**Unit II**

**ESAY QUESTION AND ANSWERS:**

Set theory has its own notations and symbols that can seem unusual for many. In this tutorial, we look at some solved examples to understand how set theory works and the kind of problems it can be used to solve.

Definition

A set is a collection of objects.

It is usually represented in flower braces.

**For example**:  
Set of natural numbers           = {1,2,3,…..}  
Set of whole numbers             = {0,1,2,3,…..}

Each object is called an element of the set.

The set that contains all the elements of a given collection is called the universal set and is represented by the symbol ‘µ’, pronounced as ‘mu’.

For two sets A and B,

* n(AᴜB) is the number of elements present in either of the sets A or B.
* n(A∩B) is the number of elements present in both the sets A and B.
* n(AᴜB) = n(A) + (n(B) – n(A∩B)

For three sets A, B and C,

* n(AᴜBᴜC) = n(A) + n(B) + n(C) – n(A∩B) – n(B∩C) – n(C∩A) + n(A∩B∩C)

Consider the following example:

**1.Question: In a class of 100 students, 35 like science and 45 like math. 10 like both. How many like either of them and how many like neither?**

Solution:

Total number of students, n(µ) = 100

Number of science students, n(S) = 35

Number of math students, n(M) = 45

Number of students who like both, n(M∩S) = 10

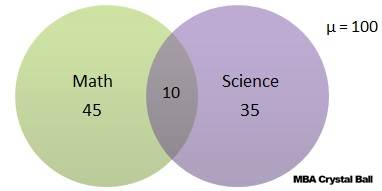
Number of students who like either of them,

n(MᴜS) = n(M) + n(S) – n(M∩S)

→ 45+35-10 = 70

Number of students who like neither = n(µ) – n(MᴜS) = 100 – 70 = 30

The easiest way to solve problems on sets is by drawing Venn diagrams, as shown below.



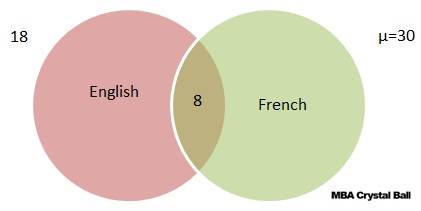
As it is said, one picture is worth a thousand words. One Venn diagram can help solve the problem faster and save time. This is especially true when more than two categories are involved in the problem.

Let us see some more solved examples.

**Problem 1:** There are 30 students in a class. Among them, 8 students are learning both English and French. A total of 18 students are learning English. If every student is learning at least one language, how many students are learning French in total?

Solution:

The Venn diagram for this problem looks like this.



Every student is learning at least one language. Hence there is no one who fall in the category ‘neither’.

So in this case, n(EᴜF) = n(µ).

It is mentioned in the problem that a total of 18 are learning English. This DOES NOT mean that 18 are learning ONLY English. Only when the word ‘only’ is mentioned in the problem should we consider it so.

Now, 18 are learning English and 8 are learning both. This means that 18 – 8 = 10 are learning ONLY English.

n(µ) = 30, n(E) = 10

n(EᴜF) = n(E) + n(F) – n(E∩F)

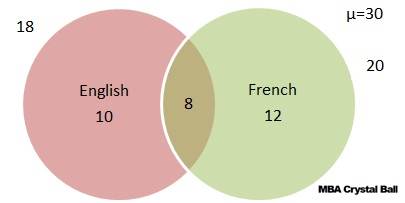
30 = 18+ n(F) – 8

n(F) = 20

Therefore, total number of students learning French = 20.

**Note**: The question was only about the total number of students learning French and not about those learning ONLY French, which would have been a different answer, 12.

Finally, the Venn diagram looks like this.



   
**Problem 2: Among a group of students, 50 played cricket, 50 played hockey and 40 played volley ball. 5 played both cricket and hockey, 10 played both hockey and volley ball, 5 played cricket and volley ball and 10 played all three. If every student played at least one game, find the number of students and how many played only cricket, only hockey and only volley ball?**

Solution:

n(C) = 50, n(H) = 50, n(V) = 40

n(C∩H) = 5

n(H∩V) = 10

n(C∩V) = 5

n(C∩H∩V) = 10

No. of students who played atleast one game

n(CᴜHᴜV) = n(C) + n(H) + n(V) – n(C∩H) – n(H∩V) – n(C∩V) + n(C∩H∩V)

= 50 + 50 + 40 – 5 – 10 – 5 + 10

Total number of students = 130.

