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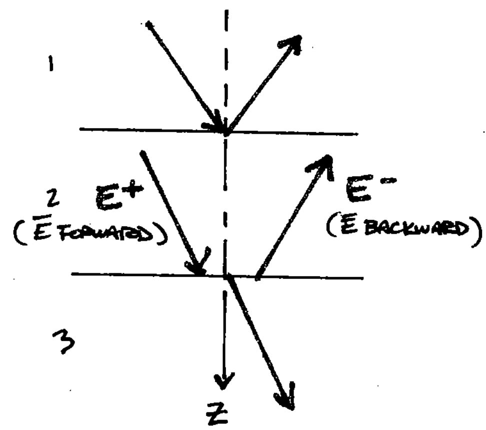
LAST TIME:



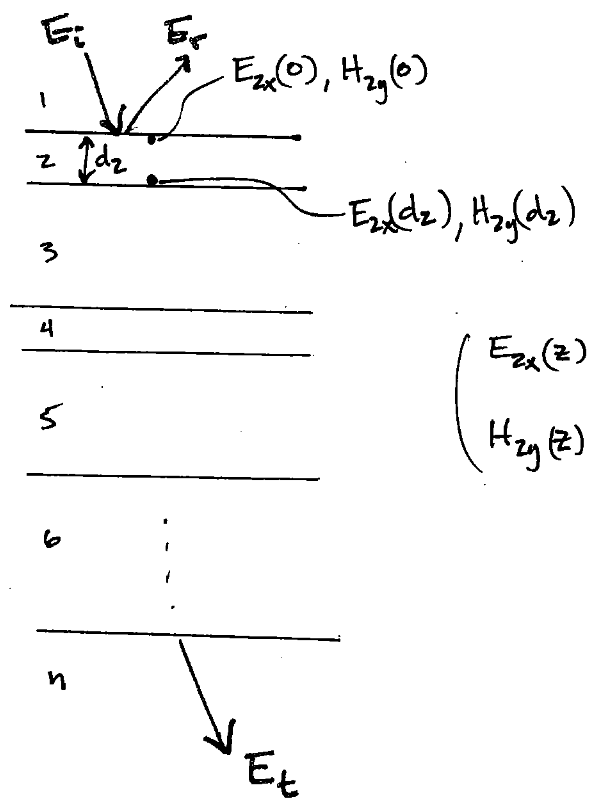
"RESULTANT WAVE METHOD"

$\bar{E}$ TANG ACROSS INTERFACE
$\bar{H}$ " " "

CAN WRITE $\bar{H}$ IN TERMS OF $\bar{E}$

$$r = \frac{r_{12} + r_{23} e^{2i\phi_2}}{1 + r_{12} r_{23} e^{2i\phi_2}} \quad ; \quad \phi_2 = \frac{2\pi \cos\theta_2 N_2 d_2}{\lambda_0}$$

FOR MULTIPLE LAYERS, TRANSFER MATRIX METHOD IS EASIEST



$$\begin{pmatrix} E_{2x}(z) \\ H_{2y}(z) \end{pmatrix} = \begin{pmatrix} \cos(\phi(z)) & i p_2 \sin(\phi(z)) \\ \frac{i}{p_2} \sin(\phi(z)) & \cos\phi(z) \end{pmatrix} \begin{pmatrix} E_{2x}(0) \\ H_{2y}(0) \end{pmatrix}$$

$$\begin{pmatrix} E_{2x}(0) \\ H_{2y}(0) \end{pmatrix} = \begin{pmatrix} \cos\phi_2 & -ip_2 \sin\phi_2 \\ -\frac{i}{p_2}\sin\phi_2 & \cos\phi_2 \end{pmatrix} \begin{pmatrix} E_{2x}(d_2) \\ H_{2y}(d_2) \end{pmatrix} = \mathbb{M}_2 \begin{pmatrix} E_{2x}(d_2) \\ H_{2y}(d_2) \end{pmatrix}$$

$$A \begin{pmatrix} E_i \\ E_r \end{pmatrix}$$

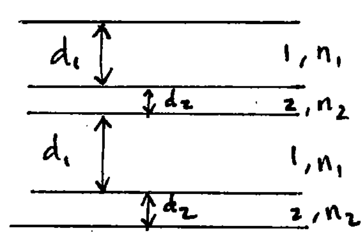
$$= \mathbb{M}_2 \mathbb{M}_3 \begin{pmatrix} E_{3x}(d_3) \\ H_{3y}(d_3) \end{pmatrix}$$

