Harvard-MIT Division of Health Sciences and Technology
HST.523J: Cell-Matrix Mechanics
Prof. Ioannis Yannas

Mechanical behavior of Tendon and Skin

1. The nonlinear mechanical behavior of tendon.
2. The extensibility of skin.

1. The nonlinear mechanical behavior of tendon.

Seven steps: Data to be explained. Rough geometry. Stresses. Strains. Detailed geometry. Constitutive equation. Limiting cases. Structure-function analysis.

Tendon connects muscle to bone and supports axial forces

Diagram illustrating extension and flexion removed for copyright reasons.

Vander et al., *Human Physiology*. McGraw-Hill. 1970

What is the orientation of 100-nm thin collagen fibers in 200-μm thick tendon fibers? Are they helically wound round axis of tendon fiber? Is their axis parallel to tendon fiber axis?

Photo removed for copyright reasons.

Tendon fibers

Yannas and Huang, 1970

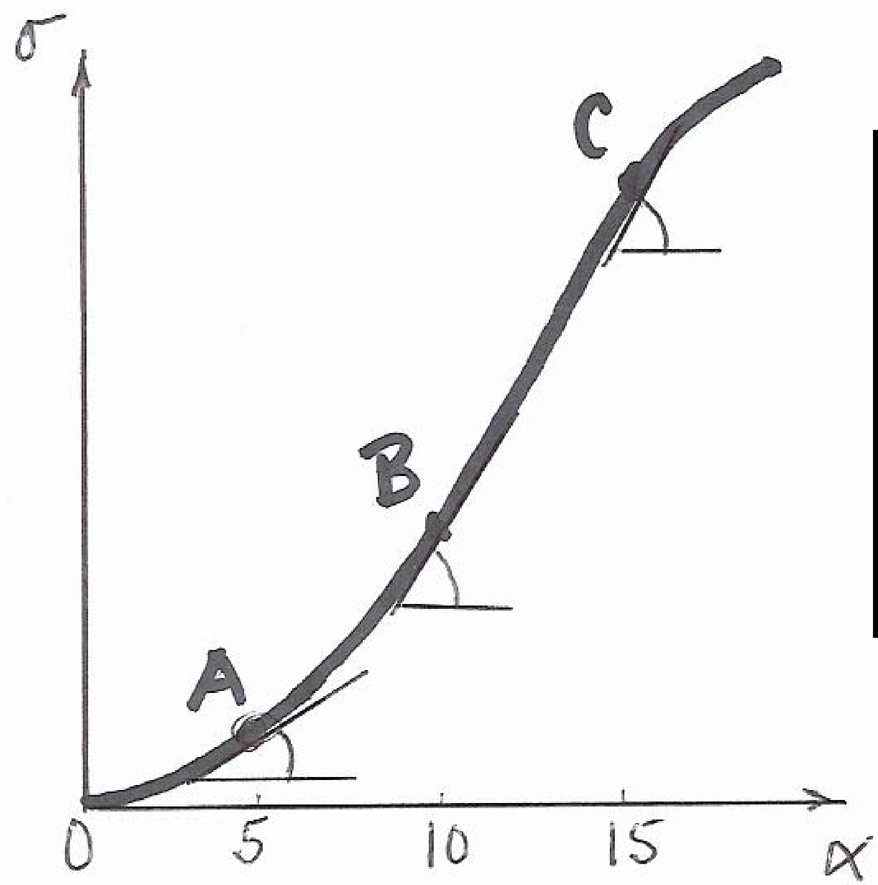
Detail of fracture surface for dry tendon showing parallel arrangement of fibers along major fiber axis. Individual fibers, about 100 nm diameter, are single crystals of collagen.

Photo removed for copyright reasons.

normal to
fracture surface

Yannas and Huang, 1970

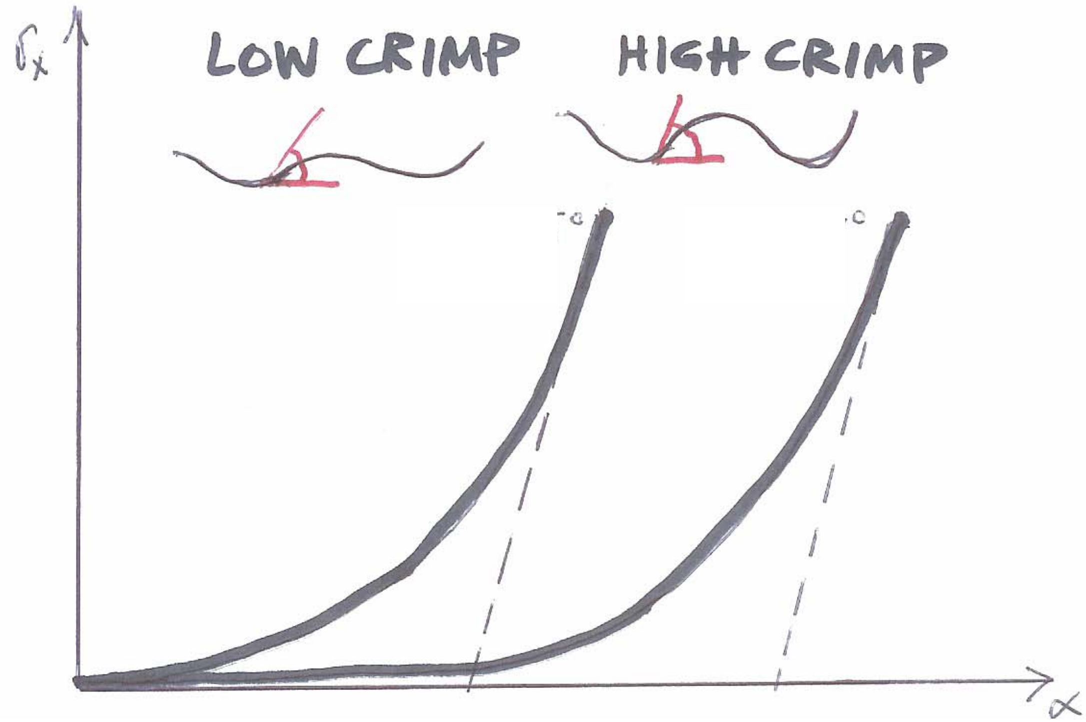
<u>Step 1</u>. Measure the nonlinear mechanical behavior of tendon. It needs to be explained.



POINT	STRAIN, α	E, Pa
O	~ 0	5×10^6
A	5	$\sim 10^7$
B	10	$\sim 10^8$
C	15	2×10^8

The curvature $\partial^2\sigma/\partial^2\varepsilon$ is positive (up to C where failure occurs)

Other data that need to be explained: The extensibility of tendon increases with the crimp angle ($\bar{\theta}_o$).



<u>Step 2</u>. The stresses. Select representative experimental volume. Tendon is made up of many individual collagen fibers. The axes of fibers are aligned along the direction of tendon axis. (Fibers are not helically twisted round tendon axis, as in rope!)

Therefore, the nonlinearity of tendon (several mm in diameter), measured in uniaxial tension, can be reproduced when a <u>single aligned fiber</u> (about 100 μm in diameter) is loaded in uniaxial tension.

$$\sigma_x = E\varepsilon_x$$

$$\sigma_y = \sigma_z = 0$$

$$\tau_{xy} = \tau_{xz} = \tau_{zx} = 0$$

Step 3. The strains. Summary of stresses and strains (so far) on collagen fiber in uniaxial tension

$\sigma_x = E\varepsilon_x$

$\sigma_y = 0$

$\sigma_z = 0$

$\tau_{xy} = \tau_{xz} = \tau_{zx} = 0$

$\varepsilon_x = (1/E)[\sigma_x - \nu(\sigma_y + \sigma_z)] = \sigma_x/E$ (axial strain)

$\varepsilon_y = (1/E)[\sigma_y - \nu(\sigma_x + \sigma_z)] = -\nu\sigma_x/E$

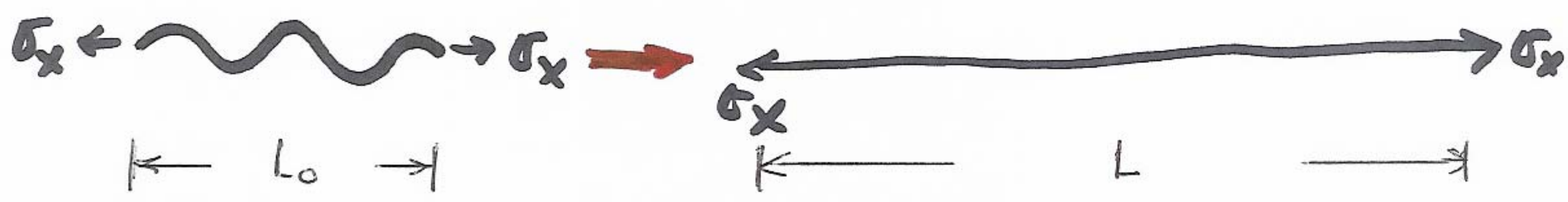
$\varepsilon_z = (1/E)[\sigma_z - \nu(\sigma_x + \sigma_y)] = -\nu\sigma_x/E$

