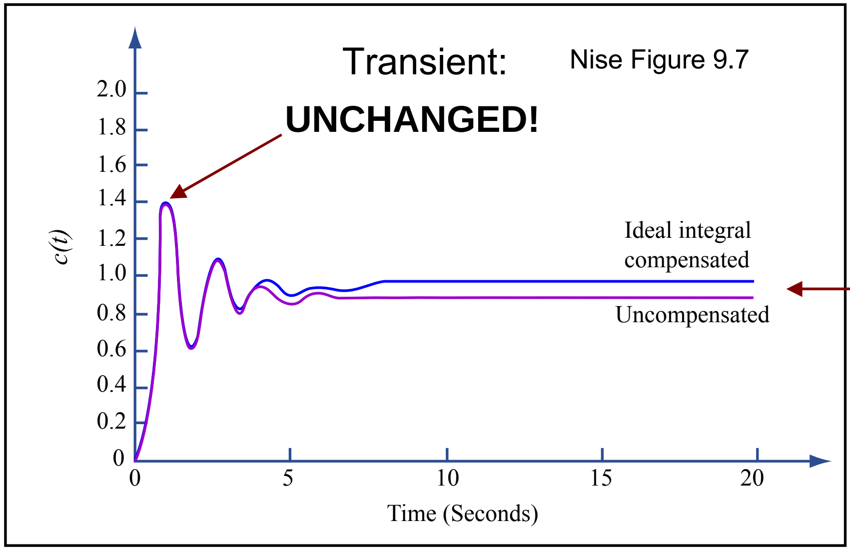
Today’s goals

- **So far**
  - Frequency domain
    - Bode plots
    - Stability
    - Transient and steady-state characteristics from frequency response
    - Proportional control in the frequency domain
- **Today**
  - Simultaneous control of transient and steady-state error characteristics: Proportional-Integral-Derivative (PID) controller
  - Design using root locus
- **Friday**
  - The PID controller in the frequency domain

Reminder #1: PI control

Image removed due to copyright restrictions.

Please see: Fig. 9.5 and 9.6 in Nise, Norman S. *Control Systems Engineering*. 4th ed. Hoboken, NJ: John Wiley, 2004.



Transient: Nise Figure 9.7

UNCHANGED!

Steady-state error=0

FIXED!

Nise Figure 9.6

Compensator TF:

zero near the integrator

$$G_c(s) = \frac{K(s+0.1)}{s}.$$

integrator

Figure by MIT OpenCourseWare.