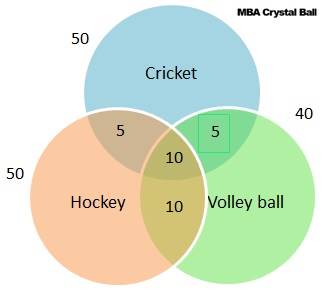
No. of students who played only cricket = n(C) – [n(C∩H) + n(C∩V) + n(C∩H∩V)] = 50 – (5+5+10) = 30.

No. of students who played only hockey = n(H) – [n(C∩H) + n(H∩V) + n(C∩H∩V)] = 50 – (5+10+10) = 25.

No. of students who played only volley ball = n(V) – [n(H∩V) + n(C∩V) + n(C∩H∩V)]=40-(10+5+10) = 15.

Alternatively, we can solve it faster with the help of a Venn diagram.

The Venn diagram for the given information looks like this.



Subtracting the values in the intersections from the individual values gives us the number of students who played only one game.

SETOPERATIONS :

**1. Intersection**: The common elements of two sets: A ∩ B = {x | (x ∈ A) ∧ (x ∈ B)} . If A ∩ B = ∅, the sets are said to be disjoint.

**2. Union**: The set of elements that belong to either of two sets: A ∪ B = {x | (x ∈ A) ∨ (x ∈ B)} . 2.1. SET THEORY 23

**3. Complement**: The set of elements (in the universal set) that do not belong to a given set: A = {x ∈ U | x 6∈ A} .

**4. Difference or Relative Complement**: The set of elements that belong to a set but not to another: A − B = {x | (x ∈ A) ∧ (x 6∈ B)} = A ∩ B .

**5. Symmetric Difference**: Given two sets, their symmetric difference is the set of elements that belong to either one or the other set but not both. A ⊕ B = {x | (x ∈ A) ⊕ (x ∈ B)} . It can be expressed also in the following way: A ⊕ B = A ∪ B − A ∩ B = (A − B) ∪ (B − A).

**Properties of Sets.**

**The set operations verify the following properties**:

1. Associative Laws: A ∪ (B ∪ C) = (A ∪ B) ∪ C A ∩ (B ∩ C) = (A ∩ B) ∩ C

2. Commutative Laws: A ∪ B = B ∪ A A ∩ B = B ∩ A

3. Distributive Laws: A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C) A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

4. Identity Laws: A ∪ ∅ = A A ∩ U = A

5. Complement Laws: A ∪ A = U A ∩ A = ∅

6. Idempotent Laws: A ∪ A = A A ∩ A = A

7. Bound Laws: A ∪ U = U A ∩ ∅ = ∅

8. Absorption Laws: A ∪ (A ∩ B) = A A ∩ (A ∪ B) = A

9. Involution Law: A = A 2.1. SET THEORY 25

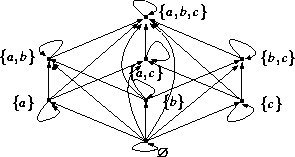
10. 0/1 Laws: ∅ = U U = ∅

11. DeMorgan’s Laws: A ∪ B = A ∩ B A ∩ B = A ∪ B n

***Hasse diagrams*** are meant to present partial order relations in equivalent but somewhat simpler forms by removing certain deducible ``noncritical'' parts of the relations. For better motivation and understanding, we'll introduce it through the following examples.

**Examples**

1. The relation in example 2 can be drawn as



It is somewhat ``messy'' and some arrows can be derived from transitivity anyway. If we

http://staff.scm.uws.edu.au/~zhuhan/dm_notes/tex/blacktriangleright.gif

omit all loops,

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omit all arrows that can be inferred from transitivity,

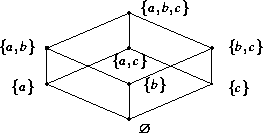
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draw arrows without heads,

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understand that all arrows point upwards,

then the above graph simplifies to



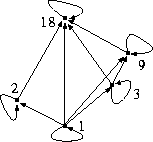
This type of graph is called a **Hasse diagram**, it is often used to represent a partially ordered set.

1. Let *A ={ 1,2,3,9,18}* and for any *a,b http://staff.scm.uws.edu.au/~zhuhan/dm_notes/tex/in.gif A*,

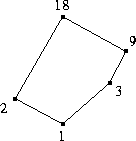
*a http://staff.scm.uws.edu.au/~zhuhan/dm_notes/tex/preceq.gif b*  iff *a* | *b* .