WE KNOW

$$\left.\begin{aligned} E_{3x}(0) &= E_{2x}(d_2) \\ H_{3y}(0) &= H_{2y}(d_2) \end{aligned}\right\} \text{MATCH BCs.}$$

$$= \mathbb{M}_2 \cdots \mathbb{M}_n \cdot \mathbb{B} E_t$$

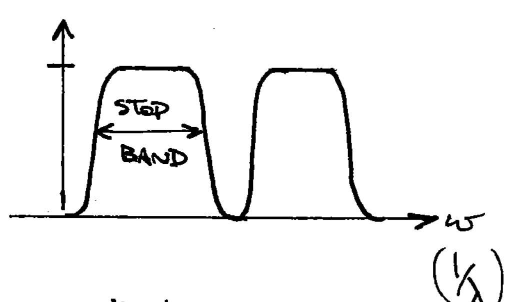
HOW THIN OR THICK CAN THE LAYERS BE?



$$\Rightarrow \phi_2 = \frac{4\pi n_2 \cos\theta_2 d_2}{\lambda_0} = \pi \cdot m$$

$$\cos\theta_2 d_2 = \frac{\lambda_0}{4n_2} \quad \text{FOR } m =$$

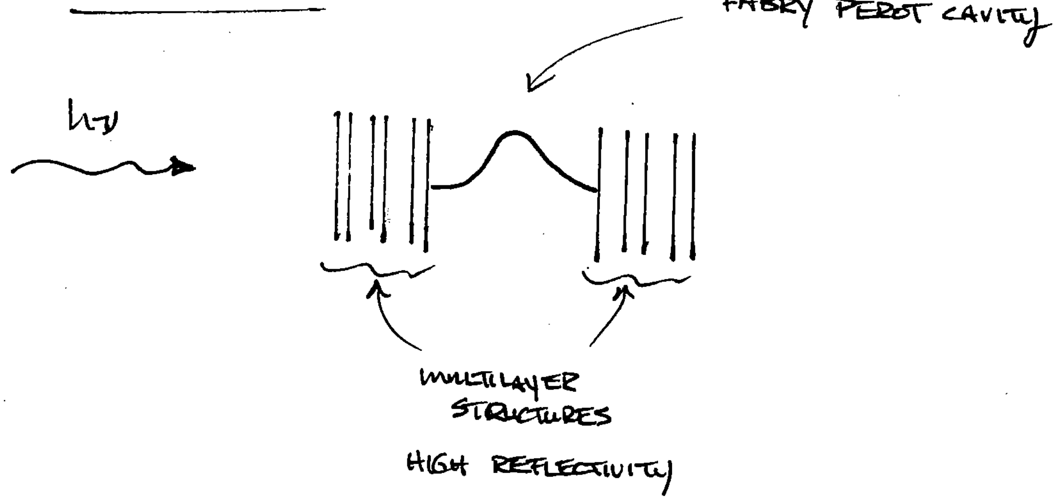
... GET A BAND~~PASS~~



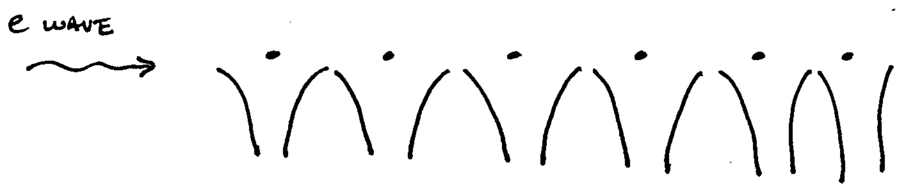
n_1, n_2

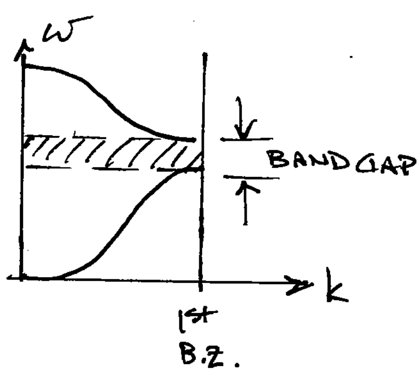
$$R = \left| \frac{n_1 - n_2}{n_1 + n_2} \right|^2$$