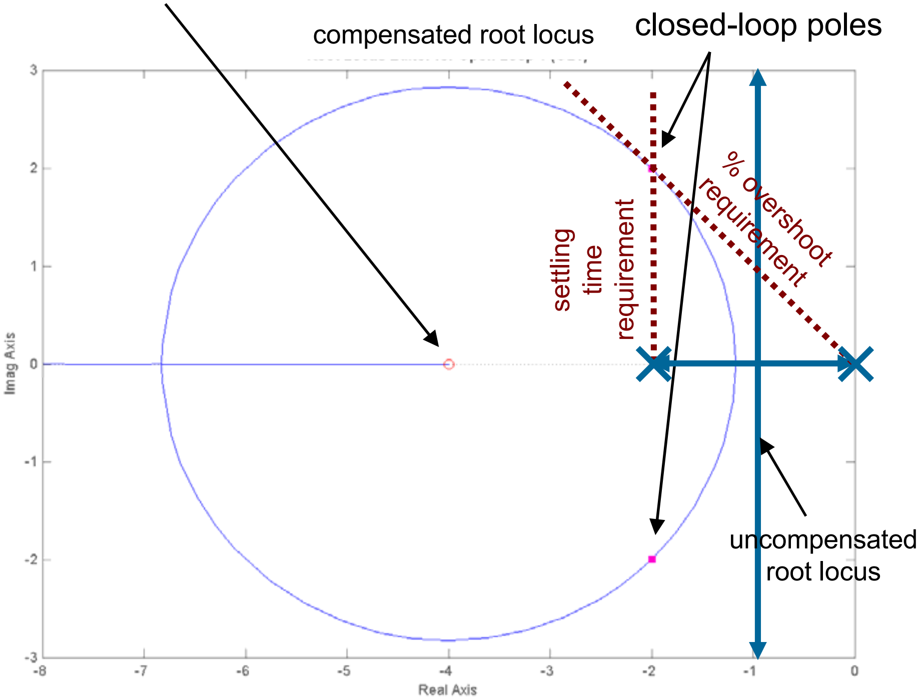
Reminder #2: PD control

Compensator TF:

$$G_c(s) = K(s+4).$$

zero location found by the requirement that the desired complex pole be on the root locus