Draw the Hasse diagram of the relation.

**Solution** First it is easy to verify that the relation *http://staff.scm.uws.edu.au/~zhuhan/dm_notes/tex/preceq.gif* defined above is a partial ordering. The directed graph of relation *http://staff.scm.uws.edu.au/~zhuhan/dm_notes/tex/preceq.gif* is



and the Hasse diagram is



1. (a)

Let set *A* be given by

*A={ 3, 4, 5, 6, 10, 12 }*

and a binary relation *R* on *A* be defined by

*(x, y) http://staff.scm.uws.edu.au/~zhuhan/dm_notes/tex/in.gif R*  iff *x* | *y* ,

i.e. *(x,y) http://staff.scm.uws.edu.au/~zhuhan/dm_notes/tex/in.gif R* if and only if *x* divides *y*. Give explicitly *R* in terms of its elements and draw the corresponding Hasse diagram.

(b)

Let a new binary relation *R'* on the set *A* given in (a) be defined by

*(x, y) http://staff.scm.uws.edu.au/~zhuhan/dm_notes/tex/in.gif R'*  if and only if either *x* | *y* or *y* | *x*

and *R''* be the transitive closure of *R'*. Use directed graphs to represent *R*, *R'* and *R''* respectively. Which of the three relations *R*, *R'* and *R''* is an equivalence relation? For the equivalence relation, give all the distinct equivalence classes.

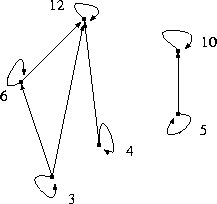
**Solution**

(a)

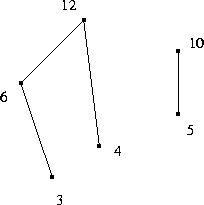
Among all the elements of set *A={ 3, 4, 5, 6, 10, 12 }*, obviously *3 http://staff.scm.uws.edu.au/~zhuhan/dm_notes/tex/in.gif A*, for instance, divides *3*, *6* and *12*. Hence by the definition of the relation *R* specified by the question we conclude *(3,3)*,*(3,6)* and *(3,12)* are all elements of the relation *R*. Likewise we can show that *(4,4)*, *(4,12)*, *(5,5)*, *(5,10)*, *(6,6)*, *(6,12)*, *(10,10)*, *(12,12)* are all elements of *R*. In fact we have

*R = { (3,3), (3,6), (3,12), (4,4), (4,12), (5,5), (5,10), (6,6), (6,12), (10,10), (12,12) } .*

Hence the digraph for *R* is

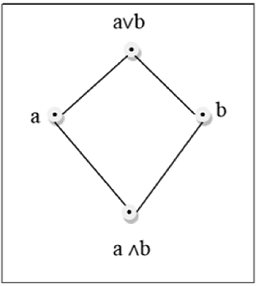


which induces the following Hasse diagram

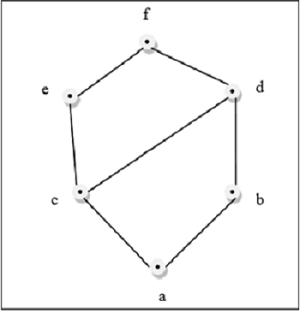


## Lattice

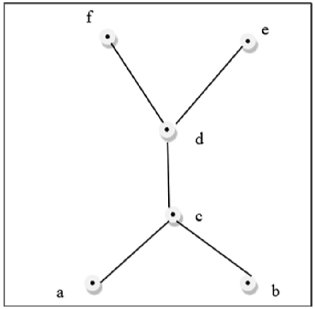
A lattice is a poset (L,≤)(L,≤) for which every pair {a,b}∈L{a,b}∈L has a least upper bound (denoted by a∨ba∨b) and a greatest lower bound (denoted by a∧ba∧b). LUB ({a,b})({a,b}) is called the join of a and b. GLB ({a,b})({a,b}) is called the meet of a and b.



### Example



This above figure is a lattice because for every pair {a,b}∈L{a,b}∈L, a GLB and a LUB exists.



This above figure is a not a lattice because GLB(a,b)GLB(a,b) and LUB(e,f)LUB(e,f) does not exist.

Some other lattices are discussed below −

### Bounded Lattice

A lattice L becomes a bounded lattice if it has a greatest element 1 and a least element 0.