$\gamma_{xy} = \gamma_{yz} = \gamma_{zx} = 0$

To account for crimped fiber geometry, define <u>two kinds of axial strain</u> for tendon

- Define an <u>apparent</u> strain (macroscopic strain):

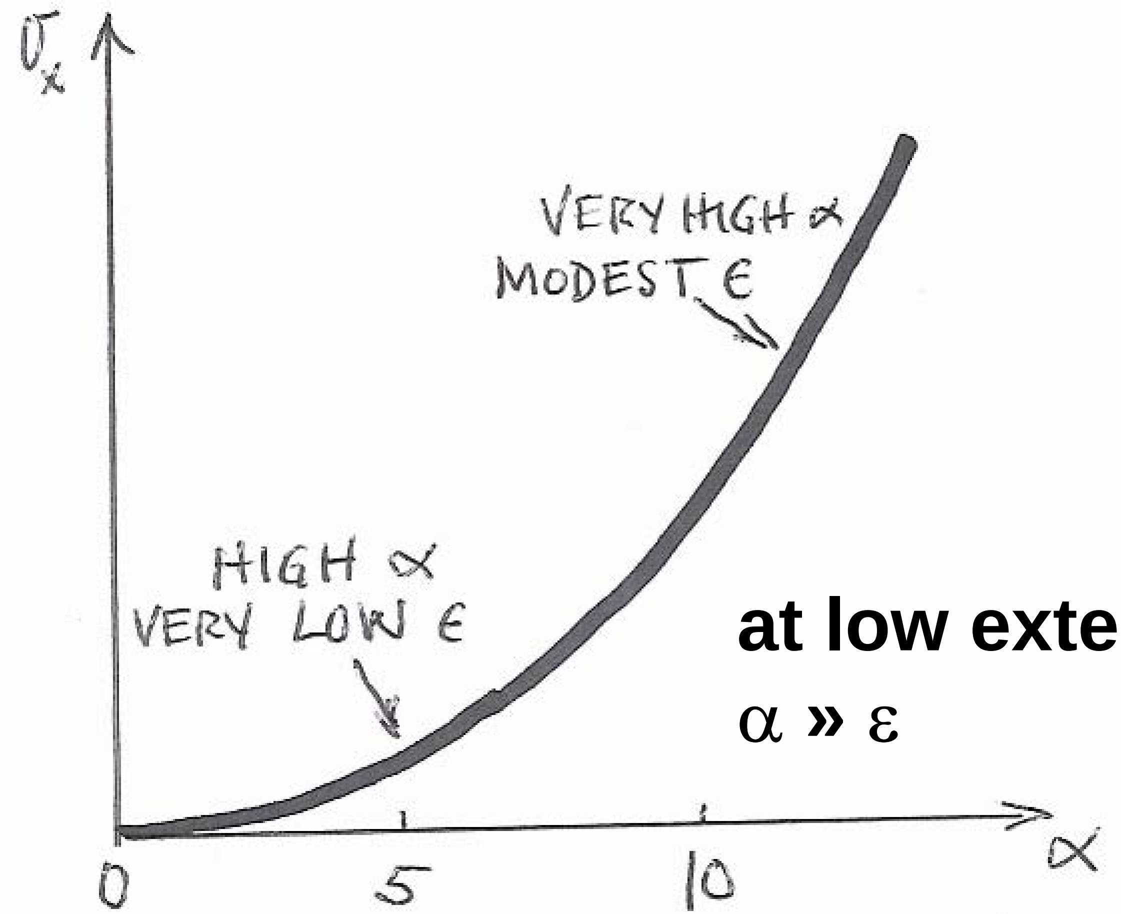
  $\alpha = (L - L_o)/L_o = \Delta L/Lo$



- and a <u>material</u> (microscopic) strain:

  $\varepsilon_x = \sigma_x/E$ (tendon fiber defined as a linear elastic material)

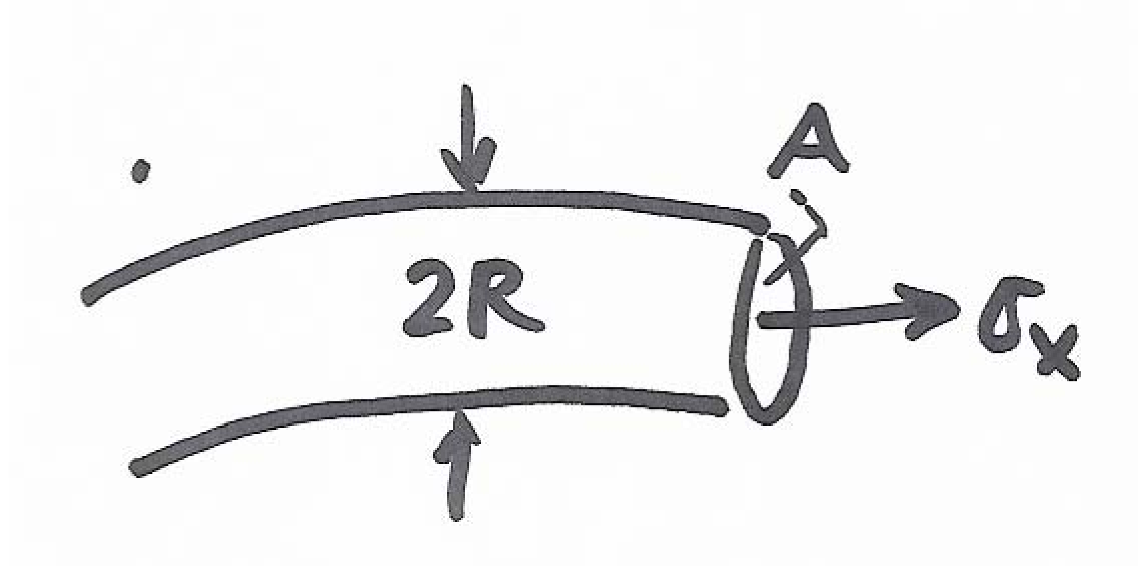
As tendon is stretched to 5% strain (walking strain on Achilles tendon), the apparent strain, α , increases rapidly while the material strain, ε, lags behind