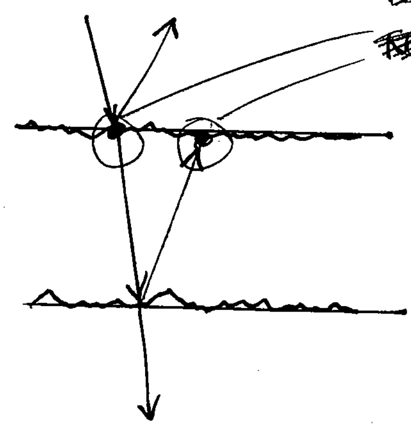
BRAGG REFLECTOR



ELECTRONS IN SOLIDS







FOR $\lambda = 0.5\mu m$ $\Rightarrow$ ~ 1000 periods IN THE WINDOW

∴ • ALL RAYS WILL NO LONGER BE //

• PHASES OF ALL SINUSOIDS WILL BE SLIGHTLY DIFFERENT

$\Rightarrow$ SUPERPOSITION OF SINUSOIDS OF ~~VARIES~~ VARIOUS PHASES WILL BE ZERO

$\Rightarrow$ NEGLECT PHASE INFORMATION

★ IF STRUCTURE ~~IS~~ IS MUCH LARGER THAN λ OR IF SURFACE IS "ROUGH", WE CAN ASSUME PHASE INFO IS LOST AND NEGLECT WAVE EFFECTS.

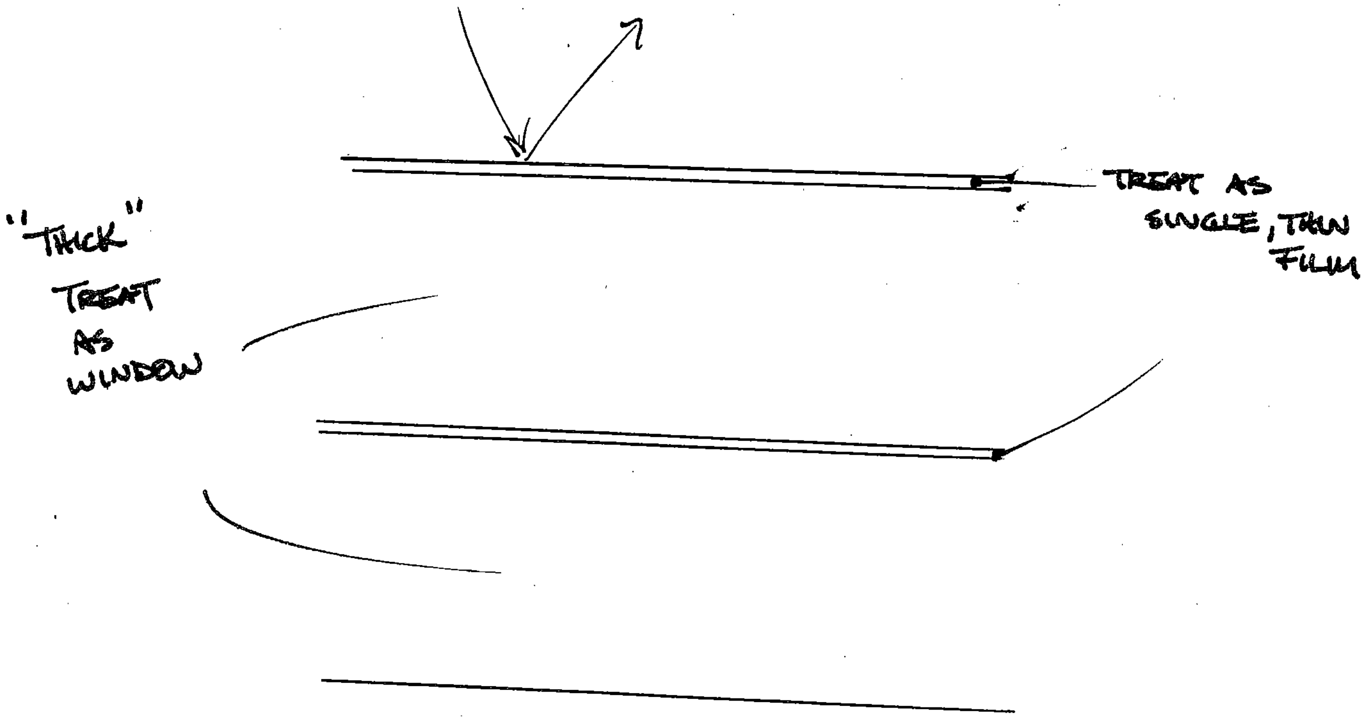
$\boxed{h \sim \frac{1}{10\lambda}}$ RULE OF THUMB   WHAT? WRONG UNITS!