### Complemented Lattice

A lattice L becomes a complemented lattice if it is a bounded lattice and if every element in the lattice has a complement. An element x has a complement x’ if ∃x(x∧x′=0andx∨x′=1)∃x(x∧x′=0andx∨x′=1)

### Distributive Lattice

If a lattice satisfies the following two distribute properties, it is called a distributive lattice.

* a∨(b∧c)=(a∨b)∧(a∨c)a∨(b∧c)=(a∨b)∧(a∨c)
* a∧(b∨c)=(a∧b)∨(a∧c)a∧(b∨c)=(a∧b)∨(a∧c)

### Modular Lattice

If a lattice satisfies the following property, it is called modular lattice.

a∧(b∨(a∧d))=(a∧b)∨(a∧d)a∧(b∨(a∧d))=(a∧b)∨(a∧d)

## Properties of Lattices

### Idempotent Properties

* a∨a=aa∨a=a
* a∧a=aa∧a=a

### Absorption Properties

* a∨(a∧b)=aa∨(a∧b)=a
* a∧(a∨b)=aa∧(a∨b)=a

### Commutative Properties

* a∨b=b∨aa∨b=b∨a
* a∧b=b∧aa∧b=b∧a

### Associative Properties

* a∨(b∨c)=(a∨b)∨ca∨(b∨c)=(a∨b)∨c
* a∧(b∧c)=(a∧b)∧ca∧(b∧c)=(a∧b)∧c

## Dual of a Lattice

The dual of a lattice is obtained by interchanging the '∨∨' and '∧∧' operations.

### Example

The dual of [a∨(b∧c)] is [a∧(b∨c)][a∨(b∧c)] is [a∧(b∨c)]

# Discrete Mathematics - Counting Theory

In daily lives, many a times one needs to find out the number of all possible outcomes for a series of events. For instance, in how many ways can a panel of judges comprising of 6 men and 4 women be chosen from among 50 men and 38 women? How many different 10 lettered PAN numbers can be generated such that the first five letters are capital alphabets, the next four are digits and the last is again a capital letter. For solving these problems, mathematical theory of counting are used. **Counting** mainly encompasses fundamental counting rule, the permutation rule, and the combination rule.

## The Rules of Sum and Product

The **Rule of Sum** and **Rule of Product** are used to decompose difficult counting problems into simple problems.

* **The Rule of Sum** − If a sequence of tasks T1,T2,…,TmT1,T2,…,Tm can be done in w1,w2,…wmw1,w2,…wm ways respectively (the condition is that no tasks can be performed simultaneously), then the number of ways to do one of these tasks is w1+w2+⋯+wmw1+w2+⋯+wm. If we consider two tasks A and B which are disjoint (i.e. A∩B=∅A∩B=∅), then mathematically |A∪B|=|A|+|B||A∪B|=|A|+|B|
* **The Rule of Product** − If a sequence of tasks T1,T2,…,TmT1,T2,…,Tm can be done in w1,w2,…wmw1,w2,…wm ways respectively and every task arrives after the occurrence of the previous task, then there are w1×w2×⋯×wmw1×w2×⋯×wm ways to perform the tasks. Mathematically, if a task B arrives after a task A, then |A×B|=|A|×|B||A×B|=|A|×|B|

### Example

**Question** − A boy lives at X and wants to go to School at Z. From his home X he has to first reach Y and then Y to Z. He may go X to Y by either 3 bus routes or 2 train routes. From there, he can either choose 4 bus routes or 5 train routes to reach Z. How many ways are there to go from X to Z?

**Solution** − From X to Y, he can go in 3+2=53+2=5 ways (Rule of Sum). Thereafter, he can go Y to Z in 4+5=94+5=9 ways (Rule of Sum). Hence from X to Z he can go in 5×9=455×9=45 ways (Rule of Product).

## Permutations

A **permutation** is an arrangement of some elements in which order matters. In other words a Permutation is an ordered Combination of elements.

### Examples

* From a set S ={x, y, z} by taking two at a time, all permutations are −

xy,yx,xz,zx,yz,zyxy,yx,xz,zx,yz,zy.