at low extension

$\alpha \gg \varepsilon$

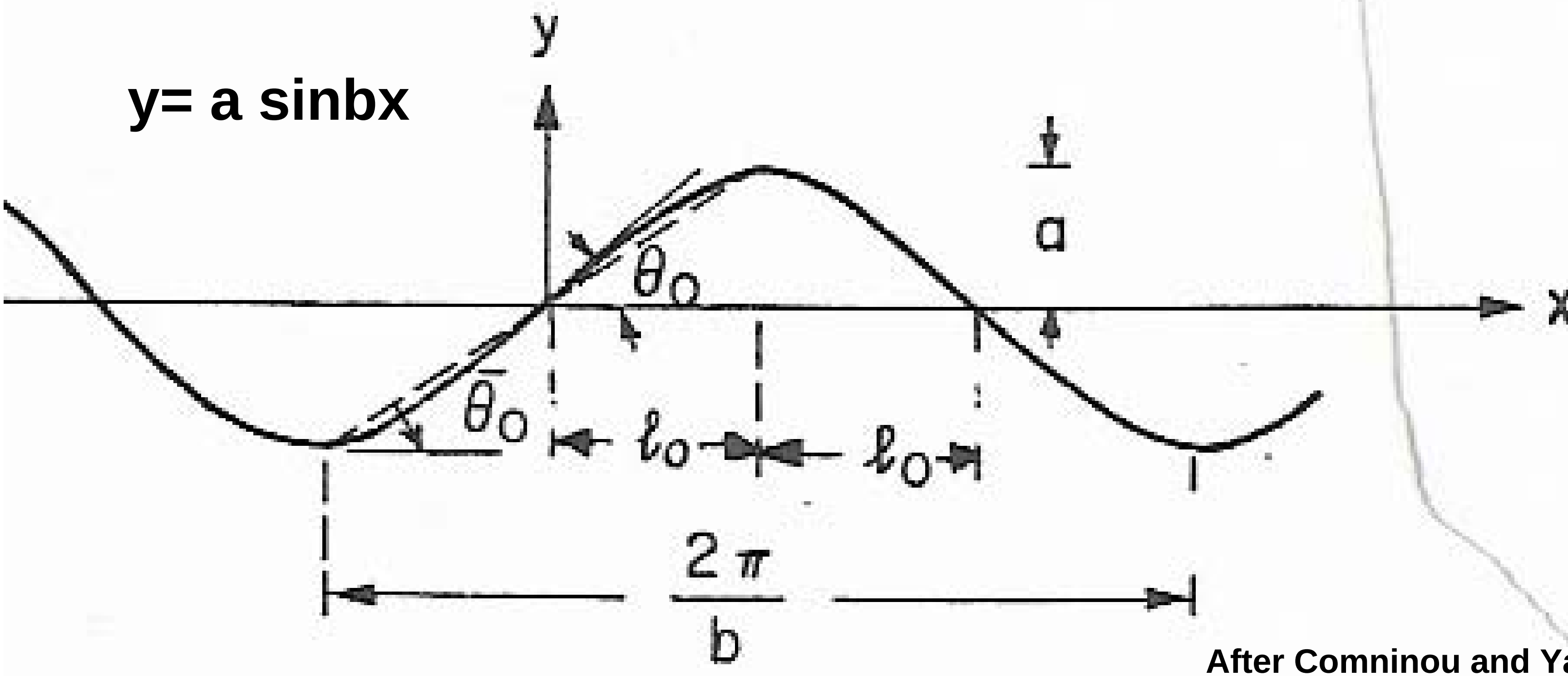
A tendon fiber in uniaxial tension



Young's modulus, E = σ_x/ε_x

moment of inertia, I = $R^2A/4$

bending rigidity, EI = $ER^2A/4$



After Comninou and Yannas, *J. Biomech*. , 1976

<u>Step 4</u>. Model fiber geometry as sine wave. The horizontal projection of the fiber is » λ

amplitude, a

frequency, $b = 2\pi/\lambda$

wavelength, $\lambda = 2\pi/b = 4l_o$

crimp angle, $\bar{\theta}_o$

$a/l_o = \tan \bar{\theta}_o$

$ab = 2\pi a/\lambda = (\pi/2)[\tan \bar{\theta}_o]$

Step 5: Derive constitutive equation for crimped structure of tendon fibers in uniaxial tension. Equation satisfies (a) conditions of equilibrium, (b) geometric compatibility and (c) Hookean behavior for the material (collagen): $\sigma_x = E\varepsilon_x$. Neglect all stresses except σ_x. [see detailed derivation of tendon model in handout from J. Biomechanics 9:427-433 (1976)].

The constitutive equation relates stress extending tendon, σ_x, to the apparent (uncrimping) strain, α:

$$\sigma_x = E\alpha - [Ea^2b^2/4]\cdot[\Lambda(\Lambda+2)/(\Lambda+1)^2]$$

where $\Lambda = 4\varepsilon_x/b^2R^2 = A\sigma_x/b^2EI$ **and** $I = R^2A/4$

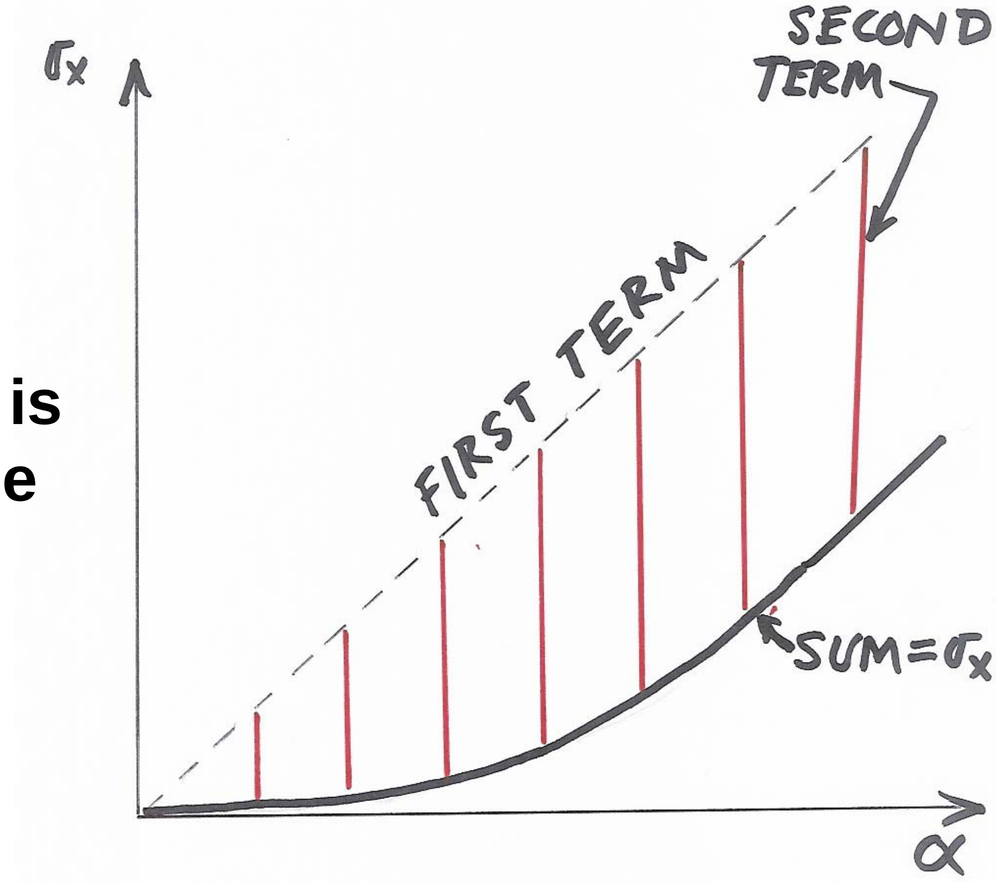
(notice that this nonlinear constitutive equation is an implicit function in σ_x; σ_x is present both on the left- and the right-hand side)

Graphic for $\sigma_x = E\alpha - [Ea^2b^2/4]\cdot[\Lambda(\Lambda+2)/(\Lambda+1)^2]$

Stress = first term – second term

where $\Lambda = 4\varepsilon_x/b^2R^2 = A\sigma_x/b^2EI$ and $I = R^2A/4$

Nonlinearity is entirely in the second term



Step 6. Examine limiting cases of the constitutive equation.

- **The constitutive equation is relatively complicated. Hard to tell at a glance what is happening to the extensibility when the crimp angle is increased.**
- **To get a simple picture, study constitutive equation at small stresses ($\Lambda \ll 1$) and large stresses ($\Lambda \gg 1$).**
- $\Lambda \propto \varepsilon_x = \sigma_x/E$

Small stresses. $\Lambda \ll 1$

$$\Lambda(\Lambda + 2)/(\Lambda + 1)^2 \cong 2\Lambda$$

$$\sigma_x = E\alpha - [Ea^2b^2/4][2\Lambda] =$$

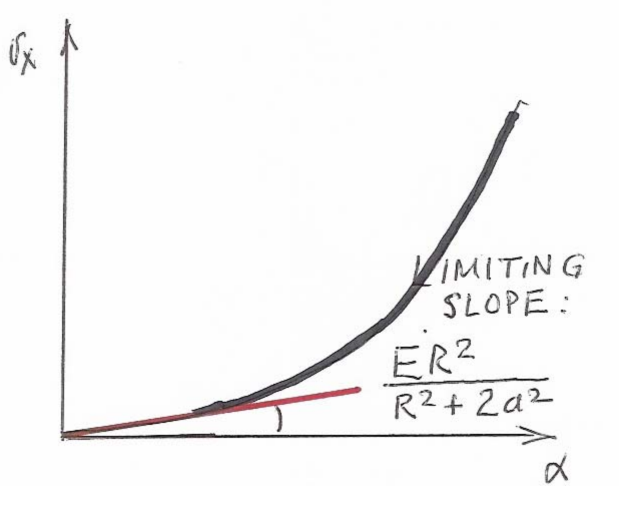
$$= E\alpha - [Ea^2b^2/4][8\varepsilon/b^2R^2] =$$

$$= E\alpha - (Ea^2/R^2)(2\sigma_x/E) =$$

$$= E\alpha - 2a^2\sigma_x/R^2$$

this is another implicit function in σ_x; solve for σ_x to get simpler expression:

$$\sigma_x = [ER^2/(R^2 + 2a^2)]\alpha$$

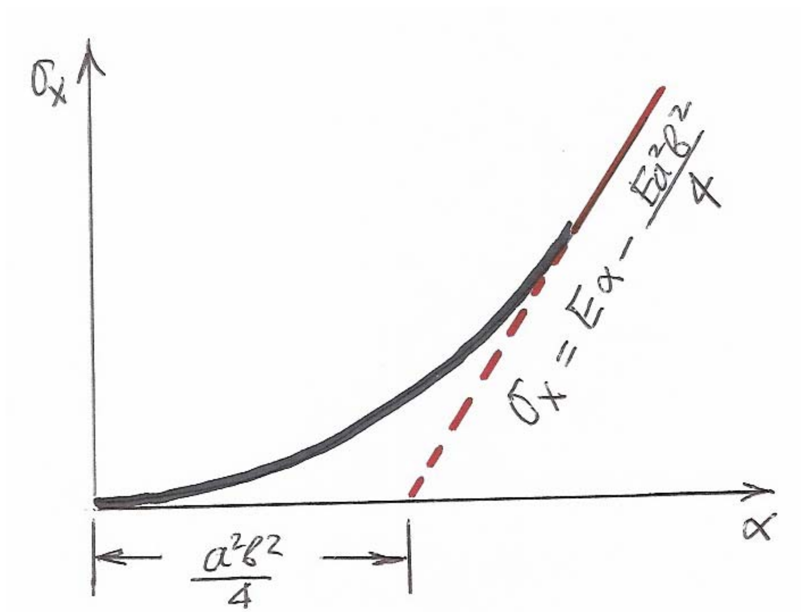


Large stresses. Λ » 1

$$\Lambda(\Lambda + 2)/(\Lambda + 1)^2 \cong \Lambda(\Lambda)/\Lambda^2 = 1$$

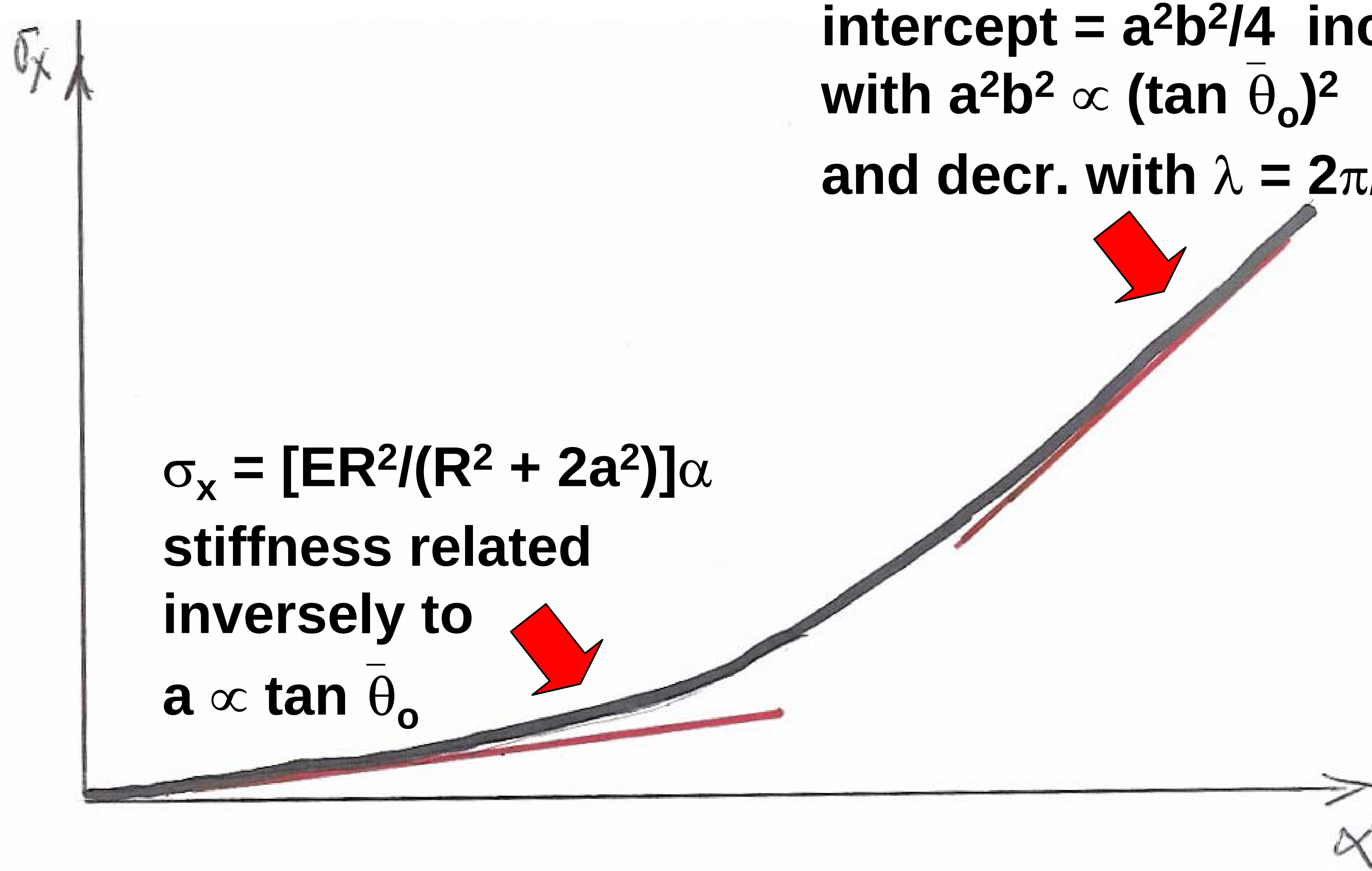
$$\sigma_x = E\alpha - [Ea^2b^2/4]$$

at $\sigma_x = 0$, intercept $= a^2b^2/4$



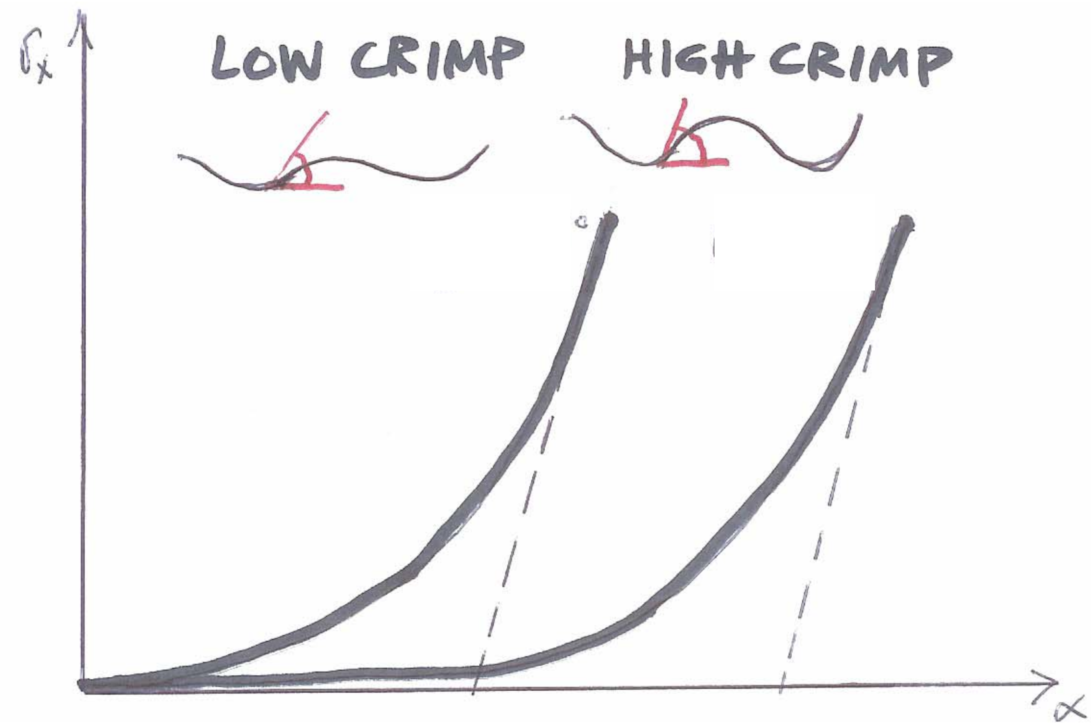
Summary of limiting cases.