** $\frac{1}{4\lambda}$ IS NOT enough

$h \sim 10\lambda$ i.e., $h \gg \lambda$

WHAT IF THIN FILM & THICK FILM SYSTEM

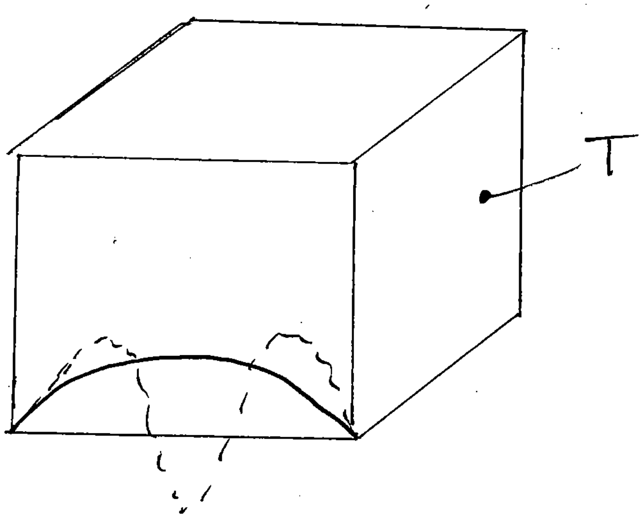
⟹ COMBINE SINGLE LAYER FILM AND WINDOW FORMULA



PLANCK'S LAW DERIVATION

WAVES IN A CAVITY, FOR HOMOG. BC'S

$\Longrightarrow$ STANDING WAVE SOLUTIONS



$$k_x^2 + k_y^2 + k_z^2 = \frac{\omega^2}{c^2}$$

$$k_x = \pm \frac{2\pi}{\lambda_x} = \pm \frac{2\pi n_x}{L_x}$$

$$n_x \lambda_x = L_x$$

$$k_y = \pm \frac{2\pi n_y}{L_y}$$

$$k_z = \pm \frac{2\pi n_z}{L_z}$$

n CORRESPONDS TO VIBRATION "MODE"

3 dirs, SO MANY MODES CAN EXIST
AT SAME TIME FOR A GIVEN FREQ. (DEGENERACY)