* We have to form a permutation of three digit numbers from a set of numbers S={1,2,3}S={1,2,3}. Different three digit numbers will be formed when we arrange the digits. The permutation will be = 123, 132, 213, 231, 312, 321

### Number of Permutations

The number of permutations of ‘n’ different things taken ‘r’ at a time is denoted by nPrnPr

nPr=n!(n−r)!nPr=n!(n−r)!

where n!=1.2.3.…(n−1).nn!=1.2.3.…(n−1).n

**Proof** − Let there be ‘n’ different elements.

There are n number of ways to fill up the first place. After filling the first place (n-1) number of elements is left. Hence, there are (n-1) ways to fill up the second place. After filling the first and second place, (n-2) number of elements is left. Hence, there are (n-2) ways to fill up the third place. We can now generalize the number of ways to fill up r-th place as [n – (r–1)] = n–r+1

So, the total no. of ways to fill up from first place up to r-th-place −

nPr=n(n−1)(n−2).....(n−r+1)nPr=n(n−1)(n−2).....(n−r+1)

=[n(n−1)(n−2)...(n−r+1)][(n−r)(n−r−1)…3.2.1]/[(n−r)(n−r−1)…3.2.1]=[n(n−1)(n−2)...(n−r+1)][(n−r)(n−r−1)…3.2.1]/[(n−r)(n−r−1)…3.2.1]

Hence,

nPr=n!/(n−r)!nPr=n!/(n−r)!

### Some important formulas of permutation

* If there are *n* elements of which a1a1 are alike of some kind, a2a2 are alike of another kind; a3a3 are alike of third kind and so on and arar are of rthrthkind, where (a1+a2+...ar)=n(a1+a2+...ar)=n.

Then, number of permutations of these n objects is = n!/[(a1!(a2!)…(ar!)]n!/[(a1!(a2!)…(ar!)].

* Number of permutations of n distinct elements taking n elements at a time = nPn=n!nPn=n!
* The number of permutations of n dissimilar elements taking r elements at a time, when x particular things always occupy definite places = n−xpr−xn−xpr−x
* The number of permutations of n dissimilar elements when r specified things always come together is − r!(n−r+1)!r!(n−r+1)!
* The number of permutations of n dissimilar elements when r specified things never come together is − n!–[r!(n−r+1)!]n!–[r!(n−r+1)!]
* The number of circular permutations of n different elements taken x elements at time = npx/xnpx/x
* The number of circular permutations of n different things = npn/nnpn/n

## Combinations

A **combination** is selection of some given elements in which order does not matter.

The number of all combinations of n things, taken r at a time is −

nCr=n!r!(n−r)!nCr=n!r!(n−r)!

**Problem 1**

Find the number of subsets of the set {1,2,3,4,5,6}{1,2,3,4,5,6} having 3 elements.

**Solution**

The cardinality of the set is 6 and we have to choose 3 elements from the set. Here, the ordering does not matter. Hence, the number of subsets will be 6C3=206C3=20.

**Problem 2**

There are 6 men and 5 women in a room. In how many ways we can choose 3 men and 2 women from the room?

**Solution**

The number of ways to choose 3 men from 6 men is 6C36C3 and the number of ways to choose 2 women from 5 women is 5C25C2

Hence, the total number of ways is − 6C3×5C2=20×10=2006C3×5C2=20×10=200

**Problem 3**

How many ways can you choose 3 distinct groups of 3 students from total 9 students?

**Solution**

Let us number the groups as 1, 2 and 3

For choosing 3 students for 1st group, the number of ways − 9C39C3

The number of ways for choosing 3 students for 2nd group after choosing 1st group − 6C36C3

The number of ways for choosing 3 students for 3rd group after choosing 1stand 2nd group − 3C33C3

Hence, the total number of ways =9C3×6C3×3C3=84×20×1=1680=9C3×6C3×3C3=84×20×1=1680

## Pigeonhole Principle

In 1834, German mathematician, Peter Gustav Lejeune Dirichlet, stated a principle which he called the drawer principle. Now, it is known as the pigeonhole principle.

**Pigeonhole Principle** states that if there are fewer pigeon holes than total number of pigeons and each pigeon is put in a pigeon hole, then there must be at least one pigeon hole with more than one pigeon. If n pigeons are put into m pigeonholes where n > m, there's a hole with more than one pigeon.

### Examples

* Ten men are in a room and they are taking part in handshakes. If each person shakes hands at least once and no man shakes the same man’s hand more than once then two men took part in the same number of handshakes.
* There must be at least two people in a class of 30 whose names start with the same alphabet.