$\sigma_x = E\alpha - [Ea^2b^2/4]$
intercept = $a^2b^2/4$ incr. with $a^2b^2 \propto (\tan \bar{\theta}_o)^2$
and decr. with $\lambda = 2\pi/b$



$\sigma_x = [ER^2/(R^2 + 2a^2)]\alpha$
stiffness related inversely to
$a \propto \tan \bar{\theta}_o$

The low-strain stiffness is related inversely to the crimp angle. The high-strain stiffness is independent of the crimp angle but the α-axis intercept increases with the square of tan (crimp angle).



<u>Step 7</u>. Which collagen structure carries the load when a person walks? Study collagen crystallinity at different strains with X-ray diffraction.

<u>Wide-angle</u> x-ray diffraction is used to study short distances, typically periodicities in the range 0.1-1 nm in the collagen crystals. Provides information at the molecular level. The repeat distance (or "rise per amino acid residue") of the triple helix is about 0.29 nm. The banding period of collagen is about 67 nm along the axial direction.

<u>Small-angle</u> diffraction is used to measure long periodicities of about 5-100 nm. Ordered lateral packing of triple helical molecules ("interhelical distance") can be observed at these angles.

Set up for measurement of X-ray diffraction from rat tail tendon (oriented vertically). The 0.29-nm meridional reflection indicates the rise per amino acid residue. The 0.95-nm reflection indicates the pitch of the triple helix. Short repeat distances are scattered over wide angles ("reciprocal lattice"); and vice versa.

Schematic removed for copyright reasons.

tendon specimen, S, oriented vertically

X-ray beam scattered from specimen and captured by photo film

From Brodsky

photo film

X-ray
diffraction
pattern of
rat tail tendon.
Meridional (M)
and
equatorial (E)
directions.
Meridional
reflections show
banding period
(67 nm).
Equatorial
maxima
show ordered
lateral packing
of molecules.

From Brodsky

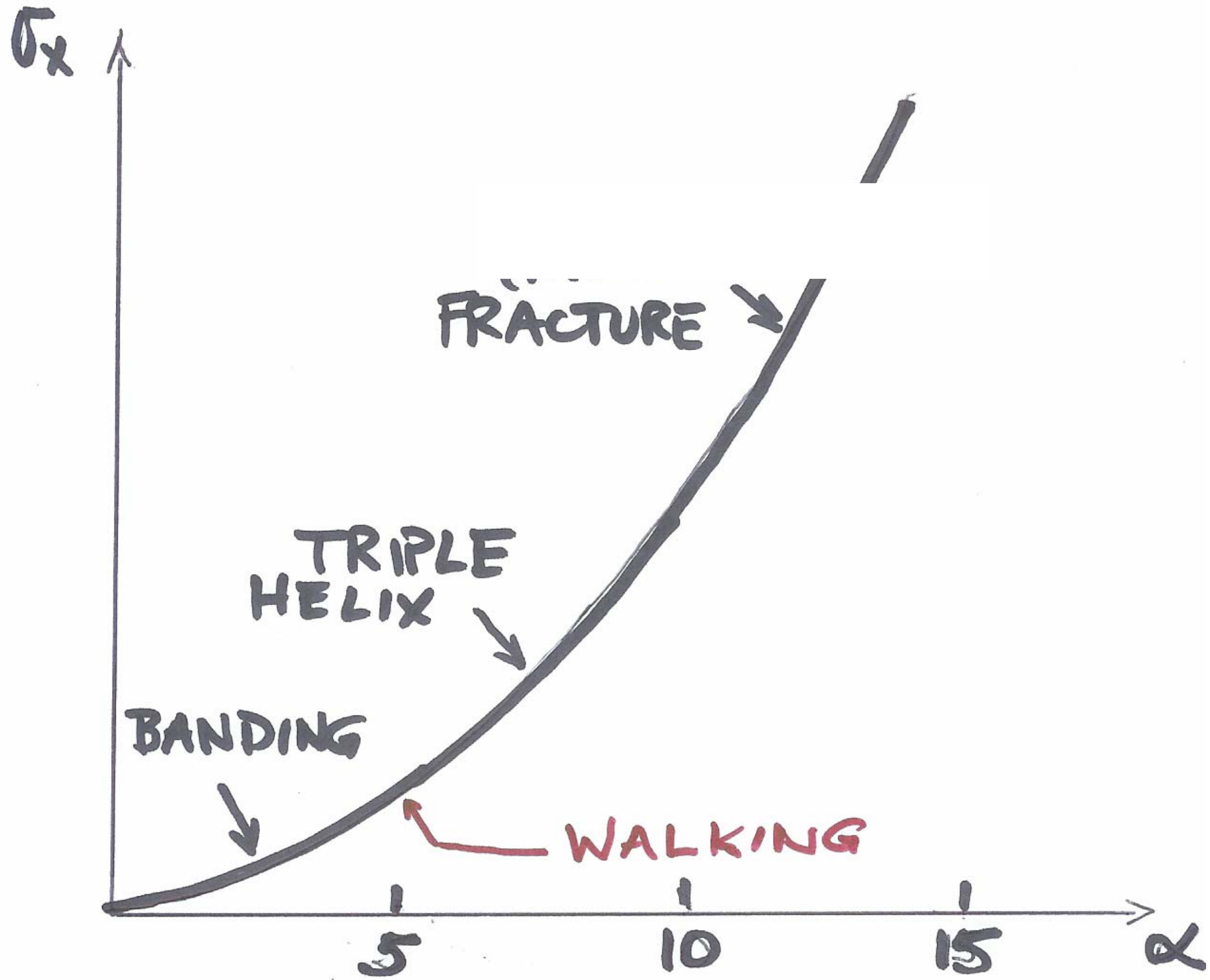
Changes in distances with Achilles tendon strain. No change in meridional reflection (rise per amino acid residue) observed until about 5% strain (approx. walking). Triple helix probably not deformed during walking.

Increase in banding period (small angle reflections) observed even at very small strains.

Graphs removed for copyright reasons.

Cowan data

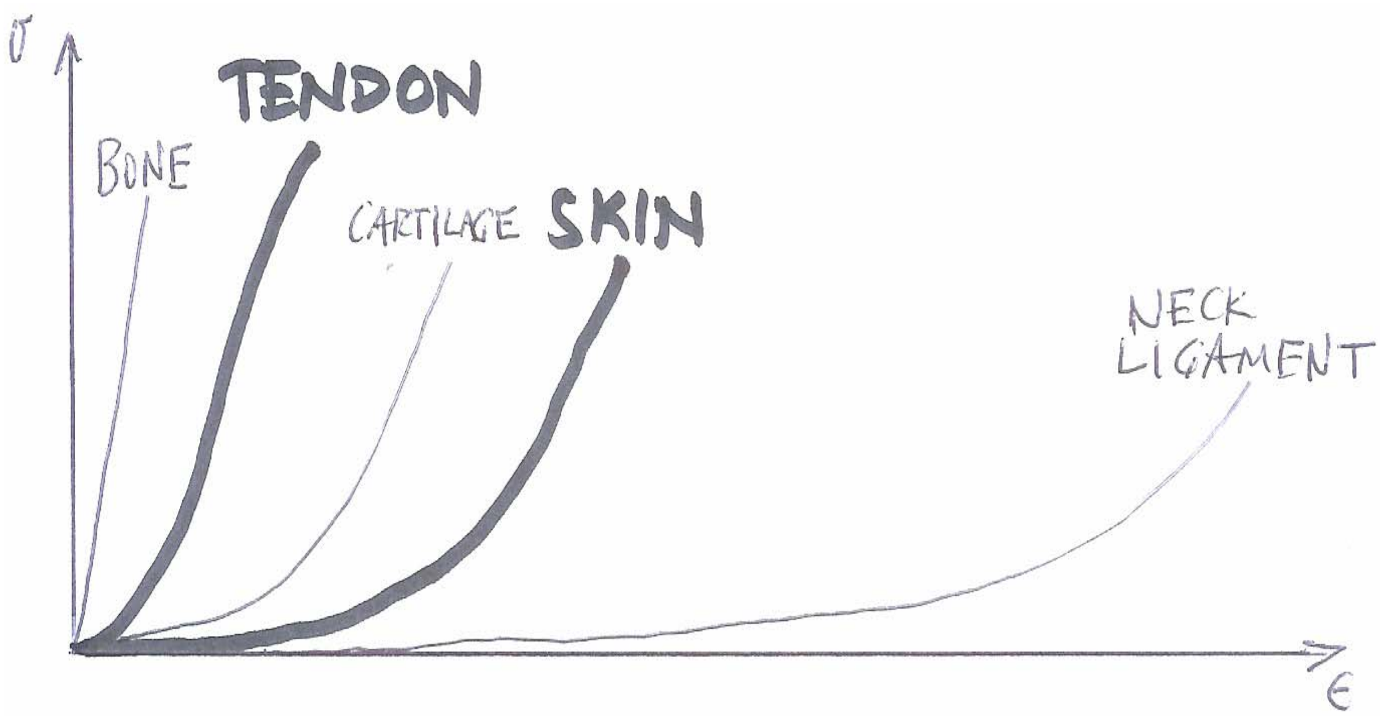
Structural features of collagen fibers that carry load at different levels of strain



2. The extensibility of skin.

- Data to explain.
- Kinematic analysis.
- Prediction of model vs. data.