PD compensator zero



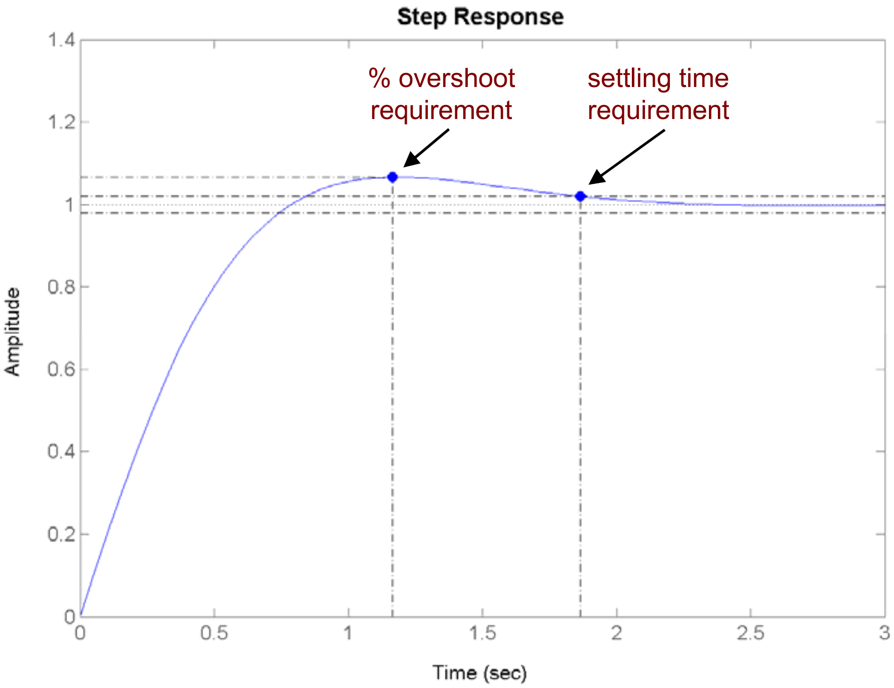
compensated root locus

closed-loop poles

settling time requirement

% overshoot requirement

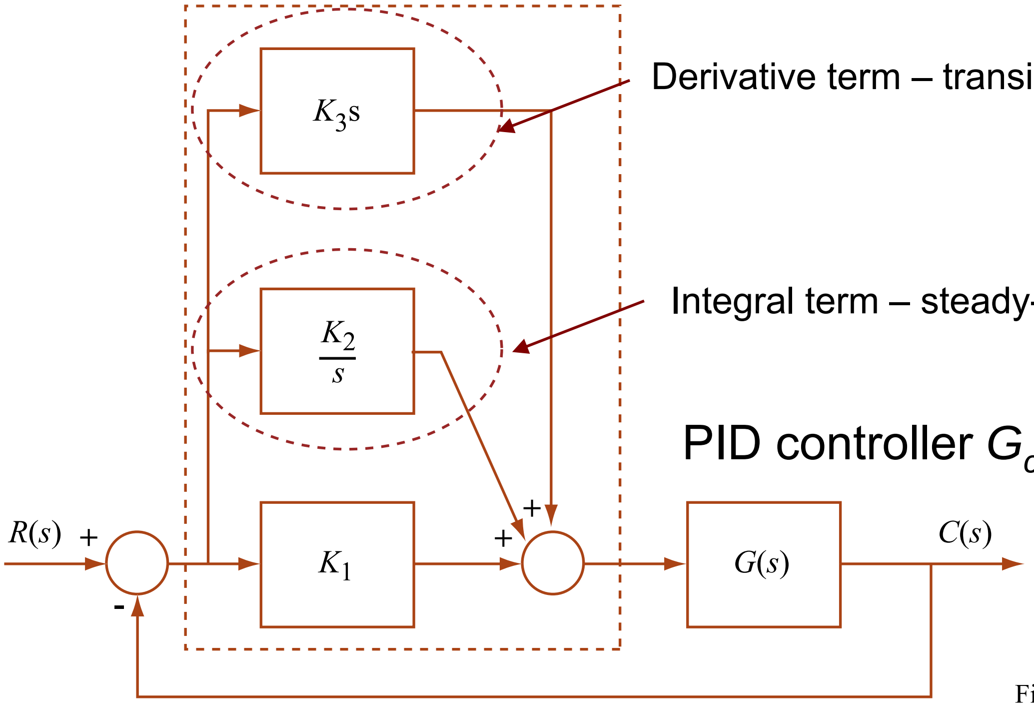
uncompensated root locus



% overshoot requirement

settling time requirement

Improving both transient response & steady-state error



Derivative term – transient response compensation

Integral term – steady-state error compensation

PID controller $G_c(s)$

Figure by MIT OpenCourseWare.

Figure 9.30

Integral term – steady-state error compensation

Derivative term – transient response compensation

$$G_c(s) = K_1 + \frac{K_2}{s} + K_3 s = \frac{K_1 s + K_2 + K_3 s^2}{s}.$$

Example

uncompensated
system

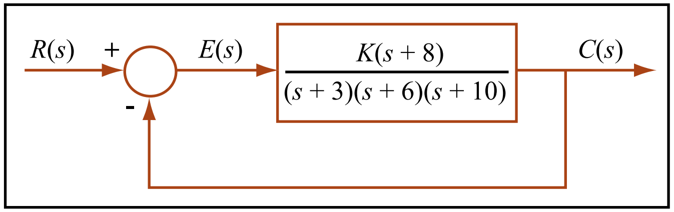


Figure by MIT OpenCourseWare.

Compensator requirements: for step input,
(1) maintain 20% overshoot
(2) reduce peak time to 2/3 of the uncompensated system's peak time;
(3) eliminate steady-state error