## The Inclusion-Exclusion principle

The **Inclusion-exclusion principle** computes the cardinal number of the union of multiple non-disjoint sets. For two sets A and B, the principle states −

|A∪B|=|A|+|B|−|A∩B||A∪B|=|A|+|B|−|A∩B|

For three sets A, B and C, the principle states −

|A∪B∪C|=|A|+|B|+|C|−|A∩B|−|A∩C|−|B∩C|+|A∩B∩C||A∪B∪C|=|A|+|B|+|C|−|A∩B|−|A∩C|−|B∩C|+|A∩B∩C|

The generalized formula -

|⋃ni=1Ai|=∑1≤i<j<k≤n|Ai∩Aj|+∑1≤i<j<k≤n|Ai∩Aj∩Ak|−⋯+(−1)π−1|A1∩⋯∩A2||⋃i=1nAi|=∑1≤i<j<k≤n|Ai∩Aj|+∑1≤i<j<k≤n|Ai∩Aj∩Ak|−⋯+(−1)π−1|A1∩⋯∩A2|

**Problem 1**

How many integers from 1 to 50 are multiples of 2 or 3 but not both?

**Solution**

From 1 to 100, there are 50/2=2550/2=25 numbers which are multiples of 2.

There are 50/3=1650/3=16 numbers which are multiples of 3.

There are 50/6=850/6=8 numbers which are multiples of both 2 and 3.

So, |A|=25|A|=25, |B|=16|B|=16 and |A∩B|=8|A∩B|=8.

|A∪B|=|A|+|B|−|A∩B|=25+16−8=33|A∪B|=|A|+|B|−|A∩B|=25+16−8=33

**Problem 2**

In a group of 50 students 24 like cold drinks and 36 like hot drinks and each student likes at least one of the two drinks. How many like both coffee and tea?

**Solution**

Let X be the set of students who like cold drinks and Y be the set of people who like hot drinks.

So, |X∪Y|=50|X∪Y|=50, |X|=24|X|=24, |Y|=36|Y|=36

|X∩Y|=|X|+|Y|−|X∪Y|=24+36−50=60−50=10|X∩Y|=|X|+|Y|−|X∪Y|=24+36−50=60−50=10

Hence, there are 10 students who like both tea and coffee.

# Discrete Mathematics - Probability

Closely related to the concepts of counting is Probability. We often try to guess the results of games of chance, like card games, slot machines, and lotteries; i.e. we try to find the likelihood or probability that a particular result with be obtained.

**Probability** can be conceptualized as finding the chance of occurrence of an event. Mathematically, it is the study of random processes and their outcomes. The laws of probability have a wide applicability in a variety of fields like genetics, weather forecasting, opinion polls, stock markets etc.

## Basic Concepts

Probability theory was invented in the 17th century by two French mathematicians, Blaise Pascal and Pierre de Fermat, who were dealing with mathematical problems regarding of chance.

Before proceeding to details of probability, let us get the concept of some definitions.

**Random Experiment** − An experiment in which all possible outcomes are known and the exact output cannot be predicted in advance is called a random experiment. Tossing a fair coin is an example of random experiment.

**Sample Space** − When we perform an experiment, then the set S of all possible outcomes is called the sample space. If we toss a coin, the sample space S={H,T}S={H,T}

**Event** − Any subset of a sample space is called an event. After tossing a coin, getting Head on the top is an event.

The word "probability" means the chance of occurrence of a particular event. The best we can say is how likely they are to happen, using the idea of probability.

Probabilityofoccurenceofanevent=TotalnumberoffavourableoutcomeTotalnumberofOutcomesProbabilityofoccurenceofanevent=TotalnumberoffavourableoutcomeTotalnumberofOutcomes

As the occurrence of any event varies between 0% and 100%, the probability varies between 0 and 1.

### Steps to find the probability

Step 1 − Calculate all possible outcomes of the experiment.

Step 2 − Calculate the number of favorable outcomes of the experiment.

Step 3 − Apply the corresponding probability formula.

### Tossing a Coin

If a coin is tossed, there are two possible outcomes − Heads (H)(H) or Tails (T)(T)

So, Total number of outcomes = 2

Hence, the probability of getting a Head (H)(H) on top is 1/2 and the probability of getting a Tails (T)(T) on top is 1/2

### Throwing a Dice

When a dice is thrown, six possible outcomes can be on the top − 1,2,3,4,5,61,2,3,4,5,6.