Step 1. Data. Tendon is one of the stiffer connective tissues. Skin is much more extensible. Max. extensibility for skin is about 60% (tendon has about 15%). Why? Is the origin of skin extensibility the uncrimping of collagen fibers? or is it the alignment of fibers along the direction of principal stress?



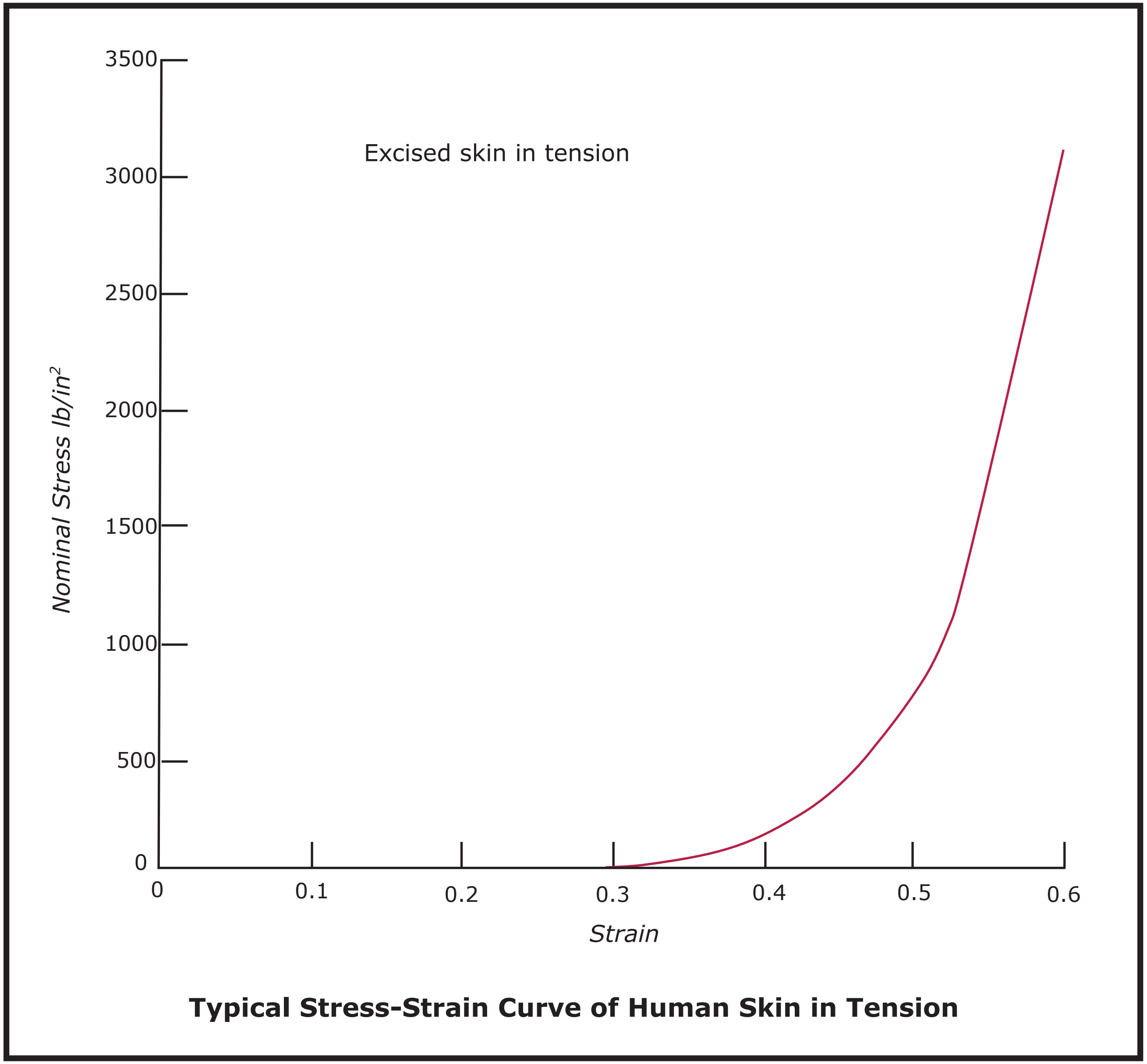
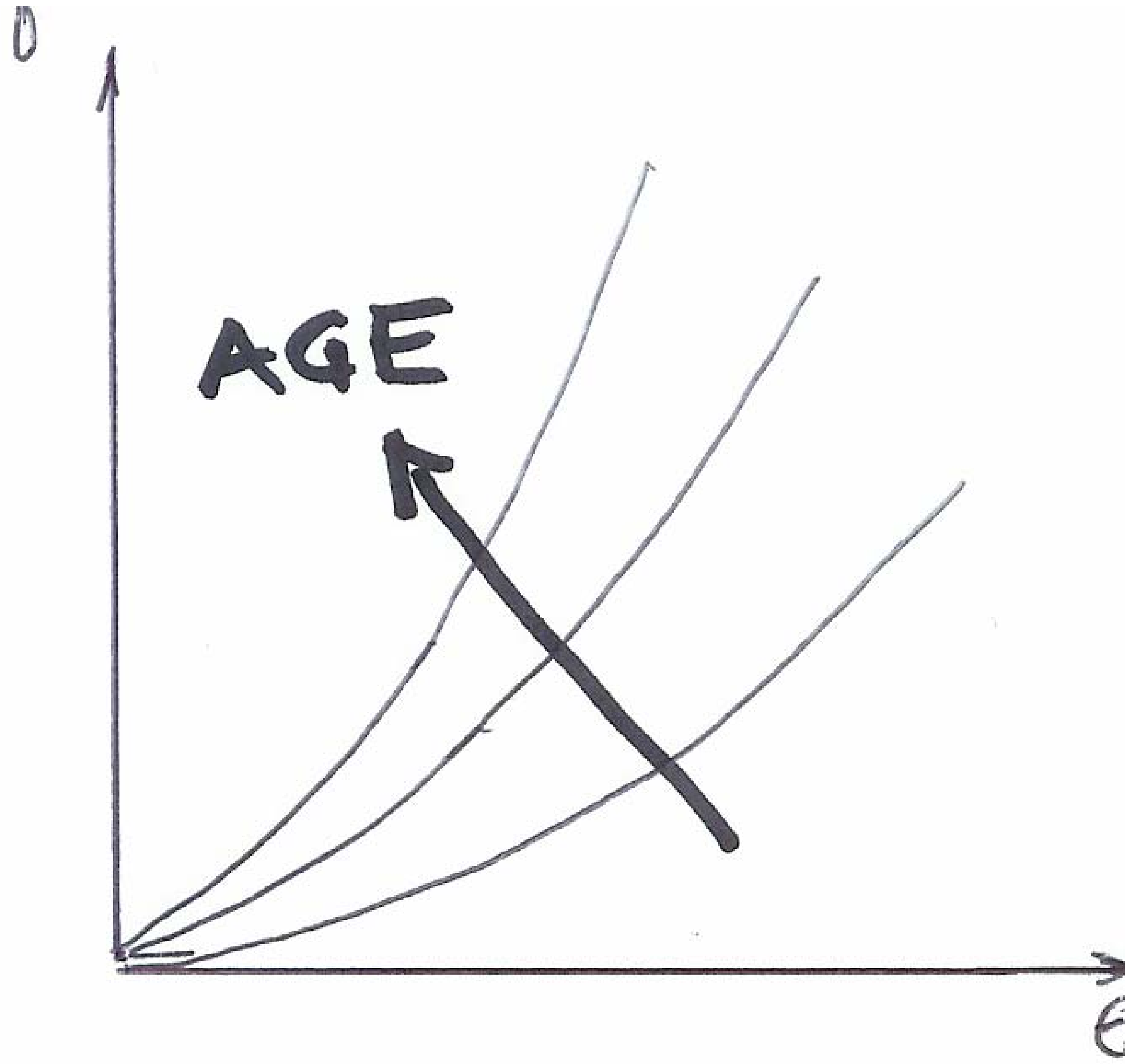
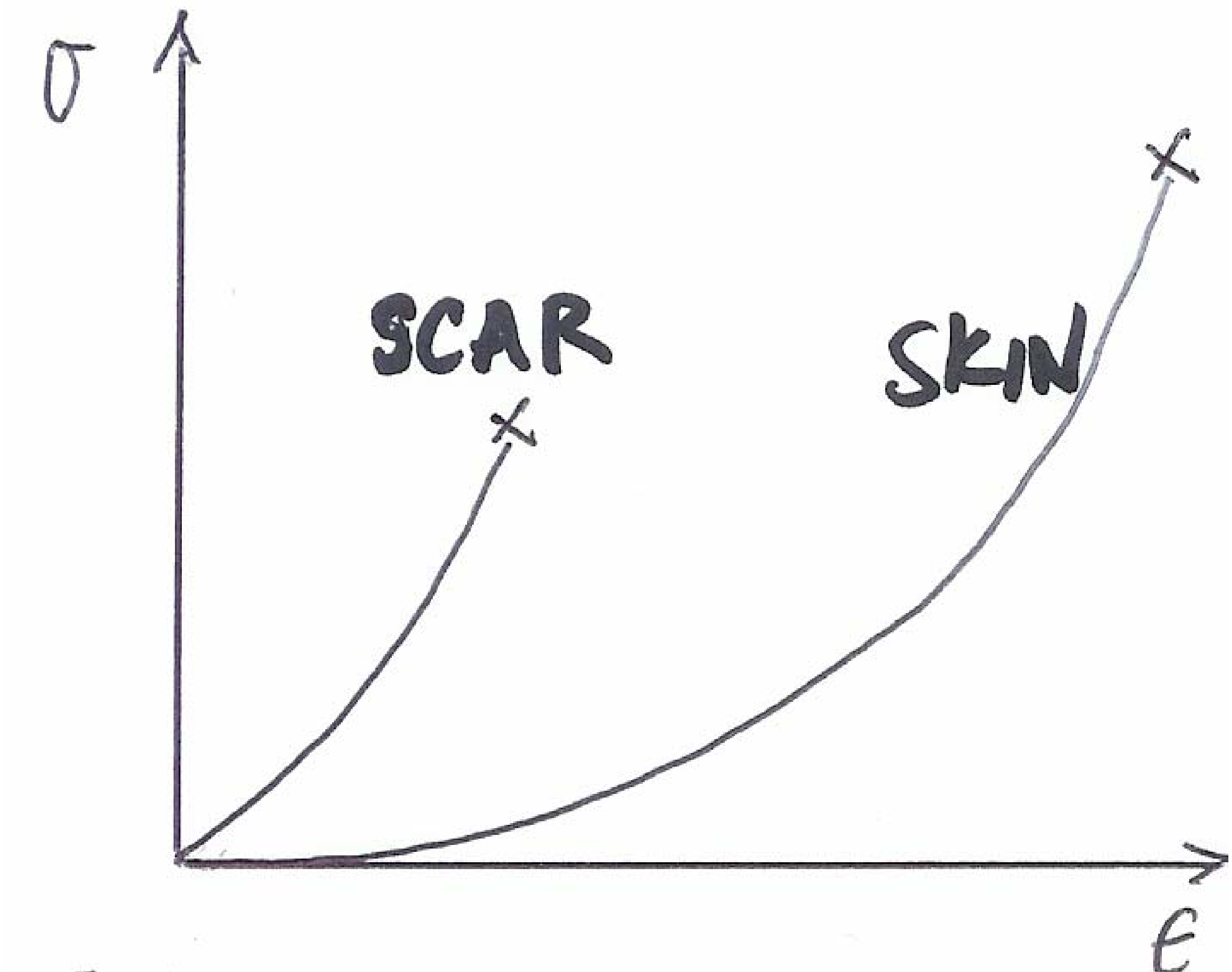