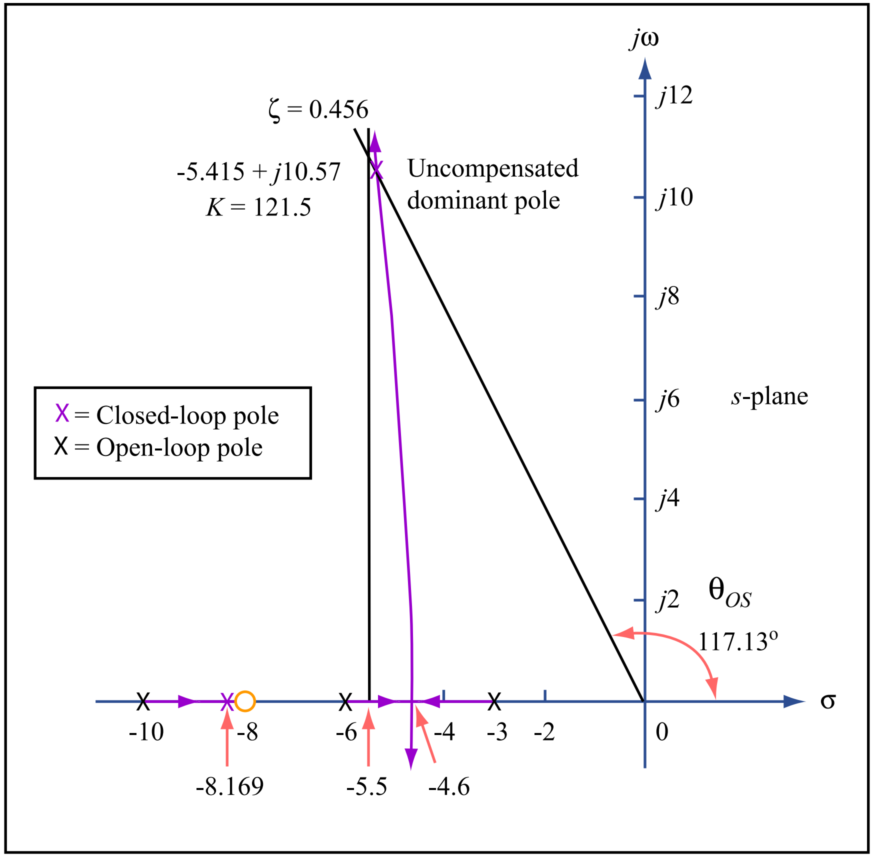


Figure by MIT OpenCourseWare.

Uncompensated system performance
(1) 20% overshoot ↔ $\zeta=0.456$ line crosses the RL;
proportional gain $K=121$ achieves overshoot target;
(2) $T_p=0.297$sec, must be reduced to ~0.2sec
(3) $e(\infty)=0.156$, must be reduced to 0

Example

uncompensated system

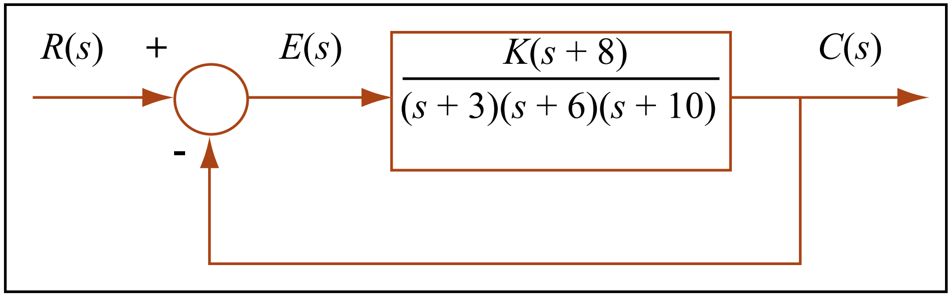


Figure 9.31

Figure by MIT OpenCourseWare.

We begin by compensating the system transient. Since we seek to make the system faster, we muse use PD compensation. The desired dominant pole location is found from the two requirements:

(1) peak time requirement⇒imaginary part

$$\omega_d = \frac{\pi}{T_d} \Rightarrow \omega_d = \frac{\pi}{0.2} \approx 15.87$$

(2) overshoot requirement⇒diagonal to s–plane's origin

$$\sigma_d = \frac{\omega_d}{\tan\theta_{OS}} = \frac{\omega_d}{\tan 117^\circ} = -8.13.$$

We find the necessary location of the compensator's zero by requiring that the resulting angular contributions to the desired dominant pole at $-8.13 \pm j15.87$ add up to 180°. The contributions from the system's open–loop poles and zeros add up to 198.37°; therefore, the compensator zero's contribution must be 18.37°. From the geometry (see figure on right)

$$\frac{15.87}{z_c - 8.13} = \tan 18.37^\circ \Rightarrow z_c = 55.92.$$

PD compensator design sketch

Image removed due to copyright restrictions.

Please see: Fig. 9.33 in Nise, Norman S. *Control Systems Engineering.* 4th ed. Hoboken, NJ: John Wiley, 2004.