FOR ONE MODE, ENERGY (AT FREQ ω)

$$E_n = \hbar\omega \cdot n \quad \text{(neglect zero point NRG)}$$

BOLTZMANN STATISTICS GIVES

$$P(E_n) = A \exp\left(\frac{-E_n}{k_B T}\right)$$

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GEOM. SERIES

$$\sum_{n=0}^{\infty} A \exp\left(\frac{-\hbar\omega n}{k_B T}\right) = 1$$

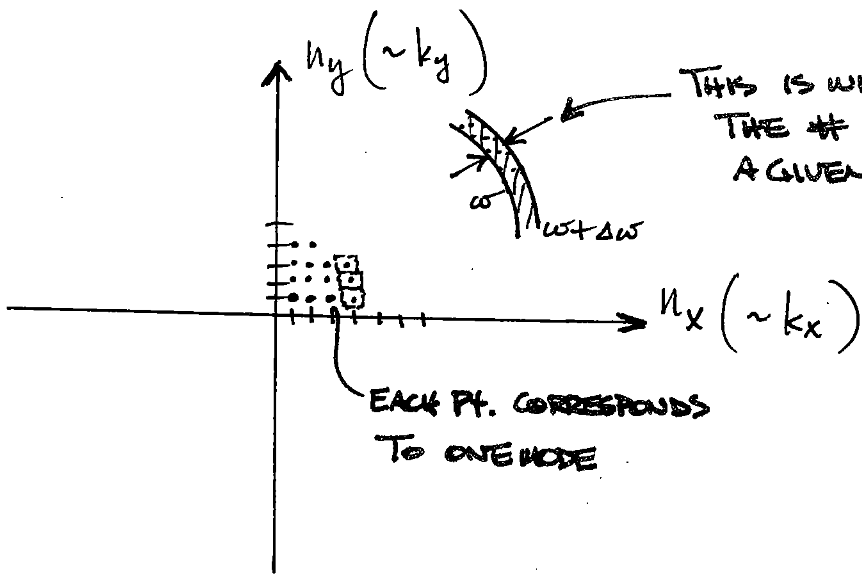
$$A = 1 - \exp\left(-\frac{\hbar\omega}{k_B T}\right)$$

$$\langle n \rangle = \sum n P_n(E_n) = \frac{1}{\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1}$$

" BOSE-EINSTEIN DIST. "

SO, HOW MANY MODES?

e.g., $\frac{2\pi n_x}{L_x} = \frac{\omega}{c} \longrightarrow n \sim \frac{\omega}{2\pi c} L_x$ ($\therefore$ n IS A "HUGE" #)



VOLUME OF ONE MODE $\frac{2\pi}{L_x} \cdot \frac{2\pi}{L_y} \cdot \frac{2\pi}{L_z} = \frac{8\pi^3}{V}$

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$\omega \rightarrow \omega + \Delta\omega$

TE & TM WAVE

$$\Delta N \equiv \#\text{ OF MODES} = \frac{\text{SHELL}}{\text{TOTAL VOL}} = \frac{\Delta k_x \Delta k_y \Delta k_z}{8\pi^3/V} \cdot 2$$

COORD CHANGE CARTESIAN → POLAR

$$\Delta k_x \Delta k_y \Delta k_z = 4\pi k^2 \Delta k$$

$$= \frac{V k^2 \Delta k}{\pi^2}$$

$k^2 = \frac{\omega^2}{c^2}$

$$\frac{\Delta N}{\Delta\omega} = \frac{V k^2}{\pi^2}\frac{\Delta k}{\Delta\omega} = \frac{V\left(\frac{\omega}{c}\right)^2}{\pi^2}\cdot\frac{1}{c} = \frac{V\omega^2}{\pi^2 c^3} = D(\omega)$$

ENERGY

$$U = \hbar\omega \langle n \rangle \cdot D(\omega) = \frac{\hbar\omega}{\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1} \cdot \frac{\omega^2}{\pi^2 c^3} \quad \left[\frac{J}{m^3}\right]$$

INTENSITY

$$I = \frac{d\,\text{POWER}}{d\lambda\, d\Omega\, dA} = \frac{\text{ENERGY} \cdot \text{SPEED}}{d\lambda\, d\Omega\, dA}$$

THUS

$$I_\omega = \frac{U \cdot c}{4\pi} = \frac{\hbar\omega^3}{c^2 4\pi^3 \left(e^{\hbar\omega/kT} - 1\right)}$$

$$I_\lambda = \frac{\Delta\lambda}{\Delta\omega} I_\omega$$

WHEN TO USE RAY VS. WAVE

λ VS. ROUGHNESS SIZE