Figure by MIT OCW. After T. Gibson and R. M. Kenedi.

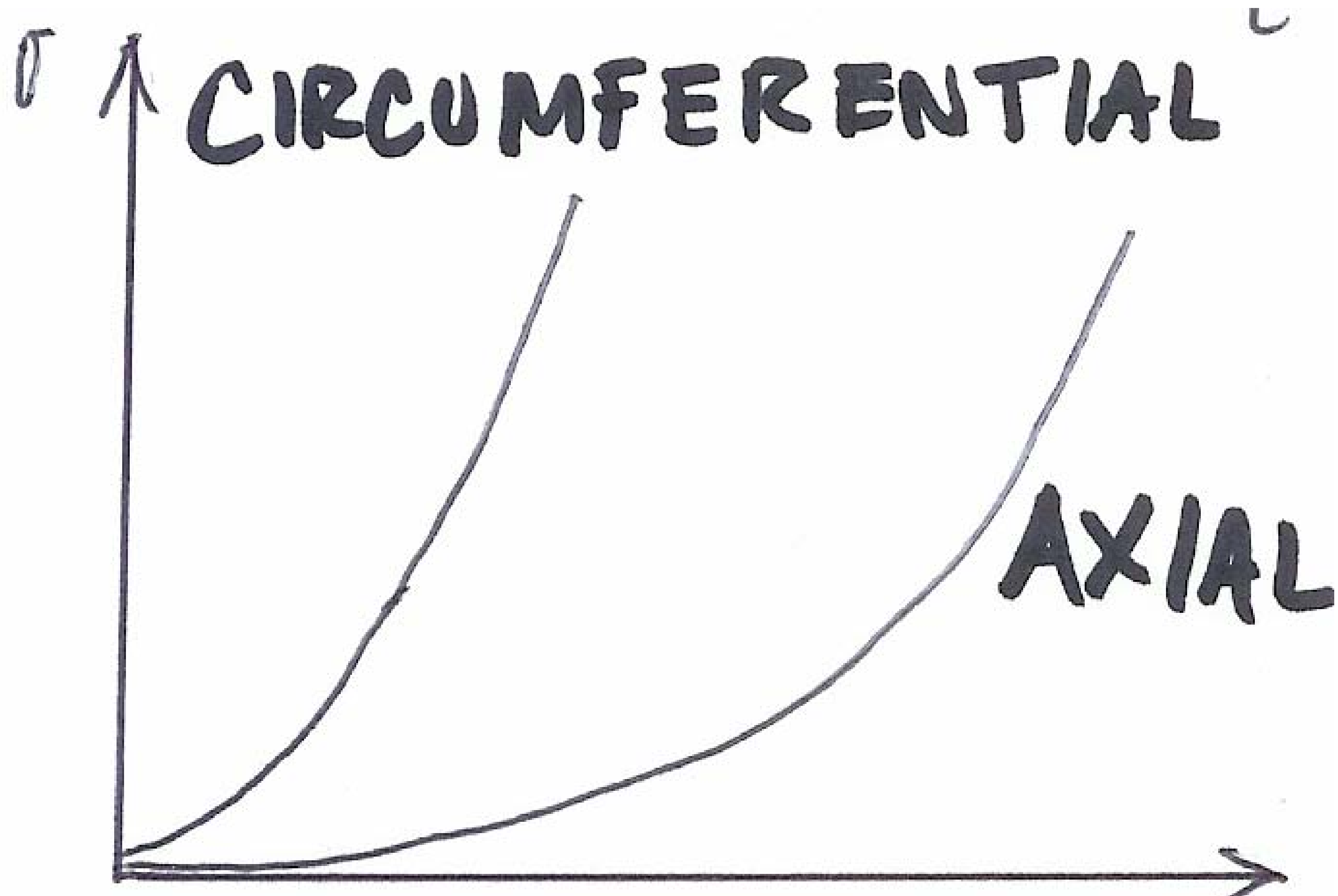
Effect of age on mechanical behavior of skin.