The probability of any one of the numbers is 1/6

The probability of getting even numbers is 3/6 = 1/3

The probability of getting odd numbers is 3/6 = 1/3

### Taking Cards From a Deck

From a deck of 52 cards, if one card is picked find the probability of an ace being drawn and also find the probability of a diamond being drawn.

Total number of possible outcomes − 52

Outcomes of being an ace − 4

Probability of being an ace = 4/52 = 1/13

Probability of being a diamond = 13/52 = 1/4

## Probability Axioms

* The probability of an event always varies from 0 to 1. [0≤P(x)≤1][0≤P(x)≤1]
* For an impossible event the probability is 0 and for a certain event the probability is 1.
* If the occurrence of one event is not influenced by another event, they are called mutually exclusive or disjoint.

If A1,A2....AnA1,A2....An are mutually exclusive/disjoint events, thenP(Ai∩Aj)=∅P(Ai∩Aj)=∅ for i≠ji≠j and P(A1∪A2∪....An)=P(A1)+P(A2)+.....P(An)P(A1∪A2∪....An)=P(A1)+P(A2)+.....P(An)

## Properties of Probability

* If there are two events xx and x¯¯¯x¯which are complementary, then the probability of the complementary event is −

p(x¯¯¯)=1−p(x)p(x¯)=1−p(x)

* For two non-disjoint events A and B, the probability of the union of two events −

P(A∪B)=P(A)+P(B)P(A∪B)=P(A)+P(B)

* If an event A is a subset of another event B (i.e. A⊂BA⊂B), then the probability of A is less than or equal to the probability of B. Hence, A⊂BA⊂B implies P(A)≤p(B)P(A)≤p(B)

**SHORT QUESTION AND ANSWERS**

1**:**Let A and B be two finite sets such that n(A) = 20, n(B) = 28 and n(A ∪ B) = 36, find n(A ∩ B).

**Solution:** Using the formula n(A ∪ B) = n(A) + n(B) - n(A ∩ B).

then n(A ∩B) = n(A) + n(B) - n(A ∪B)

= 20 + 28 - 36

= 48 - 36

= 12

**2:** In a group of 60 people, 27 like cold drinks and 42 like hot drinks and each person likes at least one of the two drinks. How many like both coffee and tea?

**Solution:** Let A = Set of people who like cold drinks B = Set of people who like hot drinks Given,

(A ∪B) = 60     n(A) = 27     n(B) = 42 then;

n(A ∩ B) = n(A) + n(B) - n(A ∪ B)

= 27 + 42 - 60

= 69 - 60 = 9

= 9

Therefore, 9 people like both tea and coffee.

**3:**In a competition, a school awarded medals in different categories. 36 medals in dance, 12 medals in dramatics and 18 medals in music. If these medals went to a total of 45 persons and only 4 persons got medals in all the three categories, how many received medals in exactly two of these categories?

**Solution:**Let A = set of persons who got medals in dance.

B = set of persons who got medals in dramatics.

C = set of persons who got medals in music.

Given,

n(A) = 36

n(B) = 12

n(C) = 18

n(A ∪ B ∪C) = 45

n(A ∩ B ∩ C) = 4

We know that number of elements belonging to exactly two of the three sets A, B, C

= n(A ∩ B) + n(B ∩ C) + n(A ∩ C) - 3n(A ∩ B ∩ C)

= n(A ∩ B) + n(B ∩ C) + n(A ∩ C) - 3 × 4 ……..(i)

n(A ∪ B ∪ C) = n(A) + n(B) + n(C) - n(A ∩ B) - n(B ∩ C) - n(A ∩ C) + n(A ∩ B ∩C)

Therefore, n(A ∩ B) + n(B ∩ C) + n(A ∩ C) = n(A) + n(B) + n(C) + n(A ∩ B ∩ C) - n(A ∪ B ∪ C)

From (i) required number

= n(A) + n(B) + n(C) + n(A ∩ B ∩ C) - n(A ∪ B ∪ C) - 12

= 36 + 12 + 18 + 4 - 45 - 12

= 70 - 67

= 3

**4:** In a group of 100 persons, 72 people can speak English and 43 can speak French. How many can speak English only? How many can speak French only and how many can speak both English and French?

**Solution:** Let A be the set of people who speak English.

B be the set of people who speak French.

A - B be the set of people who speak English and not French.

B - A be the set of people who speak French and not English.

A ∩ B be the set of people who speak both French and English.

