g(x') - g(x)]\, dx'$$

$$+\, g(x) \underline{\int_a^b K(x, x')\, dx'}$$

can be integrated analytically or Numerically.

back to the problem



more heat loss at 2 edges.