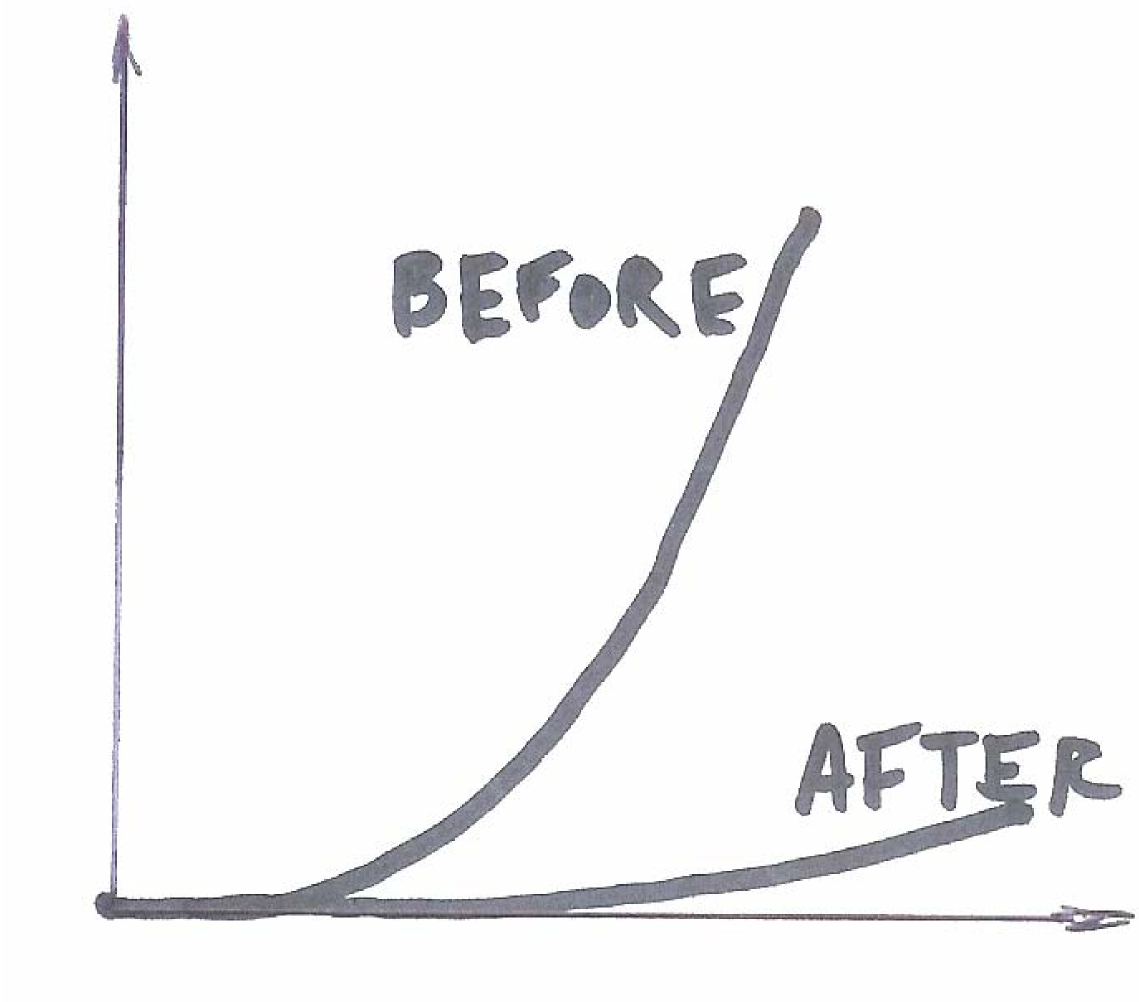
Effect of scar formation on mechanical behavior of skin.



Effect of direction on mechanical behavior of skin. Circumferential implies around the male chest. Axial on the chest implies along the vertical body axis.



Effect of treatment of skin with the enzyme collagenase, which degrades collagen.

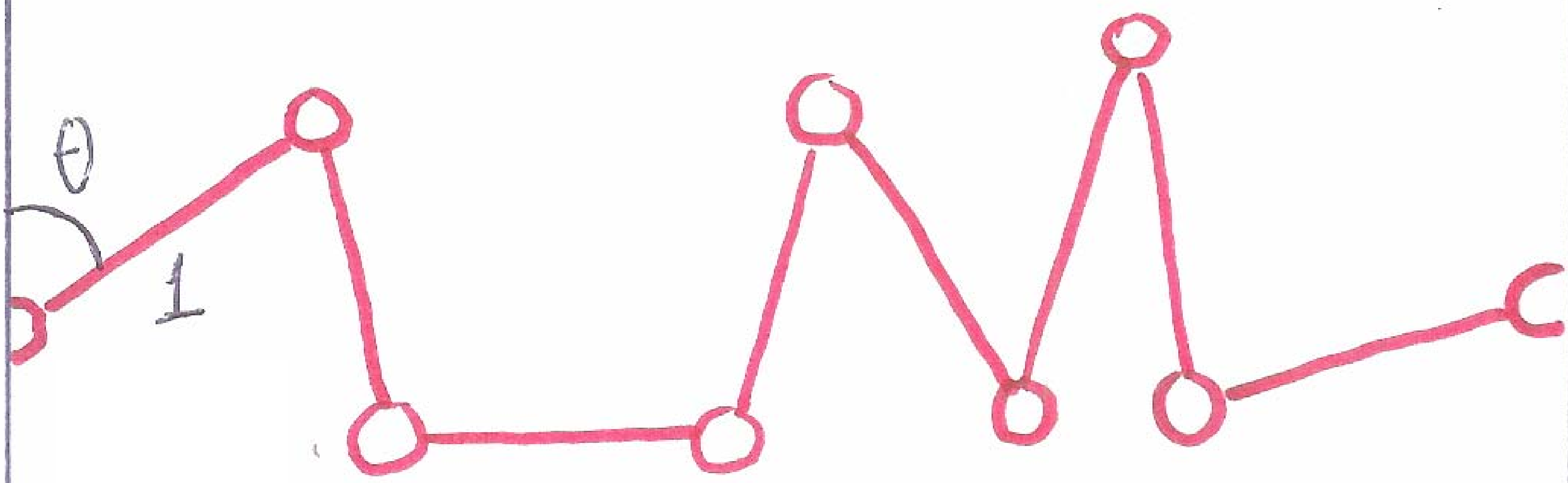


Step 2. Develop model

- **Microscopy shows more-or-less randomly oriented, kinked fibers in the plane of the epidermis.The geometry is two-dimensional. There is very little out-of-plane orientation of collagen fibers.**

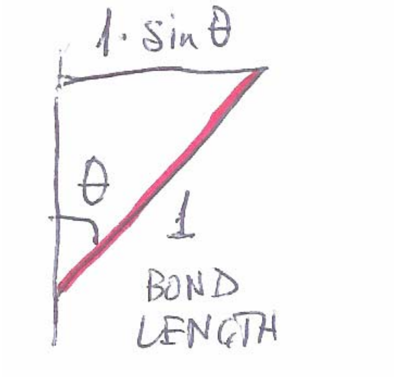
- **Develop a “kinematic” model that takes into account these data from microscopy. Then calculate the maximum extension.**

Model of skin extensibility



All fibers in the plane. Total length, L_o. Efficient load transfer between fiber ends. Each of n fibers has unit length. Angle θ varies randomly but from 0^o to only 180^o.

L_o



First calculate the projected bond length = $1 \cdot \sin\theta$.

Average value of the unextended bond length =

= n(average value of projected bond length) =

$$= nE\left[1 \cdot \sin\theta\right] = n\left[1/(\pi - 0)\right] \int_0^{\pi} \sin\theta \, d\theta =$$

$$= n\left[1/(\pi - 0) \left| -\cos\theta \right|_0^{\pi}\right] =$$

$$= (n/\pi)(-\cos\pi + \cos 0) = (n/\pi)[-(-1)+1] = 2n/\pi$$

NB. The symbol E denotes the “expected value” or arithmetic average of the function in parentheses.

Calculate the maximum extension for two models (n =100)

- **Random walk.**

ε_{max} = (L – L_o)/L_o = (n·1- $n^{1/2}$ ·1)/$n^{1/2}$ ·1 =

= [100 – $(100)^{1/2}$]/$(100)^{1/2}$ = 900%

- **Model for skin.**

The average value of the unextended bond length (see preceding graphic) = 2n/π

ε_{max}= (L – L_o)/L_o = (n·1- 2n/π)/(2n/π) = (π-2)/2 =57%

Step 3. Compare model predictions with data

Uncrimping of fibers alone cannot predict such a large extensibility.

A max. extensibility of 60% can be explained entirely by preferential alignment of collagen fibers along the direction of principal stress.

The increases in curvature with age, scar formation and anatomical direction can also be explained by preferential alignment of collagen fibers along the direction of principal stress. The increase in curvature reduces the extensibility of skin. Microscopic study of skin in these three conditions confirms the prediction of the model.

Summary of “Tendon and Skin”

- Both trendon and skin show nonlinear behavior but for different reasons. With increasing deformation, the prealigned collagen fibers in tendon lose their crimp and straighten out. In skin, extensibility is gained by aligning the randonly oriented crimped collagen fibers along the direction of major deformation.