Example

uncompensated system

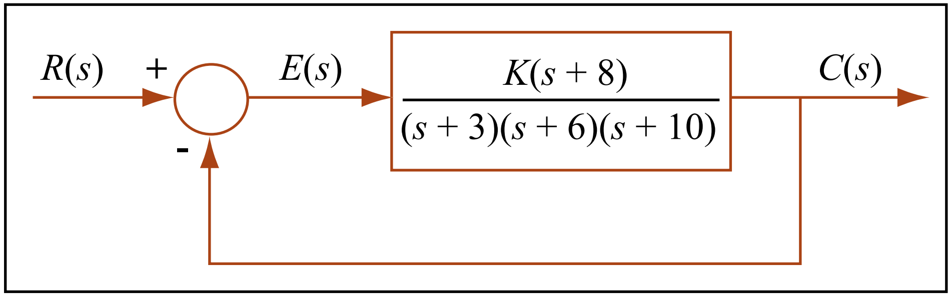


Figure 9.31

Figure by MIT OpenCourseWare.

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Please see: Fig. 9.34 and 9.36 in Nise, Norman S. *Control Systems Engineering*. 4th ed. Hoboken, NJ: John Wiley, 2004.

Root locus for PD-compensated system

New root locus after cascading an integral compensator to eliminate the steady-state error

$$K(s+55.92)\times\frac{(s+8)}{(s+3)(s+6)(s+10)}$$

$$K(s+55.92)\frac{s+0.5}{s}\times\frac{(s+8)}{(s+3)(s+6)(s+10)}$$

Example

uncompensated system

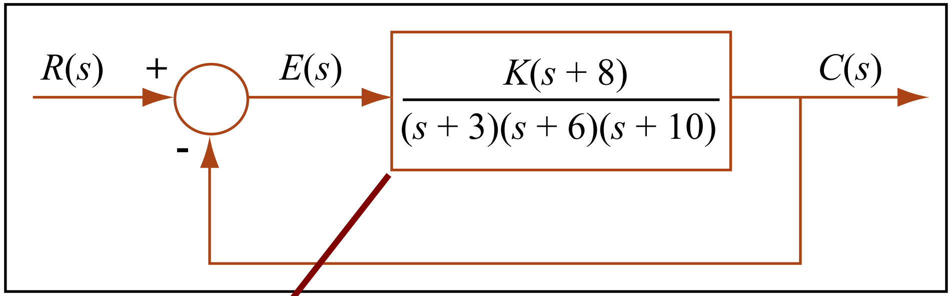

Figure 9.31

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PID compensator block diagram

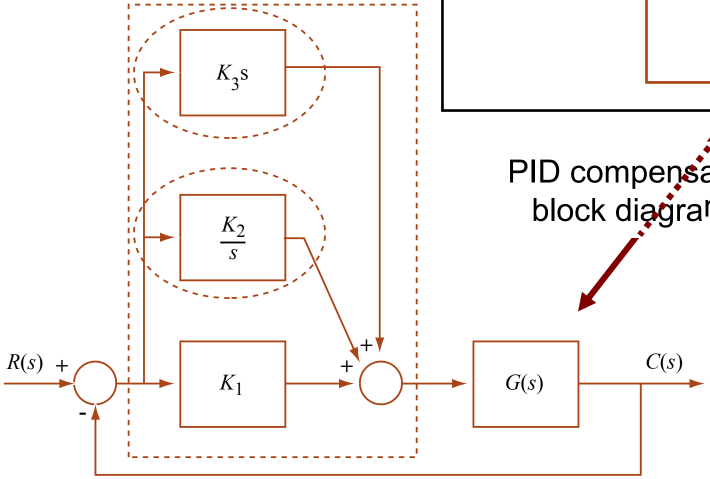

Figure by MIT OpenCourseWare.

Figure 9.30

PID compensator gains

$$K(s+55.92)\frac{s+0.5}{s} = \frac{4.6\left(s^2+56.42s+27.96\right)}{s}$$

$$\Rightarrow \begin{cases} K_1 = 259.5 \\ K_2 = 128.6 \\ K_3 = 4.6 \end{cases}$$

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Please see: Fig. 9.35 and 9.36 in Nise, Norman S. *Control Systems Engineering*. 4th ed. Hoboken, NJ: John Wiley, 2004.

Final PID compensated system root locus