Given,

n(A) = 72

n(B) = 43

n(A ∪ B) = 100

Now, n(A ∩ B) = n(A) + n(B) - n(A ∪ B)

= 72 + 43 - 100

= 115 - 100

= 15

Therefore, Number of persons who speak both French and English = 15

n(A) = n(A - B) + n(A ∩ B) ⇒

n(A - B) = n(A) - n(A ∩ B)

= 72 - 15

= 57

and n(B - A) = n(B) - n(A ∩ B)

= 43 - 15

= 28

Therefore, Number of people speaking English only = 57

Number of people speaking French only = 28

ON LINE BITS:

In a group, there were 115 people whose proofs of identity were being verified. Some had passport, some had voter id and some had both. If 65 had passport and 30 had both, how many had voter id only and not passport?

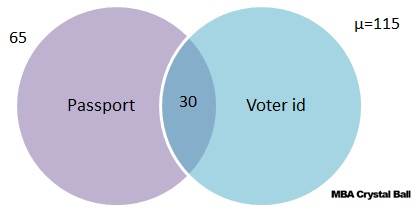
A. 30  
B. 50  
C. 80  
D. None of the above

**Answer 1**

B.

**Explanation**

Let us draw the Venn diagram for the given information.



n(PᴜV) = n(P) + n(V) – n(P∩V)

115 = 65+n(V) – 30

n(V) = 80

People with only voter id = 80-30 = 50

**Problem 2**

**Answer 2**

C.

**Explanation**:

n(RᴜBᴜG) = n(R) + n(B) + n(G) – n(R∩B) – n(B∩G) – n(R∩G) + n(R∩G∩B)

86 = 40+30+30-5-10-7+ n(R∩G∩B)

Solving this gives 8.

3.In a country 50% of all teenagers own a cycle and 30% of all teenagers own a bike and cycle. What is the probability that a teenager owns bike given that the teenager owns a cycle?

Let us assume A is the event of teenagers owning only a cycle and B is the event of teenagers owning only a bike.

So, P(A)=50/100=0.5P(A)=50/100=0.5 and P(A∩B)=30/100=0.3P(A∩B)=30/100=0.3 from the given problem.

P(B|A)=P(A∩B)/P(A)=0.3/0.5=0.6P(B|A)=P(A∩B)/P(A)=0.3/0.5=0.6

Hence, the probability that a teenager owns bike given that the teenager owns a cycle is 60%.

4.In a class, 50% of all students play cricket and 25% of all students play cricket and volleyball. What is the probability that a student plays volleyball given that the student plays cricket?

Let us assume A is the event of students playing only cricket and B is the event of students playing only volleyball.

So, P(A)=50/100=0.5P(A)=50/100=0.5 and P(A∩B)=25/100=0.25P(A∩B)=25/100=0.25 from the given problem.

P⟮B|A⟯=P⟮A∩B⟯/P⟮A⟯=0.25/0.5=0.5P⟮B|A⟯=P⟮A∩B⟯/P⟮A⟯=0.25/0.5=0.5

Hence, the probability that a student plays volleyball given that the student plays cricket is 50%.

5.Six good laptops and three defective laptops are mixed up. To find the defective laptops all of them are tested one-by-one at random. What is the probability to find both of the defective laptops in the first two pick?

Let A be the event that we find a defective laptop in the first test and B be the event that we find a defective laptop in the second test.

Hence, P(A∩B)=P(A)P(B|A)=3/9×2/8=1/21P(A∩B)=P(A)P(B|A)=3/9×2/8=1/21

5.From a bunch of 6 different cards, how many ways we can permute it?

As we are taking 6 cards at a time from a deck of 6 cards, the permutation will be 6P6=6!=7206P6=6!=720

6. In how many ways can the letters of the word 'READER' be arranged?

There are 6 letters word (2 E, 1 A, 1D and 2R.) in the word 'READER'.

The permutation will be =6!/[(2!)(1!)(1!)(2!)]=180.=6!/[(2!)(1!)(1!)(2!)]=180.

7.In how ways can the letters of the word 'ORANGE' be arranged so that the consonants occupy only the even positions?

There are 3 vowels and 3 consonants in the word 'ORANGE'. Number of ways of arranging the consonants among themselves =3P3=3!=6=3P3=3!=6. The remaining 3 vacant places will be filled up by 3 vowels in 3P3=3!=63P3=3!=6 ways. Hence, the total number of permutation is 6×6=366×6=36